

Applied Maths - III

QP Code : 30542



(3 Hours)

[ Total Marks : 80

- N.B. : (1) Question No.1 is compulsory  
 (2) Answer any three from remaining  
 (3) Figures to the right indicate marks.

1. (a) Find laplace transform of  $\frac{\sin^2 2t}{t}$  5  
 (b) Find the orthogonal trajectory of the family of curves  $e^{-x} \cos y + xy = \alpha$  where  $\alpha$  is a real constant in the  $xy$  plane. 5  
 (c) Find complex form of fourier series  $f(x) = e^{3x}$  in  $0 < x < 3$  5  
 (d) Show that the function is analytic and find their derivative  $f(z) = ze^z$  5
2. (a) Using laplace transform solve:  $\frac{d^2y}{dt^2} + y = t$   $y(0) = 1$   $y'(0) = 0$  6  
 (b) Using Crank Nicholson method 6
- Solve :  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$   
 $u(0, t) = 0$   $u(4, t) = 0$   
 $u(x, 0) = \frac{x}{3}(16 - x^2)$  find  $u_{ij}$   
 for  $i = 0, 1, 2, 3, 4$  and  $j = 0, 1, 2$
- (c) Show that the set of functions  $1, \sin \frac{\pi x}{L}, \cos \frac{\pi x}{L}, \sin \frac{2\pi x}{L}, \cos \frac{2\pi x}{L}, \dots$  8  
 form an orthogonal set in  $(-L, L)$  and construct an orthonormal set.

TURN OVER

3. (a) Find the bilinear transformation that maps points 0, 1,  $\infty$  of the z plane into -5, -1, 3 of w plane. 6

(b) By using Convolution theorem find inverse laplace transform of 6

$$\frac{1}{(s-2)^4(s+3)}$$

(c) Find the Fourier series of f(x)

$$f(x) = \begin{cases} \cos x & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$$

4. (a) Find half range sine series for  $x \sin x$  in  $(0, \pi)$  and hence deduce 6

$$\frac{\pi^2}{8\sqrt{2}} = \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} \dots$$

(b) Evaluate and prove that 6

$$\int_0^{\infty} e^{-\sqrt{2}t} \frac{\sin t \sinh t}{t} dt = \frac{\pi}{8}$$

(c) Obtain Laurent's series for the function  $f(z) =$  8

$$\frac{-7z-2}{z(z-2)(z+1)} \text{ about } z = -1$$

5. (a) Solve :  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$  subject to the conditions  $u(0, t) = 0$ ,  $u(5, t) = 0$  6  
 $u(x, 0) = x^2(25 - x^2)$  taking  $h = 1$  upto 3 seconds only by Bender schmidt formula.

(b) Construct an analytic function whose real part is  $\frac{\sin 2x}{\cosh 2y + \cos 2x}$  6

(c) Evaluate  $\int_0^{\pi} \frac{d\theta}{3+2\cos\theta}$  8

6. (a) An elastic string is stretched between two points at a distance  $l$  apart. In its equilibrium position a point at a distance  $a$  ( $a < l$ ) from one end is displaced through a distance  $b$  transversely and then released from this position. Obtain  $y(x, t)$  the vertical displacement if  $y$  satisfies the equation. 6

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

- (b) Evaluate :  $\int_0^{1+i} z^2 dz$  along

- (i) The line  $y = x$   
 (ii) The parabola  $x = y^2$

Is the line integral independent of path? Explain. 6

- (c) Find fourier expansion of 8

$$f(x) = \left( \frac{\pi - x}{2} \right)^2$$

in the interval  $0 \leq x \leq 2\pi$  and  $f(x+2\pi) = f(x)$   
 and also deduce

(i)  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots$

(ii)  $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \dots$

---