

- N.B. 1) All questions are compulsory.  
 2) Figures to the right indicate marks.  
 3) Illustrations, in-depth answers and diagrams will be appreciated.  
 4) Mixing of sub-questions is not allowed.

**Q. 1 Attempt the following questions****(15M)**

- (a) Choose the best choice for the following questions
- (5M)**

- i) The absolute value of  $3 + 4i$  is:  
 a) 4      b) 5      c) 6      d) zero
- ii) In GF(2) field,  $1 + 1$  is equal to  
 a) 1      b) 0      c) both a) and b)      d) none of these
- iii) How to declare the complex number in Python?  
 a) (3, 4)      b) Complex(3, 4)      c) Complex (3, 4i)      d) None of these
- iv) If a matrix is  $R \times C$  and a vector is a  $C$  vector then the product is called  
 a) Matrix-Matrix      b) Vector-Matrix  
 c) Vector-Vector      d) Matrix-Vector
- v) Suppose  $t = (1, 2, 4, 3)$ , which of the following is incorrect?  
 a) `print(t[3])`      b) `t[3] = 45`  
 c) `print(max(t))`      d) `print(len(t))`

- (b) Fill in the blanks for the following questions
- (5M)**

- i) Any complex number multiplying by  $i$ , rotate it by \_\_\_\_\_  
 ii) Set of all linear combinations of vectors is called \_\_\_\_\_  
 iii) A rectangular array of  $m$  rows and  $n$  columns is called a \_\_\_\_\_  
 iv) Norm of Vector  $(1, 2, 3)$  is \_\_\_\_\_  
 v) Every Subset of a linearly independent set is linearly \_\_\_\_\_

- (c) Answer the following questions
- (5M)**

- i) Solve:  $1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 1$   
 ii) Find dot product of  $(1, 2)$ ,  $(3, 4)$   
 iii) Show with example matrix representation in python  
 iv) Define the term Basis  
 v) Define the term Inner Product Space

**Q. 2 Attempt the following (Any THREE)(Each of 5Marks)****(15M)**

- (a) Find the square root of complex number  $-5 + 12i$   
 (b) Show that vectors  $v_1 = (1, 0, 1)$ ,  $v_2 = (2, 1, 4)$  and  $v_3 = (1, 1, 3)$  do not span vector space.  
 (c) Write a Python program to rotate a complex no by  $90^\circ$ ,  $180^\circ$  and  $270^\circ$

- (d) Check whether the vectors are linearly dependent  
 $v_1=(1, -2, 1)$ ,  $v_2=(2, 1, -2)$  and  $v_3=(7, -4, 1)$ .
- (e) Express  $[(3 + 2i)/(2 + i)(1 - 3i)]$  in the form  $x + iy$
- (f) Check whether the set of all pairs of real numbers of the form  $(1, x)$  with operation  $(1, y) + (1, y') = (1, y + y')$  and  $k(1, y) = (1, ky)$  is a vector space.

**Q. 3 Attempt the following (Any THREE) (Each of 5Marks) (15M)**

(a) Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 4 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \quad c = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \quad D = [2 \ 4 \ 3 \ 1]$$

Compute the following if they exists.

- a)  $A + B$     b)  $3A$     c)  $B + 2D$
- (b) Find the dimension of the vector space spanned by the vectors  $(1, 1, -2, 0, -1)$ ,  $(1, 2, 0, -4, 1)$ ,  $(0, 1, 3, -3, 2)$ ,  $(2, 3, 0, -2, 0)$  and also find the basis.
- (c) Check whether the set of functions are Linearly independent?  
 $2 - x + 4x^2$ ,  $3 + 6x + 2x^2$ ,  $2 + 10x - 4x^2$ .
- (d) Explain Matrix-Vector and Vector-Matrix multiplication with example.
- (e) Write a python program to enter a matrix and check if it is invertible.  
 if invertible exists then find inverse.
- (f) Show that vector  $\{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$  of  $R^3$  form a basis of  $R^3$

**Q. 4 Attempt the following (Any THREE) (Each of 5Marks) (15M)**

- (a) If  $u = (2, 3, -1)$  and  $v = (6, -3, -2)$   
 Find a)  $d(u, v)$     b)  $u - v$     c)  $2u + 3v$
- (b) Verify Pythagorean Theorem for  $u = (1, 0, 2, -4)$  and  $v = (0, 3, 4, 2)$
- (c) If  $x, y, z \geq 0$   
 Show that  $(x^2 + y^2 + z^2)^{1/2} \geq (1/13)(3x + 4y + 12z)$
- (d) Find inner product, angle, orthogonality for  
 $P = -5 + 2x - x^2$ ,  $q = 2 + 3x^2$
- (e) Find the vector orthogonal to both  $u = (-6, 4, 2)$  and  $v = (3, 1, 5)$
- (f) Write a python program to find orthogonal projection  $u$  on  $v$ .

**Q. 5 Attempt the following (Any THREE) (Each of 5Marks) (15M)**

- (a) Express the following as a linear combination of  $v_1=(-2, 1, 3)$ ,  $v_2=(3, 1, -1)$  and  $v_3=(-1, -2, 1)$  with  $w=(6, -2, 5)$

- (b) Write a python program to convert a  $2 \times 2$  matrix to row echelon form  
(c) Verify Cauchy's Schwartz's inequality  $u = (1, 2, -1)$  and  $v = (3, 2, -1)$   
(d) Find eigen Values and eigen vectors of

$$A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

- (e) Construct an orthonormal basis of  $\mathbb{R}^2$  by Gram Schmitt Process  
 $S = \{(3, 1), (4, 2)\}$
-