Paper / Subject Code: 49372 / Discrete Structures & Graph Theory

June 6, 2024   02:30 pm - 05:30 pm   1T01873 - S.E. Computer Science & Engineering (Artificial Intelligence & Machine Learning) (R-2019)(C-Scheme) SEMESTER - III / 49372 - Discrete Structures & Graph Theory   QP CODE: 10056489

Duration: 3 Hours   **Total Marks: 80**

N.B. : 1) Question Number 1 is compulsory

2) Solve any three questions from the remaining questions

3) Make suitable assumptions if needed

4) Assume appropriate data whenever required. State all assumptions clearly.

.

|			
Q.1		**Solve any four of the following questions.**	
	a.	What is a tautology? Check whether the following logical expression is tautology? $[(p{\to}r) \wedge (\sim q{\to}p) \wedge \sim r] \to q$	**5**
	b.	State the Pigeonhole principle and show that if any five numbers form 1 to 8 are chosen, then two of them will add to 9.	**5**
	c.	Convert the following into CNF form. $(A{\to}B) \to ((B{\to}C) {\to}(A{\to}C))$	**5**
	d	Given S = {1, 2, ..., 10} and a relation R on S, where R = {(x, y)| x + y = 10}. Is it reflexive, symmetric, transitive, antisymmetric?	**5**
	e	Define the following terms 1. Planer graph 2. Cut Vertex 3.Chain 4. Monoid 5.Group	**5**
Q.2	**a**	Let A= {p,q,r,s} and let R={(p,p),(p,q),(p,r),(q,p),(q,q),(r,p),(q,r),(r,q),(r,r),(s,s)}. Show that R is an equivalence relation . Determine the equivalence classes and find the rank of R.	**8**
	b.	Show that A={0,3,6,9,12} is a ring w.r.t. the operation of addition & multiplication modulo 15.	**8**
	c	Explain two different types of Quantifiers with example? Represent the following sentences using First Order logic i) Some students took GenAI. ii)Every student who takes GenAI passes it.	**4**
Q.3	**a**	What is an Abelian Group? Let G = { 1,2,3,4,5,6,7} i) Prepare the composition table w.r.t the operation of multiplication modulo 8. ii) Check whether it is an Abelian group? Justify your answer.	**8**
	b	Define minimum hamming distance. Find the code words generated by the parity check matrix H given below.	**8**

$$H= \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

c   **4**

Determine whether the following graph is Eulerian or Hamiltonian or both. Justify your answer.
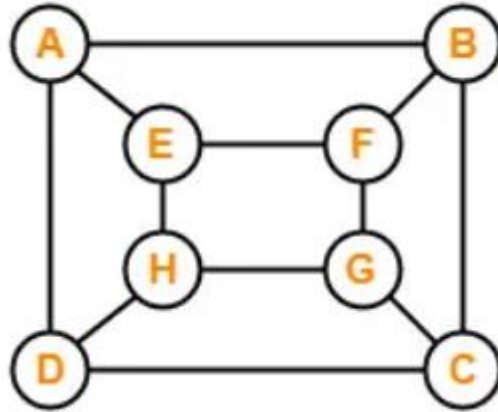
Q.4   **a**   Define function. What are three different types of functions.. Consider the function   f and g from N x N to N given by $f(x,y) = 2x+y$ and $g(x,y)= xy$,identify its type.   **8**

     b   Let A= {a,b,c,d,e} and let R be a relation on A.
Let R={(a,a),(a,c),(b,b),(c,d),(c,e),(d,a),(e,b),(e,e)}
Compute transitive closure using Warshall's algorithm   **8**

     c   Prove using Mathematical Induction that sum of cubes of three consecutive integers is divisible by 9.   **4**

Q.5   **a**   Let X={1,2,3,4,6,24,36,72} and R ={(x,y) ∈ R | x divides y}
i) Write the pairs in a relation set R.
ii) Construct Hasse diagram.
iii) Mention Chains and Anti Chains from above set.
iv) Is it a lattice?   **8**
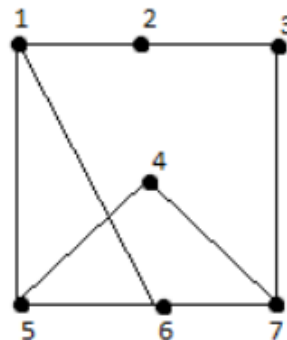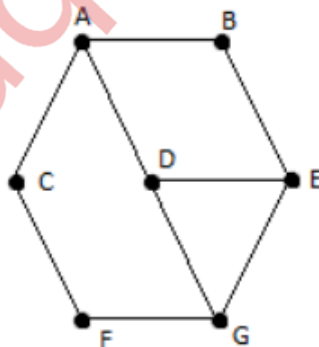
     b   Find the number of integers between 1 to 500 that are not divisible by 5,6, or 8?   **8**

     c   Check whether the following graphs are Isomorphic or not? Justify   **4**

Q.6. **a** Draw the Hasse Diagram of D_{72} **8**

i) Find the complement of each element

ii) Check whether it is a Distributive Lattice

b Let $f(x) = x + 3$, $g(x) = x - 3$ and $h(x) = 3x$ for $x \in R$, where R is the set of real numbers. **8**

Find i) $g \circ h$ ii) $f \circ g$.   i) $g \circ h \circ f$ ii) $f \circ h \circ g$

c Find the generating functions for the following sequences: **4**

    a. 0, 0, 0, 1, 2, 3, 4, 5, 6, 7,…………………………

    b. 6, -6, 6, -6, 6, -6,…………………………………….

_____