

(REVISED COURSE)  
(3 Hours)

[Total Marks: 80]

N.B. :

- 1) **Question 1 is compulsory.** Answer any **three** more from the remaining questions.
- 2) Assume data if necessary and **specify the assumptions** clearly.
- 3) Draw neat sketches wherever required.
- 4) Answers to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) The flow of a particular fluid is described by its velocity components as: [05]

$$u = -\frac{V_0}{l}x, \quad v = -\frac{V_0}{l}y$$

where  $V_0$  and  $l$  are constants. Derive an expression to calculate the rate of change of the density of the fluid  $\rho(x, y, z, t)$  with respect to time, along a flowing fluid particle.

- (b) Consider the following equation: [05]

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + D \frac{\partial^2 C}{\partial x^2} = 0$$

Derive an expression to solve the equation numerically using an implicit scheme.

- (c) Consider the following function: [05]

$$f(x, y) = \exp(-x) + \exp(-y) + 0.5 * \cos(xy)$$

Use backward differences to calculate:

$$\frac{\partial f(x, y)}{\partial x} \text{ and } \frac{\partial f(x, y)}{\partial y}$$

at  $x = 0.1$ , and  $y = 0.1$ . Also calculate the error. Take

$$\Delta x = 0.05 \text{ and } \Delta y = 0.05$$

- (d) Explain the following terms with examples. [05]

- Convergence.
- Consistency.

2. (a) An insulated metal rod has an initial temperature profile given by: [10]

$$T(x, 0) = 20^\circ C$$

At time  $t = 0$ , a hot reservoir at a temperature of  $T = 200^\circ C$  is brought into contact with the left end of the rod. Simultaneously the right end is exposed to a hot reservoir at a temperature of  $300^\circ C$ . The length of the rod is  $1 \text{ m}$ , and for the metal  $\alpha = 1.0 \times 10^{-5} \text{ m}^2/\text{sec}$ . Using the finite difference FTCS scheme, calculate the temperature across the rod for the next  $1600 \text{ sec}$ . Assume  $r = 0.2$  and  $\Delta x = 0.2 \text{ m}$ .

- (b) Solve the following equation using the weighted residual method: [10]

$$\frac{d^2u}{dx^2} = -(x + 1)$$

with  $u = 0$  at  $x = 0$  and  $\frac{du}{dx} = 1$  at  $x = 2$  Use the trial function  $u = a_1x + a_2x^2$

3. Consider the steady, one dimensional heat conduction, in an insulated metal rod, of length  $0.8\text{ m}$ . The ends of the rod are maintained at constant temperatures of  $200^\circ\text{C}$  and  $400^\circ\text{C}$  respectively. Applying the finite volume method on eight control volumes of equal length, determine the temperature profile across the rod. Thermal conductivity of the metal is  $k = 1000\text{ W/m.K}$  and the cross-sectional area of the rod is  $A = 0.01\text{ m}^2$  [20]

4. (a) A company produces a perishable product in a factory at  $x = 0$ , and sells it along the distribution route  $x > 0$ . The selling price of the product,  $p$ , is a function of the length of time after it was produced,  $t$ , and the location at which it is sold,  $x$ , i.e.  $p = p(t, x)$ . At a given location the price decreases in time at a rate of  $-8\$/\text{hr}$ . In addition, because of shipping costs, the price increases with distance from the factory at a rate of  $0.1\$/\text{km}$ . If the manufacturer wants to sell the product at the same price of  $\$100$  everywhere, determine how fast he must travel along the distribution route. [10]

- (b) Consider the following equation: [10]

$$\frac{dy}{dx} + 2x + \sin x = 0$$

subject to the B.C.:

$$y(0) = 1$$

Solve the equation using the Subdomain method. Use the approximate solution:

$$y = 1 + \sum_{j=1}^3 a_j x^j$$

5. (a) Consider the following set of data: [10]

$i$	$x$	$y$
0	0	1
1	0.08	0.9057
2	0.015	0.8327
3	0.21	0.7767
4	0.31	0.6956
5	0.38	0.6472

Using quadratic interpolation calculate the value of  $y$  at  $x = 0.10$  and at  $x = 0.35$ .

- (b) Consider the cooling of a circular fin by convective heat transfer along its length. The cylindrical fin has a uniform cross-sectional area  $A$ , and perimeter  $P$ . The base of the fin is at a temperature of  $T_B = 300^\circ\text{C}$ , and the free end is insulated. The fin is exposed to an ambient temperature  $T_A = 30^\circ\text{C}$ . Calculate the temperature distribution along the fin, using the finite volume method, with five control volumes, and compare with the analytical solution: [10]

$$\frac{T - T_A}{T_B - T_A} = \frac{\cosh[n(L - x)]}{\cosh(nL)}$$

$$n^2 = \frac{hP}{kA} = 25 \text{ m}^{-2} \quad L = 1 \text{ m}$$

6. Consider the steady two-dimensional heat conduction in a metal slab of size  $2m \times 2m$ . [20]  
The boundary conditions are given by:

$$T(x, 0) = 20^\circ\text{C}, \quad T(0, y) = 20^\circ\text{C}, \quad T(2, y) = 20^\circ\text{C}, \quad \text{and } T(x, 2) = 400^\circ\text{C}$$

Using five equally spaced grid points on each side, and ADI scheme, calculate the distribution of  $T(x, y)$  across the metal slab.

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