Q.P. Code: 24629

Duration: 3hrs [Total Marks: 80]

- 1) Question no.1 is compulsory.
- 2) Solve any three questions out of remaining five questions.
- 3) All questions carry equal marks as indicated by figures to the right.
- 4) Assume appropriate data whenever required. State all assumptions clearly.
- Q.1 a) Prove by induction that n<sup>2</sup>+n is an even number, for every natural number n. (05M)
  - b) Find the generating function for the following finite sequences

(05M)

- i) 2,2,2,2,2,2
- ii) 1,1,1,1,1,1
- c) Let A={a,b,c,d,e} and R={(a,a),(a,b),(b,c),(c,e),(c,d),(d,e)} Compute  $R^2$  and  $R^\infty$

(05M)

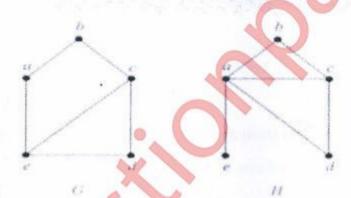
Compute K and K

d) Define Lattice. Check if the following diagram is a lattice or not?

(05M)

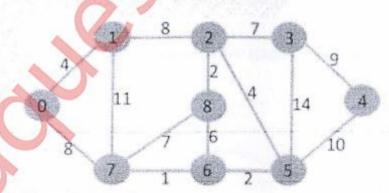


Q.2 a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic. If yes, find the one-to-one correspondence between the vertices. If not justify your answer. (08M)



b) Find Minimum spanning tree for the following graph using Kruskal's Algorithm.

(M80)



c) Prove  $(p \lor q) \land (p \land \neg q) \lor q] \leftrightarrow p \lor q$ 

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- Q. 3 a) Prove that set  $G = \{0,1,2,3,4,5\}$  is a finite abelian group of order 6 with respect to addition modulo (08M)6.
  - b) Let A={1,2,3,4}, let R={(1,2),(2,3),(3,4),(2,1)} Find Transitive closure of R using Warshall's (08M)Algorithm.
  - c) Test whether the following function is one-to-one, onto or both.

(04M)

$$f: Z \rightarrow Z$$
,  $f(x)=x^2+x+1$ 

Q.4 a) Show that the (2,6) encoding function  $e:B^2 \rightarrow B^{\bullet}$  defined by

(08M)

e(01)=011110

e(11)=111000 is a group code.

Find Minimum distance. How many errors will it detect and correct?

b) Let H = 
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Be a parity check matrix. Decode the following words relative to a maximum likelihood decoding (08M)function associated with e<sub>H</sub>. 1) 01111 2)01110 3)11001

- c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month? (04M)
- Q.5 a) Let G be a set of rational numbers other than 1. Let \* be an operation on G defined by a\*b=a+bab for all a,b €G. Prove that (G,\*) is a group.
  - b) Solve the recurrence relation  $a_r$ - $a_{r-1}$ - $6a_{r-2}$ =-30 given  $a_0$ =20,  $a_1$ =-5
- (M80) (04M)

- c) Let A={a,b,c,d,e,f,g,h}. Consider the following subsets of A  $A1=\{a,b,c,d\}$ 
  - $A2=\{a,c,e,g,h\}$
  - $A4=\{b,d\}$  $A3=\{a,c,e,g\}$

 $\rightarrow$  A5= $\{f,h\}$ 

Determine whether following is partition of A or not. Justify your answer.

- i) {A1, A2} ii) {A3, A4, A5}
- Q.6 a) Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers. (08M)
  - A={2,4,,12,24}
  - $A = \{1,3,5,15,30\}$
  - b) Let the functions f,g, and h defined as follows:

(08M)

 $f:R \rightarrow R$ , f(x)=2x+3

g:  $R \rightarrow R$ , g(x)=3x+4

h:  $R \rightarrow R$ , h(x)=4x

Find gof, fog, foh, , gofoh

c) Determine Euler Cycle and path in graph shown below



