

N.B.

1. Q.1 is compulsory. Attempt any three from the remaining questions.
2. All questions carry equal marks.
3. Figures in the Right margin indicate full marks.
3. Assume suitable data if necessary

Q.1 Attempt any four

- a. Determine the sign-definiteness of following matrices

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$$(i) F = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, (ii) G = \begin{bmatrix} -1 & -2 \\ -3 & -2 \end{bmatrix}$$

- b. Define the singular point in phase-plane. Compute the singular points for the following system.

$$\dot{x} = x^3 - 3x^2 + 2x$$

- c. Define relative degree for the system $\dot{z} = f(z) + g(z)u$ at $y = h(z)$.
 d. Linearize the following system at point $z_0 = [-1 \ 1 \ 1]^T$ with the nominal input is to be held constant at $u_0 = 1$.

$$\begin{aligned} \dot{z}_1 &= 0.5z_2^2 - 0.5 \\ \dot{z}_2 &= 0.5z_3^2 - 0.5 \\ \dot{z}_3 &= -3z_1 - z_2^2 - z_3^2 + u \end{aligned}$$

- e. Obtain the classical control 'c' from the IMC controller 'q' using block diagram reduction rules.
 f. Explain the linear and nonlinear components of the friction.

- Q.2 A. Write the steps to construct the Lyapunov function using variable gradient method. 10
 B. Obtain the describing function for saturation nonlinearity. 10

Q.3 A. Determine the stability of the system,

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$$\dot{x}_1 = -2x_1$$

$$\dot{x}_2 = -3x_2$$

using Lyapunov's equation.

B. Linearize the following system using feedback control

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$$\dot{x}_1 = -x_1 - x_2^2 + (\sin x_1 + \cos x_1)u$$

$$\dot{x}_2 = x_1$$

$$y = x_2$$

Where y is output and u is input.

Q.4 A. What is limit cycle? Explain it with Vander Pol's equation.

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B. What is jump resonance in frequency response? Explain it with an example.

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Q.5 A. Construct the phase trajectory for the system $\ddot{x} + \dot{x} + 2x = 0$ using delta method. Consider an initial condition $x(0) = 1, \dot{x}(0) = 1$.

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B. Design the optimal control for the system

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$$\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

that minimizes the performance index

$$J = \frac{1}{2} \int_0^{\infty} \left\{ x^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + u^2 \right\} dt$$

Q.6 A. Design the IMC controller for the system model

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$$\tilde{G}_p = \frac{e^{-3s}(-0.5s + 1)}{25s + 1}$$

to track the step input. Use simple factorization for design.

B. Obtain the IMC based PI controller for the model

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$$\tilde{G}_p = \frac{5}{10s + 1}$$