

(3hours)

[Total marks: 80]

- N.B.** 1) Question No. 1 is compulsory.
 2) Answer **any Three** from remaining
 3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = e^{-9t} \int_0^t u \sin 3u \, du$. 5
- b) Verify Laplace equation for $u = \left(r + \frac{a^2}{r} \right) \cos \theta$. 5
- c) Show that $\{\sin nx, n = 1, 2, 3 \dots\}$ is a set of orthogonal function over an interval $(-\pi, \pi)$. 5
- d) Evaluate $\int_0^{3+i} |z|^2 \, dz$ along the line $3y = x$ 5
2. a) Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2-4z+3}$ indicating the region of convergence. 6
- b) Find complex form of Fourier series of $f(x) = \cosh 2x$ in $(-3, 3)$. 6
- c) Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$ where $y(0) = 0, y'(0) = 1$ 8
3. a) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ with $u(0, t) = 0, u(5, t) = 0, u(x, 0) = x^2(25 - x^2)$ taking $h = 1$ up to $t = 3$ seconds by Bender -Schmidt method. 6
- b) Find the bilinear transformation which maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$. 6
- c) Obtain half range Cosine Series of $f(x) = \sin x$ in the interval $(0, \pi)$. Use Parseval's identity to prove that – 8
- $$\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots = \frac{\pi^2 - 8}{16}.$$

[TURN OVER]

4. a) Find the orthogonal trajectory of the family of curves, $x^3y - xy^3 = c$, where c is a constant. 6
- b) Obtain Fourier Series of $f(x) = |x|$ in $(-\pi, \pi)$ 6
- c) Find the inverse Laplace transform of :-
 i) $F(s) = \frac{1}{s(s^2+4)}$, using Convolution theorem, ii) $F(s) = \frac{e^{-3s}}{(s-2)^4}$. 8
5. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,
 $u(0, t) = u(1, t) = 0, u(x, 0) = 200(x - x^2)$
 taking $h = 0.25$ for one-time step. 6
- b) Find an analytic function $f(z) = u + iv$, if 6
 $u = e^{-x}\{(x^2 - y^2) \cos y + 2xy \sin y\}$
6
- c). Obtain Fourier series of $f(x) = x^2$ in $(0, 2\pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + + \dots$$
6. a) Using Residue theorem, evaluate, $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$ 6
- b) Find the Laplace transform of
 $f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$ and $f(t + 2) = f(t)$ for $t > 0$. 6
- c) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in form $y = a \sin(\pi x / l)$ from which it is released at a time $t = 0$. If the vibrations of a string is given by $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ show that the displacement of a point at a distance x from one end at time t is given by $y(x, t) = a \sin(\pi x / l) \cos(\pi ct / l)$. 8