SE Instru. Sem - III (Rev) Dec 2018

S.E(Instru) Sem-III choice Based

Q.P.Code: 37198

20/11/18

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Duration : 3 Hours Max. Marks: 80

N.B . 1) Question No. 1 is compulsory .

2) Attempt any three questions out of the remaining five questions .

3) Figures to the right indicate full marks .

1. (a) Find the Laplace transform of $f(t) = t.sinh3t.sint.cost.e^{-t}$. 5

(b) Find the Fourier series for $f(x) = x^3$ in $(-\pi, \pi)$ 5

(c) Find the directional derivative of $\emptyset(x, y, z) = yx^3 + zy^3 + xz^3$ at point A(0,-1,1) in the direction of BA where B is (1,2,3). 5

(d) Determine Constant 'm' if F(z)= $r^7\ cosm\theta + ir^m\ sin7\theta$. 5

2. (a) Find Fourier cosine integral representation of the function $f(x) = e^{-ax}$, $x \geq 0$ 6

and hence show that $\int_0^{\infty} \frac{cos\omega s}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}$, $x \geq 0$

(b) Solve using Laplace transform $(D^2 + 4)y = 1 + 9t$, 6

if $(0) = 0$, $Dy(0) = 0$.

(c) Find the Fourier series for $f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ x & 0 \leq x \leq \pi \end{cases}$ 8

Hence Find the value for $\frac{\pi^2}{8}$.

3) (a) Show that $\vec{F} = (ye^{xy}cosz)i + (xe^{xy}cosz)j - (e^{xy}sinz)k$ is irrotational , hence find its Scalar potential function . 6

(b) Find Fourier Series for the following function 6

$$f(x) = \begin{cases} (x-\pi)^2 & 0 \leq x \leq \pi \\ \pi^2 & \pi \leq x \leq 2\pi \end{cases}$$

(c) Evaluate $\int_0^{\infty} \frac{9}{e^t} \int_0^t (u^2.e^{-3u}.sin4u.)du.dt$ 8

4) (a) Find the bilinear transformation which maps the points $z = 0, 1, \infty$ onto the points $w = -1, \infty, 3$. 6

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(b) By using Stokes theorem evaluate $\int_C \vec{F}.\vec{dr}$ where

$\vec{F} = (x^2 + x)i + (2xy)j$ and C is the boundary of the region enclosed by

rectangle $x = 0, y = 0, x = p, y = q$ 6

(c) Find Inverse Laplace transform

$i)\left\{\frac{e^{-2s}s}{(s+4)^3}\right\}$ $ii)\log\left\{\frac{s^2-4}{s^2+121}\right\}$ 8

5) a) Define Orthogonal set of functions on (a,b) ,Show that the functions $\emptyset_1(x) = 1$, $\emptyset_2(x) = 4x$ are orthogonal on (-1,1). Determine the constants P , Q such that $\emptyset_3(x) = Px^2 + Qx^3 + 4$ is orthogonal to both $\emptyset_1(x)$ & $\emptyset_2(x)$ on the same interval . 6

(b) Find the analytic function $f(z) = u + iv$ in terms of Z if 6

$$7u - 4v = x^3 + x^2 - 3xy^2 - y^2 - 3yx^2 + y^3 - 2xy .$$

(c) Verify Green's theorem for $\int_C (3x^2)dx + (2xy)dy$,

C is a triangle whose vertices are A(0,2) , B(2,0) , C(4,2) in the XY-plane . 8

6) (a) Find Laplace transform of $f(t) = R\frac{t}{T}$ $for\ 0 < t < T and\ f(t) = f(t + \pi)$. 6

(b) Prove that $w = i\left(\frac{z-i}{z+i}\right)$ maps upper half of the Z-plane into the interior of the unit circle in the W-plane . 6

(c) Obtain Complex form of Fourier series for $f(x) = ccsh3x + sinh3x$ in $(-3,3)$ 8

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