Q.P. Code: 538301

(3 Hours)

[Total Marks: 80



- (2) Attempt any three questions out of the remaining five questions.
- (3) Figures to right indicate full marks.

1 a) Evaluate 
$$\int (z^2 + 2z)dz$$
 along the circle  $x^2 + y^2 = 1$  (5)

- b) Evaluate the integral using Laplace Transform  $\int_{0}^{\infty} e^{-t} \left(t\sqrt{1+\sin t}\right) dt$  (5)
- c) Determine the analytic function whose real part is  $u = -r \sin 3\theta$ . (5)
- d) A rod of length 1 has its ends A and B kept at 0 C and 100°C respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so while that of A is maintained. Find the temperature u(x,t) at a distance from A and at time t.

2 a) Find complex form of Fourier series of 
$$f(x) = e^{2x}$$
 in (0,2) (6)

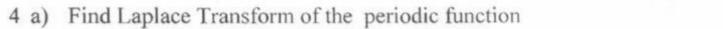
- b) Find the orthogonal trajectory of the family of curves given by  $2x x^3 + 3xy^2 = a$  (6)
- c) Using Bender Schmidt method solve  $\frac{\partial^2 u}{\partial x^2} \frac{\partial u}{\partial t} = 0$  subject to the conditions u(0,t) = 0, u(1,t) = 0,  $u(x,0) = \sin \pi x$ ,  $0 \le x \le 1$ .

  Assume h=0.2

3 a) Find k such that 
$$\frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}(\frac{kx}{y})$$
 is analytic (6)

- b) Evaluate  $\int_{C} \frac{1}{(z^3 1)^2} dz \quad \text{where C is the circle } |z 1| = 1$  (6)
- c) Show that the set of functions  $\left\{Sin\left(\frac{\pi x}{2L}\right), Sin\left(\frac{3\pi x}{2L}\right), Sin\left(\frac{5\pi x}{2L}\right), \ldots\right\}$  (8) forms an orthogonal set over the interval [0, L]. Construct corresponding orthonormal set.

(6)



$$f(t) = \begin{cases} \sin 2t \ , \ 0 < t < \frac{\pi}{2} \\ 0 \ , \qquad \frac{\pi}{2} < t < \pi \end{cases}$$
  $f(t) = (t + \pi)$ 

b) Find half range sine series for 
$$x \sin x$$
 in  $(0, \pi)$  (6)

c) Expand 
$$f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$$
 around  $z = 1$  (8)

5 a) Using residue theorem evaluate 
$$\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$$
 where  $C$  is  $|z| = 4$ 

b) Find Fourier expansion of 
$$f(x) = x + x^2$$
 in  $(-\pi, \pi)$  and (6)  $f(x + 2\pi) = f(x)$ 

c) Find i) 
$$L(e^{-4t} \int_{0}^{t} u \sin 3u du)$$
 ii)  $L^{-1} \left(\frac{1}{s} \log 1 + \frac{1}{s^{2}}\right)$  (8)

Show that the function  $w = \frac{4}{z}$  transform the straight lines x = c in the z-plane into circles in the w-plane.

b) Solve using Laplace Transform 
$$\bar{R} \frac{\partial Q}{\partial t} + \frac{Q}{c} = V$$
,  $Q = 0$  when  $t = 0$  (6)

c) Solve the Laplace equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for the following data by successive iterations (Calculate first two iterations)

50	8.7	12.1	12.8	9.0
Jc.	$u_7$	$u_8$	u <sub>9</sub>	17.0
0	u4	<i>u</i> <sub>5</sub>	<i>u</i> <sub>6</sub>	21.0
0	$u_1$	<i>u</i> <sub>2</sub>	$u_3$	21.9
0	11.1	17.0	19.7	18.6