



(3 Hours)

Total Marks: 80

- Note: 1) Question No.1 is compulsory.
2) Attempt any **THREE** from the remaining.
3) Figures to the right indicate full marks.

- Q.1 A) Find the values of constants a, b, c and d if $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic 5
B) Find the Eigen Value of $A^3 - 3A^2$ 5
Where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$
C) Find the Laplace Transform of $t \sin at$ 5
D) Find the Fourier series expansion for $f(x) = x$ defined in $(-1, 1)$ 5
- Q.2 A) If $L[f(t)] = \frac{s}{s^2 + s + 4}$ find $L[e^{-3t}f(2t)]$ 6
B) Find the Fourier series expansion for $f(x) = x$ defined in $(-\pi, \pi)$ with period 2π 6
C) Find the analytic function $f(z)$ with the real part $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ 8
- Q.3 A) Show that the function $u = x^3 - 3xy^2$ is harmonic function. 6
Hence find the corresponding analytic function and harmonic conjugate.
B) A string is stretched and fastened to two points distance L apart motion is started by displacing the string in the form $u = \alpha \sin(\frac{\pi x}{L})$ from which it is released at time $t = 0$. Show that the displacement of a point at a distance X from one end at time t is given by $u(x, t) = \alpha \sin(\frac{\pi x}{L}) \cos(\frac{\pi ct}{L})$ 6
C) Obtain the Fourier series expansion of $f(x) = |x|$ where $-\pi \leq x \leq \pi$ 8
- Q.4 A) Find Laplace transform of $e^{-4t} \int_0^t u \sin 3u du$ 6
B) Find Inverse Laplace transform of $\frac{2s+3}{s^2+2s+2}$ 6
C) Verify Cayley - Hamilton theorem for the matrix A and hence find A^{-1} & A^4 8
where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$
- Q.5 A) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0, 0 \leq x \leq 1$ 6
subject to the condition $u(0, t) = 0, u(1, t) = 100t, u(x, 0) = 0, h = \frac{1}{4}$
for one -time step.
B) Find the inverse Laplace transform of $\log \left(\frac{s+a}{s+b} \right)$ 6
C) Show that the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 7 \end{bmatrix}$ is diagonalizable. 8
Find transforming matrix and diagonal Matrix.
- Q.6 A) Evaluate $\int_0^\infty e^{-3t} t \sin t dt$ using Laplace transform. 6
B) Find the solution $u_t = u_{xx}$ subject to $u(0, t) = 0, u(5, t) = 0, u(x, 0) = x^2 (25 - x^2)$ 6
using Schmidt method taking $h = 1$ up to 3 seconds.
C) Find the inverse Laplace transform of $\frac{s}{(s^2+1)^2}$ using convolution theorem. 8

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