

(3 hours)

Total Marks:80

**N.B:** (1) Question no.1 is **compulsory**.(2) Attempt any **three** questions from remaining **five** questions.(3) **Figures** to the **right** indicate **full** marks.

(4) Assume suitable data if necessary.

1. (a) Use Laplace Transform to evaluate  $L \left\{ \frac{\sin t \cos 2t}{e^t} \right\}$  (5)

(b) If  $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ , find the Eigen values of  $A^2 + I$  (5)

(c) Show that the function  $f(z) = \frac{1}{r^2} [\cos 2\theta - i \sin 2\theta]$  is analytic. (5)

(d) A manufacturer of metal pistons finds that on average 12% of his pistons are rejected because they are either oversized or undersized. What is the probability that a batch of 10 pistons will contain not more than 2 rejects. (5)

2. (a) The average of marks scored by 32 boys is 72 with standard deviation 8. While that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than girls ( $Z_\alpha = 2.58$ ). (6)

(b) Find analytic function whose imaginary part is  $\tan^{-1} \left( \frac{y}{x} \right)$ . (6)

(c) Reduce the following quadratic form to canonical form. Also find its rank and signature.

$$x^2 + 2y^2 + 2z^2 - 2xy - 2yz + zx \quad (8)$$

3. (a) Show that the matrix A is diagonable, where  $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ . (6)

(b) Find the inverse Laplace Transform of  $\frac{1}{(S^2 + 4S + 13)^2}$  using convolution theorem. (6)

(c) (i) A continuous random variable has probability density function

$$f(x) = 6(x - x^2), \quad 0 \leq x \leq 1 \quad \text{find mean and variance} \quad (4)$$

(ii) If mean of the following distribution is 16 find m and n (4)

<b>X</b>	8	12	16	20	24
<b>P(X= x)</b>	1/8	m	n	1/4	1/12

4. (a) Evaluate  $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$  using Laplace transform (6)

(b) Find the orthogonal trajectories of the family of curves  $x^3y - xy^3 = c$  (6)

(c) 300 digits were chosen at random from a table of random number. The frequency of digits was as follows: (8)

<b>Digit</b>	0	1	2	3	4	5	6	7	8	9	<b>Total</b>
<b>Frequency</b>	28	29	33	31	26	35	32	30	31	25	300

Using  $\chi^2$  - test examine the hypothesis that the digits were distributed in equal numbers in the table at 5% LOS.

5. (a) Find the bilinear transformation which maps the points  $z=1, i, -1$  onto the points

$w=i, 0, -i$  (6)

(b) From the following data calculate Spearman's rank correlation coefficient R

X: 10 12 18 18 15 40

Y: 12 18 25 25 50 25 (6)

(c) Solve using Laplace Transform  $(D^2 + 2D + 5)y = e^{-t} \sin t,$

Where  $y(0)=0, y'(0) = 1$  (8)

6. (a) A manufacturer knows from his experience that resistance of resistors he produces is normal with mean 100 ohms and standard deviation is 2 ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms (for a standard S.N.V the area under the curve between  $Z=0$  and  $Z=1$  is 0.3413). (6)

(b) Verify the Cayley-Hamilton Theorem for matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ . (6)

(c) Find the inverse Laplace transform of the following functions :

(i)  $\log \left[ \frac{s+a}{s+b} \right],$  (ii)  $\frac{e^{-5s}}{(s-2)^4}$  (8)