(3 Hours) Total Marks: 80

Note: (1) Question No. 1 is Compulsory.

- (2) Answer any three questions from Q.2 to Q.6
- (3) Figures to the right indicate full marks.
- Q1. a) Find $L[te^{3t}sint]$

b) Find a, b, c, d if
$$f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$$
 is analytic.

- c) Find the Fourier expansion of $f(x) = x^2, -\pi \le x \le \pi$
- d)
 Find the eigen values of $A^2 5A + 4I$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$
- Q2. a) i) If $L\{f(t)\} = \frac{s}{s^2 + s + 4}$, find $L(e^{-2t})$ ii) Find $L(t^2)$ iii) Find $L(t^2)$ iii)
 - b) Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2 x^2)}{12}$, where $0 < x < \pi$.
 - c) Find analytic function f(z) whose imaginary part is $e^x \cos y + x^3 3xy^2$
- Q3. a) Solve $\frac{\partial^2 u}{\partial x^2} 32 \frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method subjected to the conditions u(0,t) = 0, u(x,0) = 0, u(1,t) = t, taking h =0.25, 0 < x < 1, upto t = 5.
 - b) Determine the Harmonic Conjugate of u if u + iv is analytic $3x^2y y^3 = u$
 - Determine the Fourier Series $f(x) = \left(\frac{\pi x}{2}\right)^2$ over $[0, 2\pi]$. Hence show that $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \dots = \frac{\pi^2}{6}$
- Q4. a) Evaluate the following Integral using Laplace Transforms. $I = \int_0^\infty e^{-t} \left(\int_0^t u \cdot \cos^2 u \, du \right) dt$
 - b) Determine inverse Laplace Transform of $\frac{s}{(s^2+1)(s^2+4)}$, using Convolution theorem.
 - c) Is the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ diagonalizable? If so find the diagonal form of A and transforming matrix of A.

Paper / Subject Code: 50821 / Engineering Mathematics-III

Q5. a) Find the Eigen value and the eigen vector of



$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & 2 & 2 \end{bmatrix}$$

b) Determine the Inverse Laplace Transform of $\log \left[\frac{s^2 + a^2}{(s+b)^2} \right]$

6

Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, by Crank-Nicholson simplified formula, where u(0,t) = 0, u(4,t) = 0, $u(x,0) = \frac{x}{3}(16 - x^2)$, find u_{ij} , for i = 0,1,2,3,4 and j = 0,1,2 taking h = 1.

Q6. a) Find the Laplace Transform of $f(t) = \frac{\cos at - \cos b}{1}$

0

b) A tightly stretched string with fixed end points x = 0, and x = l, in the shape defined by y = kx(l - x), where k is a constant, is released from this position of rest. Find y(x, t) the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

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Determine the Inverse Laplace Transform of i) $\frac{s+2}{s^2-4s+13}$ ii) $tan^{-1}(s)$

4