02/06/2025 SE CSE-AIML SEM-III C-SCHEME EM-III QP CODE: 10081973

TIME: 03 HOURS **MAX. MARKS : 80**

Note:

1. Question **No. 1** is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

Q.1 Answer the following **(Any four)** **Marks**

a. Find the Laplace transform of t sin^3t. **05**

b. Calculate the Spearman's rank correlation coefficient R. **05**

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

c. Find the constants a, b, c, d ,e if f (z) = **05**
(a $x^3 + bx\,y^2 + 3x^2 + cy^2 + x$) + i ($dx^2y - 2y^3$ + e x y + y) is analytic.

d. Find inverse Laplace transform of $tan^{-1}\left(\frac{s+a}{b}\right)$. **05**

Q.2 a. Evaluate by using Laplace transform of $\int_0^\infty \left(\frac{\sin 3t + sn\,2t}{te^t}\right)$ dt. **06**

b. If the mean of the following distribution is 16 find m, n and variance **06**

X :	8	12	16	20	24
P (X= x) :	$\frac{1}{8}$	m	n	$\frac{1}{4}$	$\frac{1}{12}$

c. Obtain the Fourier expansion of f (x) = $\left(\frac{\pi - x}{2}\right)^2$ in (0, 2π) **08**

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$ .....

Q.3 a. Find the analytic function f (z) = u + i v in terms of z **06**
if u + v = e^x (cos y + sin y) + $\frac{x-y}{x^2+y^2}$.

b. Find the coefficient of regression and hence the equations of the lines of regression for the following data **06**

X	78	36	98	25	75	82	90	62	65	39
Y	84	51	91	60	68	62	86	58	53	47

c. Using convolution theorem Find the inverse Laplace transform of **08**
$\frac{1}{(s^2 + 4s + 13)^2}$

Q.4 a. Obtain Fourier series of f (x) = $|\sin x|$ in $((-\pi, \pi)$. **06**

b. If X denotes the outcome when a fair die is tossed, find the moment generating function of x and hence find the mean and variance of X. **06**

c. Evaluate by using Laplace transforms of $\int_0^\infty e^{-t}$ (t $\int_0^t e^{-4u} \cos u \, du$)dt . **08**

Q.5 a. Find the orthogonal trajectories of family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$. **06**

b. Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. **06**

c. Fit a second-degree parabolic curve to the following data and estimate the Production in 1982. **08**

Year (X)	1974	1975	1976	1977	1978	1979	1980	1981
Production (y)(in tons)	12	14	26	42	40	50	52	53

Q.6 a. Obtain half range Sine series for f (x) = x- x^2 in $0 \leq x \leq 1$. **06**

Hence show that $\frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} \ldots\ldots$

b. Show that the function v = $e^{2x}(y \cos 2y + x \sin 2y$ *is* harmonic. **06**
And find its corresponding analytic function f (z) = u + i v.

c. Find the value of k if the function f (x) = k $x^2(1 - x^3)$, $0 \leq x \leq 1$ **08**
f (x) = 0 otherwise .
Is a probability density function. Also find p ($0 \leq x \leq \frac{1}{2}$) find mean and variance.
