Q.P. Code : 11837

[Time: Three Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B: 1. Question.No.1 is compulsory.
2. Attempt any three from the remaining.

Q.1. a) Find the extremal of $\int_{x_0}^{x_1} \frac{1+y^2}{y'^2}dx$ **(5)**

b) Is $(6,7,-4)$ a linear combination of $v_1 = (1,2,2), v_2 = (3,4,6)$ **(5)**

c) Check whether $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ is derogatory or not. **(5)**

d) Evaluate $\int_0^{1+i} z^2 dz$, along the parabola $x = y^2$ **(5)**

Q.2. a) Show that the functional $\int_0^{\pi/2} \left\{ 2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right\} dt$; such that $x(0) = 0, x\left(\frac{\pi}{2}\right) = -1,$

$y(0) = 0, y\left(\frac{\pi}{2}\right) = 1$ is stationary if x=-sint, y=sint. **(6)**

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}dx, a > 0, b > 0$ **(6)**

c) Reduce the quadratic form $x^2 - 2y^2 + 10z^2 - 10xy + 4xz - 2zy$ to canonical form and hence, find its rank, index and signature and value class. **(8)**

Q.3. a) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} & A^4 **(6)**

b) Using Residue theorem evaluate $\int_c \frac{e^z}{z^2+\pi^2}dz$ where C is |z|=4. **(6)**

c) Find the singular value decomposition of $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ **(8)**

Q.4. a) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, prove that 3tanA=Atan3 **(6)**

b) Find the sum of the residues at singular points of $f(z) = \frac{z-4}{z(z-1)(z-2)}$ **(6)**

c) Check whether the set of real numbers (x,0) with operation $(x_1, 0) + (x_2, 0) = (x_1 + x_2, 0)$, and $k(x_1, 0) = (kx_1, 0)$ is a vector space. **(8)**

Q.5. a) Find the extremum of $\int_{x_0}^{x_1} (2xy - y''^2)\,dx$. **(6)**

b) Construct an orthonormal basis of R^3 using Gram Schmidt process to S = {(3,0,4),(-1,0,7),(2,9,11)} **(6)**

c) Find all possible Laurent's expansions of $\frac{2z-3}{z^2-4z-3}$ about $z = 4$. **(8)**

Q.6. a) Find the linear transformation Y=AX which carries $X_1 = (1,1,-1)', X_2 = (1,-1,1)', X_3 = (-1,1,1)'$ onto $Y_1 = (2,1,3)', Y_2 = (2,3,1)', Y_3 = (4,1,3)'$ **(6)**

b) Show that the vectors $v_1 = (1,2,4), v_2 = (2,-1,3), v_3 = (0,1,2)$ are linearly independent. Express $v_4 = (-3,7,2)$ in terms of v_1, v_2, v_3 **(6)**

c) If C is circle |z|=1, using the integral $\int_c \frac{e^{kz}}{z}\,dz$ where k is real, show that

$$\int_0^{\pi} e^{k\cos\theta}\cos(k\sin\theta)\,d\theta = \pi$$ **(8)**
