14/11/2024 CSE-AIML SEM-III C SCHEME DSGT QP CODE: 10066224

Time:3 Hrs **Marks:100**

N.B. : (1) Question Number 1 is compulsory
2)Solve any three questions from the remaining questions
3)Make suitable assumptions if needed
4)Assume appropriate data whenever required. State all assumptions clearly.

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1. a. Define the following with suitable example — 5
a) Power Set b) Group c) Euler Graph d) Existential Quantifier

b. Construct the Truth Table and check if the following statement is tautology. — 5
$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

c. For all sets A, B and C show that — 5
$A \times (B \cap C) = (A \times B) \cap (A \times C)$

d. Prove by mathematical induction that — 5
$1.1! + 2.2! + 3.3! + \text{--} + n.n! = (n+1)! - 1$

2 a Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on AXA defined by (a,b) R (c,d) if and only if a+d = b+c. Prove that R is an Equivalence Relation — 8

b. Let A={a, b, c, d} Find Transitive Closure of R represented by M_R using Warshall's algorithm. — 8

$$M_R = \begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

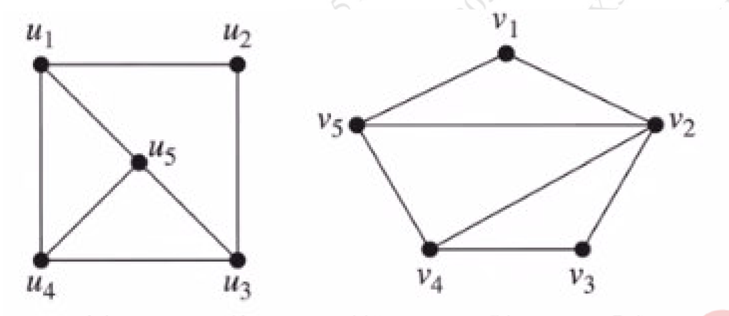
c Prove that the set A=(0,1,2,3,4,5) is a finite Abelian group under Addition modulo 6 — 4

3 a Let f,g,h be functions on real numbers R defined as follows: — 8
f(x) = 2x+5, g(x) = 5x + 3, h(x) = 3x
Find: 1) g o f 2) g o h 3) f o g o h 4) g o f o h

b Give the exponential generating function for the sequences 8

1) $\{1,1,1\ldots\}$
2) $\{1,2,3, 4,\ldots\}$
3) $\{1, a, a^2, a^3, \ldots\}$

c Determine whether the following graphs are isomorphic. Justify your answer. 4



G1 G2

4 a A Function 8

$f: R - \{(2/5)\} \to R - \{4/5\}$ is defined as $f(x) = (4x+3)/(5x-2)$

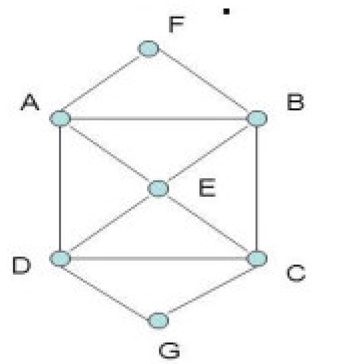
Prove that f is Bijective and find the rule for f^{-1}

b Show that (2,5) encoding function $e: B^2 \to B^5$ defined by 8

e (00) = 00000
e (01) = 01110
e (10) = 10101
e (11) = 11011
is a group code.

c Find the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2,3 or 5. 4

5 a Define Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit. Determine if the following diagram has Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit and state the path /circuit. 8



b State and explain the extended Pigeonhole principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. 8

c Find the complement of each element in D_{42} 4

6. a Draw the Hasse Diagram of D_{72} and check whether it is a Lattice. 8

b Find the complete solution of $a_n+2a_{n-1} = n+3$ for $n\geq1$ with $a_0=3$ 8

c Define the following with suitable examples. 4

a)Maximal and Minimal Element b) Partition of a set c) Sub Lattice d) Injective Function

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