1T01873 - S.E. Computer Science & Engineering (Artificial Intelligence & Machine Learning) (R-2019) (C-Scheme) SEMESTER - III / 49372 - Discrete Structures & Graph Theory

QP CODE: 10038552 DATE: 23/11/2023

Time: 3 Hours **Total Marks: 80**

N.B.

1) **Q.1 is compulsory.**
2) **Solve any 3 questions out of remaining 5 questions.**
3) **Assumptions made should be clearly stated.**
4) **Draw the figures wherever required.**

Q.1 Solve any four of the following questions.

a) Prove using Mathematical Induction that $2 + 5 + 8 + \ldots\ldots\ldots\ldots + (3n-1) = n(3n+1)/2$ **5**

b) Explain the term poset. Consider a set D_{165}. Find the elements of this set & draw the hasse diagram for this poset. **5**

c) How many strings of length 7 either begin with 2 zeros or end with 3 ones? **5**

d) Explain the term partition set with suitable example. **5**

e) State the Pigeonhole principle and show that If there are 10 marbles in the jar & you have a jar filled with red, green, and blue marbles, you'll always have at least two marbles of the same colour. **5**

Q.2

a) **10**

Let A={0, 1 ,2, 3, 4, 5}

i) Explain the term group.

ii) Prepare the composition table for the above set w.r.t. the operation of addition modulo 6.

iii) Determine whether it is a group.

iv)Whether elements of set A are invertible? If yes, then find the inverses of these elements.

v) Determine whether it is a cyclic group.

b

Let A = $\{a_1, a_2, a_3, a_4, a_5\}$ and let R be a relation on A whose matrix is : **10**

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find out transitive closure of R using Warshall's algorithm.

Q.3

a) A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.

Determine the number of computer programmers that are not proficient in any of these three languages. **8**

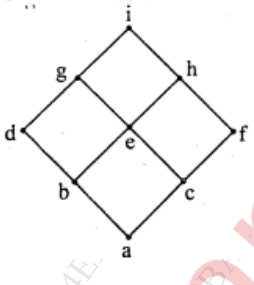
b) Explain the terms Conjunctive & Disjunctive Normal Form with suitable examples. **6**

c) **6**

Determine the sequence b_n whose recurrence relation is $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$.

Q.4

a) What is a lattice? Determine whether following hasse diagram represents a lattice. **6**



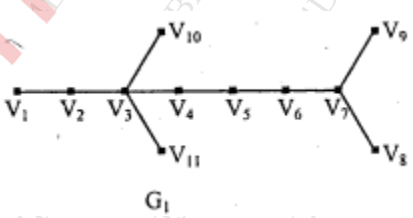
b) **6**

Consider (3, 8) an encoding function $e : B^3 \rightarrow B^8$ defined as

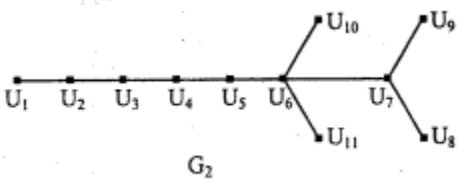
e (000) = 00000000
e (001) = 10111000
e (010) = 00101101
e (011) = 10010101
e (100) = 10100100
e (101) = 10001001
e (110) = 00011100
e (111) = 00110001

How many errors can 'e' detect & correct?

c) What are the necessary conditions for the isomorphism between 2 graphs? Determine whether following 2 graphs are isomorphic. **8**



G_1



G_2

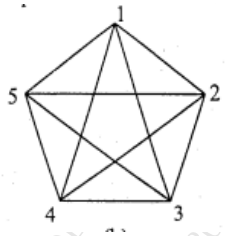
Q.5

a) If the addition & multiplication modulo 10 is defined on a set of integers A={0, 2, 4, 6, 8}. Then determine whether this algebraic system is a ring. **8**

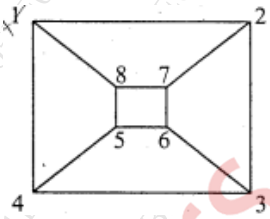
b)

A function $f : R \to R$ is defined by $f(x) = x^2$ **6**
Is it i) injective ii) surjective iii) bijective

c) Define the terms Euler path & a circuit . **6**
Determine whether following graphs have Euler path or a circuit.



a)



b)

Q.6

a) Explain the following terms with suitable example (any 4) **8**
i) Hamiltonian path & circuit
ii) Bipartite graph
iii) Adjacency matrix
iv) Equivalence relation
v) Cartesian product

b) Solve the following using the laws of logic **4**
$:p \vee q \vee (\sim p \wedge \sim q \wedge r) \leftrightarrow p \vee q \vee r$

c) **8**

$f : R \to R$ is defined by $f(x) = x^3$
$g : R \to R$ is defined by $g(x) = 4x^2 + 1$
$h : R \to R$ is defined by $h(x) = 7x - 2$
Find the rule defining
i) fog ii) gof
iii) (goh)of iv) go(hof)