F.E. sem I (CBGS). 2/12/15

Applied - Mathematics - I

Q.P. Code : 5001

(3 Hours) **[Total Marks : 100**

N.B.:-i) Q.No.1) is compulsory

ii) Attempt any THREE from remaining

iii) All questions carry equal marks

Q.No.1) a) If $\log\tan x = y$ then prove that $\sinh(n+1)y + \sinh(n-1)y = 2\sinh ny \cdot cosec2x$

b) If $z = \log(\tan x + \tan y)$ then prove that $\sin 2x\frac{\partial z}{\partial x} + \sin 2y\frac{\partial z}{\partial y} = 2$ (3)

c) If $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$ then find $\frac{\partial(r,\theta,\varphi)}{\partial(x,y,z)}$ (3)

d) Prove that $\log\sec x = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \cdots$ (3)

e) Find the values of a , b , c and A^{-1} when $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ is orthogonal (4)

f) If $y = \sin\theta + \cos\theta$ then prove that $y_n = r^n\sqrt{1 + (-1)^n \sin 2\theta}$ where $\theta = rx$ (4)

Q.No.2) a) If $z = -1 + i\sqrt{3}$ then prove that $\left(\frac{z}{2}\right)^n + \left(\frac{2}{z}\right)^n = \begin{cases} 2, & if\ n\ is\ multiple\ of 3 \\ -1, & if\ n\ is\ not\ multiple\ of\ 3 \end{cases}$ (6)

b) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ then find two non-singular matrices P & Q such that PAQ is in normal form also find $\rho(A)$ and A^{-1} (6)

c) State and prove Euler's theorem for functions of two independent variable hence prove that

$$\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right) = 0 \text{ if } x = e^u\tan v \text{ , } y = e^u\sec v$$ (8)

Q.No.3) a) Determine the values of a and b such that system $\begin{cases} 3x - 2y + z = b \\ 5x - 8y + 9z = 3 \\ 2x + y + az = -1 \end{cases}$

has i) no solution, ii) a unique solution, iii) infinite number of solutions (6)

b) Discuss the maximum and minimum of $f(x,y) = x^3 + 3xy^2 - 15(x^2 + y^2) + 72x$ (6)

c) Show that $\tan^{-1}\left(\frac{x+iy}{x-iy}\right) = \frac{\pi}{4} + \frac{i}{2}\log\left(\frac{x+y}{x-y}\right)$ (8)

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Q.No.4) a) If $u = xyz$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ then prove that $\frac{\partial x}{\partial u} = \frac{1}{(x-y)(x-z)}$ (6)

b) If $\sqrt{i}^{\sqrt{i}^{\sqrt{i}\cdots\infty}} = \alpha + i\beta$ then prove that i) $\alpha^2 + \beta^2 = e^{-\frac{\pi\beta}{2}}$ ii) $\tan\left(\frac{\beta}{\alpha}\right) = \frac{\pi\alpha}{4}$ (6)

c) Apply Crout's method to solve $\begin{cases} x - y + 2z = 2 \\ 3x + 2y - 3z = 2 \\ 4x - 4y + 2z = 2 \end{cases}$ (8)

Q.No.5) a) If $cos^6\theta + sin^6\theta = \alpha\cos 4\theta + \beta$ then prove that $\alpha + \beta = 1$ (6)

b) Find the values of a, b & c such that $\lim_{x\to 0}\frac{ae^x - be^{-x} + cx}{x - \sin x} = 4$ (6)

c) If $x = \cos\left[\log\left(y^{1/m}\right)\right]$ then prove that

$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2 + n^2)y_n = 0$ (8)

Q.No.6) a) Define linear dependence and independence of vectors, Examine for linear dependence of following set of vectors and find the relation between them if dependent

$X_1 = \begin{bmatrix}1\\-1\\1\end{bmatrix}, X_2 = \begin{bmatrix}2\\1\\1\end{bmatrix}, X_3 = \begin{bmatrix}3\\0\\2\end{bmatrix}$ (6)

b) If $z = f(u,v)$, $u = x^2 - y^2$, $v = 2xy$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4\sqrt{u^2 + v^2}\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$ (6)

c) Fit a straight line passing through points (0,1), (1,2), (2,3), (3,4.5), (4,6), (5,7.5) (8)