

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is Compulsory.

- 2) Answer any THREE questions from Q.2 to Q.6.
 3) Figures to the right indicate full marks.

Q.1 (a) Verify Cauchy-Schwartz inequality for $u = (2, 1, -3)$ $v = (3, 4, -2)$. (5)
 Also find angle between u & v .

(b) If $A = \begin{bmatrix} 2 & 0 & 0 \\ 5 & -1 & 0 \\ 2 & 3 & 3 \end{bmatrix}$ find Eigen values of $A^2 + 6A^{-1} - 3I$. (5)

(c) Evaluate $\int_C \frac{z^3 + 2z}{(z-1)^2} dz$ when C is $|z| = 2$. (5)

(d) Find the extremals of $\int_{x_1}^{x_2} (x + y')y' dx$. (5)

Q.2 (a) Verify Cayley-Hamilton theorem & hence find A^{-1} , where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. (6)

(b) Find the extremal of $\int_{x_1}^{x_2} (2xy - y''^2) dx$. (6)

(c) Obtain Laurent's series expansion of $f(z) = \frac{z+2}{(z-3)(z-4)}$ about $z = 0$. (8)

Q.3 (a) Evaluate $\int_0^{1+i} z^2 dz$ along the parabola $x = y^2$. (6)

(b) Show that $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is derogatory & find its minimal polynomial. (6)

(c) Reduce the following quadratic form into canonical form & hence find it's rank, index, signature & value class (8)
 $x^2 + 2y^2 + 3z^2 + 2yz + 2xy - 2zx$.

Q.4 (a) Find unit vector orthogonal to both $u = (-6, 4, 2)$ $v = (3, 1, 5)$. (6)

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$. (6)

(c) Show that matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. Also (8)

find its diagonal and transforming matrix.

Q.5 (a) Using Rayleigh-Ritz method find solution for the extremal of the (6)
functional $\int_0^1 (2xy + y^2 - (y')^2) dx$ given $y(0) = y(1) = 0$.

(b) Find an orthonormal basis for the subspace of \mathbb{R}^3 using Gram-Schmidt process where $s = \{(1, 0, 0), (3, 7, -2), (0, 4, 1)\}$ (6)

(c) Find the curve C of given length 'l' which encloses a maximum area. (8)

Q.6 (a) If $A = \begin{bmatrix} \pi & \frac{\pi}{4} \\ 0 & \frac{\pi}{2} \end{bmatrix}$ find $\cos A$. (6)

(b) Check whether the set of all pairs of real numbers of the form (1, x) with operations (6)

$(1, a) + (1, b) = (1, a + b)$ and $k(1, a) = (1, ka)$ is a vector space, where k is real number.

(c) Find the singular value decomposition of $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$. (8)
