N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1. (a) Prove that \( f(z) = x^2 - y^2 + 2xyz \) is analytic and find \( f'(z) \)
(b) Find the Fourier series expansion for \( f(x) = |x| \) in \((-\pi, \pi)\)
(c) Using Laplace transform solve the following differential equation with given condition \( \frac{d^2y}{dt^2} + y = t \), given that \( y(0) = 1 \) & \( y'(0) = 0 \)
(d) If \( \vec{A} = \nabla(xy + yz + zx) \), find \( \nabla \cdot \vec{A} \) and \( \nabla \times \vec{A} \)

2. (a) If \( L[J_0(t)] = \frac{1}{\sqrt{s^2 + 1}} \), prove that \( \int_0^\infty e^{-st} J_0(4t) dt = 3/500 \)
(b) Find the directional derivative of \( \phi = x^4 + y^4 + z^4 \) at \( A (1, -2, 1) \) in the direction of \( \vec{AB} \) where \( B \) is \((2, 6, -1)\). Also find the maximum directional derivative of \( \phi \) at \((1, -2, 1)\).
(c) Find the Fourier series expansion for \( f(x) = 4 - x^2 \), in \((0, 2)\)
Hence deduce that \( \frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots \ldots \ldots \ldots \)

3. (a) Prove that \( J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x \)
(b) Using Green's theorem evaluate \( \int_C (2x^3 - y^3) dx + (x^3 + y^3) dy \) where \( C \) is the boundary of the surface enclosed by the lines \( x = 0, y = 0, x = 2, y = 2 \).
(c) i) Find Laplace Transform of \( e^{-3u} \int_0^u u \sin 3u \, du \)
ii) Find the Laplace transform of \( \frac{d}{dt} \left( \frac{1 - \cos 2t}{t} \right) \)

4. (a) Obtain complex form of Fourier series for the functions \( f(x) = \sin ax \) in \((-\pi, \pi)\), where \( a \) is not an integer.
(b) Find the analytic function whose imaginary part is \( v = \frac{x}{x^2 + y^2} \cdot \cosh y \cdot \cos x \)
(c) Find inverse Laplace Transform of following
\[ \text{i)} \quad \log \left( \frac{x^2 + a^2}{\sqrt{s+b}} \right) \quad \text{ii)} \quad \frac{1}{s^3(s-1)} \]

5. (a) Obtain half-range cosine series for \( f(x) = x(2-x) \) in \( 0 < x < 2 \)
(b) Prove that \( \vec{F} = \frac{\vec{r}}{r^3} \) is both irrotational and solenoidal
(c) Show that the function \( u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 3xy \) satisfies

GN-Con. 8067-14.

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Laplace's equation and find it corresponding analytic function  

6. (a) Evaluate by Stoke's theorem \( \int \left( x \, y \, dx + x \, y^2 \, dy \right) \) where C is the square in the xy-plane with vertices \((1,0), (0,1), (-1,0), and (0,-1)\).  
(b) Find the bilinear transformation, which maps the points \(z = -1, 1, \infty\) onto the points \(w = -i, -1, i\).  
(c) Show that the general solution of \( \frac{d^2 y}{dx^2} + 4x^2 y = 0 \) is \( y = \sqrt{x} \left[ A \, J_{1/4}(x^2) + B \, J_{-1/4}(x^2) \right] \) where A and B are constants.