Q.1. (A) A bag contains 7 red and 3 black balls and another bag contains 4 red and 5 black balls. One ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag. If this ball happens to be red, find the probability that a black ball was transferred.

(B) Check whether the Random Process given by $x(t) = A \sin(t) + B \cos(t)$ is ergodic, where $A$, $B$ are Random Variables normally distributed with zero means and unit variances.

(C) Write a short note on “Markov Chain.”

(D) Find ‘$P$’ of Binomial Distribution if $n=6$ and $9P(X=4) = P(X=2)$. (05)

Q.2. (A) The Power Spectral Density of a WSS Process is given by,

$$S_x(W) = \begin{cases} \frac{b}{a} (a - |w|) & \text{if } |w| \leq a \\ 0 & \text{if } |w| > a \end{cases}$$

Find the Autocorrelation Function.

(B) Let $X_1, X_2, X_3, \ldots$ be sequence of Random variables. Define (i) Convergence almost everywhere (ii) Convergence in probability (iii) Convergence in distribution (iv) Convergence in mean square sense for the above sequence of Random variables $X$.

Q.3. (A) Prove that if input to an LTI system is Wide sense stationary (WSS) process then output is also WSS.

(B) A binary communication transmitter sends data as one of two types of signal denoted by 0 or 1. Due to noise, sometimes a transmitted 1 is received as 0 and vice versa. If the probability that a transmitted 0 is correctly received as 0 is 0.9 and the probability that the 1 is received as 1 is 0.8 and if the probability of transmitting 0 is 0.45. Find the probability that 1) A 1 is received. 2) A 0 is received. 3) 1 was transmitted given that 1 was received. 4) 0 was transmitted given that 0 was received. 5) The error has occurred.

Q.4. (A) A random variable has the following exponential probability density function: $f(x) = Ke^{-0.5x}$. Determine

i) The value of $K$ and ii) Mean and variance.

(B) The transition probability matrix of Markov Chain is given is by,

$$P = \begin{pmatrix} 1 & 0.5 & 0.4 & 0.1 \\ 2 & 0.3 & 0.4 & 0.3 \\ 3 & 0.2 & 0.3 & 0.5 \end{pmatrix}$$

Find the limiting probabilities.
Q. 5. (A) The joint probability density function of two continuous random variable X and Y is given by,

\[
f_{xy}(x, y) = \begin{cases} 
Ce^{-x}e^{-y} & 0 < x < \infty \\
0 & \text{elsewhere} 
\end{cases}
\]

Find 1) The value of C.
2) \(f_x(x), f_y(y)\).
3) \(f_{X|Y}(X/Y), f_{Y|X}(Y/X)\).
4) \(E[Y/X] = X, E[X/Y] = Y\).

(B) Write a short note on "Little’s Formula".

Q. 6. (A) State and prove Chapman-Kolmogorov equation.
(B) Write a short note on the following distributions
i) Poisson Distribution and (ii) Gaussian Distribution