N.B.: 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Verify Cauchy-Schwartz inequality for \( u = (2, 1, -3) \) \( v = (3, 4, -2) \).
Also find angle between \( u \) & \( v \).

(b) If \( A = \begin{bmatrix} 2 & 0 & 0 \\ 5 & -1 & 0 \\ 2 & 3 & 3 \end{bmatrix} \) find Eigen values of \( A^2 + 6A^{-1} - 3I \).

(c) Evaluate \( \int_C \frac{z^3 + 2z}{(z-1)^2} \, dz \) when \( C \) is \( |z| = 2 \).

(d) Find the extremals of \( \int x^2(x + y')y' \, dx \).

Q.2 (a) Verify Cayley-Hamilton theorem & hence find \( A^{-1} \), where \( A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix} \).

(b) Find the extremal of \( \int x^2(2xy - y'/2) \, dx \).

(c) Obtain Laurent’s series expansion of \( f(z) = \frac{z+2}{(z-3)(z-4)} \) about \( z = 0 \).

Q.3 (a) Evaluate \( \int_0^{1+i} z^2 \, dz \) along the parabola \( x = y^2 \).

(b) Show that \( A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \) is derogatory & find its minimal polynomial.

(c) Reduce the following quadratic form into canonical form & hence find it’s rank, index, signature & value class
\( x^2 + 2y^2 + 3z^2 + 2yz + 2xy - 2zx \).
Q.4 (a) Find unit vector orthogonal to both \( u = (-6, 4, 2) \) \( v = (3, 1, 5) \).

(b) Evaluate \( \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} \, dx \).

(c) Show that matrix \( A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \) is diagonalizable. Also find its diagonal and transforming matrix.

Q.5 (a) Using Rayleigh-Ritz method find solution for the extremal of the functional \( \int_0^1 (2xy + y^2 - (y')^2) \, dx \) given \( y(0) = y(1) = 0 \).

(b) Find an orthonormal basis for the subspace of \( \mathbb{R}^3 \) using Gram-Schmidt process where \( s = \{(1,0,0), (3,7,-2), (0,4,1)\} \).

(c) Find the curve \( C \) of given length \( 'l' \) which encloses a maximum area.

Q.6 (a) If \( A = \begin{bmatrix} \pi & \pi \\ 0 & \frac{\pi}{2} \end{bmatrix} \) find \( \cos A \).

(b) Check whether the set of all pairs of real numbers of the form \((1, x)\) with operations

\[
(1, a) + (1, b) = (1, a + b) \quad \text{and} \quad k(1, a) = (1, ka)
\]

is a vector space, where \( k \) is real number.

(c) Find the singular value decomposition of \( A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \).