

17/12/2025

Time: (3 Hours)

[Total Marks: 80]

OP-10093866

N.B.: 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Calculate Correlation coefficient between the variables x and y for the following data (5)

X	1	2	4	5	3
Y	3	3	5	8	6

(b) A random variable x has the following probability function (5)

X	1	2	3	4	5
P(x)	3c	2c	2c	c	2c

Find i) C ii)  $P(x < 3)$  iii)  $E(X)$  iv)  $V(X)$

(c) A random sample of 50 items gives the mean 6.2 and variance 10.24 .Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance ? (5)

(d) Find a,b, and c if  $\vec{F} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x - cy + 2z)\mathbf{k}$  is irrotational. (5)

Q.2 (a) Fit a straight line to the following data (6)

X	1	2	3	4	5
Y	5	8	3	9	6

(b) Find the work done in moving a particle in the force field (6)

$\vec{F} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$  along  $x = t, y = t, z = t$  from (0,0,0) to (1,1,1).

(c) Find all possible Laurent's series expansion of the function  $f(z) = \frac{3}{(z+2)(z+5)}$  about  $z = 0$  indicating region of convergence. (8)

Q.3 (a) Given:  $2x + 6y = 90$  ,  $9x + 3y = 130$  are regression lines and  $\sigma_x^2 = 16$  then find (i) mean of X and Y (ii) correlation coefficient (r) (iii)  $\sigma_y^2$  (6)

(b) Use Green's theorem to evaluate  $\int_c (x^2 - y) dx + (2y^2 + x) dy$  where c is the boundary of the region enclosed by  $y = x^2$  and  $y = 4$ . (6)

(c) Investigate the association between the darkness of eye colour in father and son from the following table using  $\chi^2$  -test (use 5% LOS) (8)

Colour of son's eyes	Colour of father's eyes		
	Dark	Not Dark	Total
Dark	48	90	138
Not Dark	80	782	862
Total	128	872	1000



Q.4 (a) Let  $X$  be a continuous random variable with probability density function  $f(x) = ke^{-x}$ ,  $x \geq 0$  Find  $k$ , mean and variance. (6)

(b) Following result were obtained from two samples each drawn from two different populations A and B (6)

Group	A	B
Sample Size	25	17
Sample SD	4	3

Test the hypothesis that variance of A is less than or equal to variance of B.

Given  $(F(0.05) = 2.24 \text{ for d.o.f. } 24 \text{ and } 16)$

(c) Show that  $\vec{F} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$  is conservative. (8)  
Find scalar potential such that  $\vec{F} = \nabla\phi$  and hence, find the work done by in displacing a particle from  $(1,2,0)$  to  $(3,3,2)$ .

Q.5 (a) A fair coin is tossed till a head appears. What is the expectation of the number of tosses required? (6)

(b) Using Stoke's Theorem to evaluate  $\int_c \vec{F} \cdot d\vec{r}$  where  $\vec{F} = x^2\mathbf{i} + xy\mathbf{j}$  and  $c$  is the boundary of the rectangle  $x = 0, y = 0, x = a, y = b$ . (6)

(c) Evaluate  $\int_c \frac{2z}{z^2-4} dz$  where  $c$  is (i)  $|z - 2| = 1$  (ii)  $|z + 2| = 1$ . (8)

Q.6 (a) Four roads lead away from a jail. A prisoner trying to escape from the jail selected a road at random. If road A is selected, the probability of escaping is  $1/8$ , for road B it is  $1/6$ , for road C it is  $1/4$  and for road D it is  $9/10$ . What is the probability that a prisoner will succeed in escaping from the jail (6)

(b) Use Gauss Divergence theorem to evaluate  $\iint_s \vec{F} \cdot \hat{n} ds$  where  $\vec{F} = 4x\mathbf{i} + 3y\mathbf{j} - 2z\mathbf{k}$  and  $s$  is the surface bounded by  $x = 0, y = 0, z = 0$  and  $2x + 2y + z = 4$  (6)

(c) Monthly salary  $X$  in a big organization is normally distributed with mean Rs 3000 and standard deviation of Rs 250. What should be the minimum salary of a worker in this organization, so that the probability that he belongs to top 5% workers? (8)