(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = e^{-9t}\int_0^t u\sin 3u\, du$. 5

b) Verify Laplace equation for $u = \left(r + \frac{a^2}{r}\right)\cos\theta$. 5

c) Show that $\{sin\, nx\, , n = 1,2,3\ldots\}$ is a set of orthogonal function over an interval $(-\pi, \pi)$. 5

d) Evaluate $\int_0^{3+i}|z|^2\, dz$ along the line $3y = x$ 5

2. a) Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2-4z+3}$ indicating the region of convergence. 6

b) Find complex form of Fourier series of $f(x) = cosh\, 2x$ in $(-3,3)$. 6

c) Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$ where $y(0) = 0, y'(0) = 1$ 8

3. a) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ with $u(0,t) = 0, u(5,t) = 0, u(x,0) = x^2(25 - x^2)$ taking $h = 1$ up to $t = 3$ seconds by Bender –Schmidt method. 6

b) Find the bilinear transformation which maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$. 6

c) Obtain half range Cosine Series of $f(x) = \sin x$ in the interval $(0, \pi)$. Use Parseval's identity to prove that –

$$\frac{1}{1^2 .3^2} + \frac{1}{3^2 .5^2} + \frac{1}{5^2 .7^2} + \ldots\ldots\ldots = \frac{\pi^2 - 8}{16}.$$ 8

[TURN OVER]

4. a) Find the orthogonal trajectory of the family of curves, $x^3y - xy^3 = c$, where c is a constant. 6

b) Obtain Fourier Series of $f(x) = |x|$ in $(-\pi, \pi)$ 6

c) Find the inverse Laplace transform of :-
i) $F(s) = \frac{1}{s(s^2+4)}$, using Convolution theorem, ii) $F(s) = \frac{e^{-3s}}{(s-2)^4}$. 8

5. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0,t) = u(1,t) = 0, u(x,0) = 200(x - x^2)$

taking $h = 0.25$ for one-time step. 6

b) Find an analytic function $f(z) = u + iv$, if 6

$$u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$$

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c). Obtain Fourier series of $f(x) = x^2$ in $(0,2\pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + + \cdots$$

6. a) Using Residue theorem, evaluate, $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ 6

b) Find the Laplace transform of

$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ and $f(t+2) = f(t)$ for $t > 0$. 6

c) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in form $y = a sin(\pi x / l)$ from which it is released at a time $t = 0$. If the vibrations of a string is given by $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ show that the displacement of a point at a distance x from one end at time t is given by $y(x,t) = a\, sin(\pi x / l)\, cos(\pi ct / l)$. 8
