

(3 Hours)

Total Marks :80

Note: 1) Question No.1 is compulsory

2) Attempt any Three from the remaining

Q1

- A) Find $L \left\{ \int_0^t e^{-u} u^n du \right\}$ 5
- B) Prove that $f(z) = e^z$ is analytic everywhere. 5
Hence find $f'(z)$
- C) Find half range sine series of $f(x) = x$ in $(0, \pi)$ 5
- D) If $A = [a_{ij}]$ is a matrix of order 3×3 such that $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$ 5

Find an eigen value of

- i) A
- ii) adjoint of A
- iii) $A^2 - 2A + 2I$

Q2

- A) If $L[f(t)] = \frac{1}{9s^2 - 3s + 1}$ then Find $L[te^t f(3t)]$ 6
- B) Find Fourier series for $f(x) = x$, if $0 < x < 2\pi$ 6
and $f(x + 2\pi) = f(x)$
- C) Find analytic function $f(z)$ in terms of z where 8
 $u = y^2 - x^2$

Q3

- A) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form 6
 $y = a \sin(\pi x / l)$ from which it is released at time $t = 0$. Show that the displacement of a point at a distance x from one end at time t is given by $y = a \sin(\pi x / l) \cos(\pi c t / l)$
- B) Prove that 6
 $u = e^x \cos y$ is harmonic function hence find its harmonic conjugate function
- C) Find the Fourier Series for $f(x)$ in $(-\pi, \pi)$ where 8

$$f(x) = |x|$$

Q4

- A) Evaluate $\int_0^\infty \left[\frac{\cos 2t - \cos 4t}{t} \right] dt$ 6
- B) Find Inverse Laplace transform of $\frac{s+1}{(s-1)^2(s-2)}$ 6
- C) Is the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ Diagonalizable? If so find the Diagonal form of A 8
and transforming matrix of A

Q5

- A) If $A = [a_{ij}]$ is a matrix of order 3×3 such that **6**
- $$a_{ij} = \begin{cases} 2, & \text{if } i = j \\ -1, & \text{if } i + j = 3 \text{ or } 5 \\ 1, & \text{if } i + j = 4 \text{ and } i \neq j \end{cases}$$

Compute: $A^9 - 6A^8 - 9A^7 - 4A^6 + A^5 - 12A^4 - 18A^3 - 8A^2 + 2A + I$

- B) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0$, **6**

$0 \leq x \leq 1$ subject to the condition $u(0, t) = 0, u(1, t) = 100t$,

$u(x, 0) = 0$ $h = 0.25$ for one-time step

- c) Find inverse Laplace transform of (i) $\log[z^2 - 4]$ (ii) $\frac{s+2}{(s+16)^2}$ **8**

Q6

- A) Find the Laplace Transform of $\int_0^t \cos(u) \sin(u) du$ **6**

- B) Find the solution of **6**

$$4 \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0, 0 < x < 8, \quad u(x, 0) = 4x - \frac{1}{2}x^2, u(0, t) = 0, u(8, t) = 0$$

Taking $h = 1, k = \frac{1}{8}$ for $0 \leq t \leq 5/8$

Where h is the step length for x axis and k is the step size in time direction using Bender-Schmidt method

- C) Find inverse Laplace transform of $\frac{1}{(s^2+16)(s^2+49)}$ using convolution theorem **8**
