

04-06-2026 SE CHEMICAL SEM-III C SCHEME EM-III QP CODE: 20014911

(Time: 3 Hours)

Total Marks: 80

Note: 1) Question No.1 is compulsory.**2) Attempt any Three from the remaining.**

- Q1 A)** Find $L \left\{ \int_0^t u^3 e^{-2u} du \right\}$ 5
- B)** Prove that $f(z) = e^{2x}(\cos 2y + i \sin 2y)$ is analytic everywhere. Hence find $f'(z)$ 5
- C)** Find half range sine series of $f(x) = x$ in $(0, \pi)$ 5
- D)** If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ then Find an eigen value of 5
- i) A
 ii) $\text{Adj}(A)$
 iii) $A^2 - 2A + I$
- Q2**
- A)** If $L[f(t)] = \frac{1}{9s^2 - 3s + 16}$ then Find $L[e^t f(3t)]$ 6
- B)** Find Fourier series for $f(x) = x$, if $0 < x < 2\pi$ and $f(x + 2\pi) = f(x)$ 6
- C)** Find analytic function $f(z)$ in terms of z where $u = 3x^2 - 3y^2 + x$ 8
- Q3**
- A)** A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form $y = a \sin(\pi x / l)$ from which it is released at time $t=0$. Show that the displacement of a point at a distance x from one end at time t is given by $y = a \sin(\pi x / l) \cos(\pi c t / l)$ 6
- B)** Prove that $u = y^3 - 3x^2 y$ is harmonic function hence find its harmonic conjugate function 6
- C)** Find the Fourier Series for $f(x) = \pi + x$ in $(-\pi, \pi)$ where 8
- Q4**
- A)** Evaluate $\int_0^\infty \left[\frac{e^{-2t} - e^{-3t}}{t} \right] dt$ 6
- B)** Find Inverse Laplace transform of $\frac{s+1}{(s-1)(s+2)}$ 6

- C) Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ Diagonalizable? If so find the Diagonal form of A and transforming matrix of A 8

Q5

- A) If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then Using Cayley Hamilton theorem evaluate $A^9 - 6A^8 + 9A^7 - 4A^6$ 6

- B) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0$, $0 \leq x \leq 1$ subject to the condition $u(0, t) = 0, u(1, t) = 100t$, $u(x, 0) = 0$ $h = 0.25$ for one-time step. 6

- c) Find inverse Laplace transform of (i) $\log \left[\frac{(s+16)}{(s+25)} \right]$ (ii) $\frac{s}{(s+4)^2}$ 8

Q6

- A) Find the Laplace Transform of $\int_0^t e^{-u} (\sin u + \cos u) du$ 6

- B) Find the solution of 6

$$4 \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0, 0 < x < 8, \quad u(x, 0) = 4x - \frac{1}{2}x^2, u(0, t) = 0, u(8, t) = 0$$

$$\text{Taking } h = 1, k = \frac{1}{8} \quad \text{for } 0 \leq t \leq 4/8$$

Where h is the step length for x axis and k is the step size in time direction using Bender –Schmidt method

- C) Find inverse Laplace transform of $\frac{1}{(s^2+25)^2}$ using convolution theorem 8
