N.B.: (1) Question No.1 is compulsory.
(2) Attempt any Three from the remaining.

1. (a) Find the extremal of the functional
\[ \int_0^1 [y'^2 + 12xy] \, dx \] subject to \( y(0) = 0 \) and \( y(1) = 1 \).

(b) Verify Cauchy-Schwartz inequality for \( u = (1,2,1) \) and \( v = (3,0,4) \).
Also find the angle between \( u \) & \( v \).

(c) If \( \lambda \) & \( X \) are eigen values and eigen vectors of \( A \) then prove that \( \frac{1}{\lambda} \) and \( X \) are eigen values and eigen vectors of \( A^{-1} \), provided \( A \) is non singular matrix.

(d) Evaluate \( \int \frac{e^{2z}}{C(z+1)^4} \, dz \) where \( C : |z| = 2 \)

2. (a) Find the extremal that minimises the integral
\[ \int_{x_0}^{x_1} (16y^2 - y'^2) \, dx \]

(b) Find eigen values and eigen vectors of \( A^3 \)
\[ A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix} \]

(c) Obtain Taylor's and two distinct Laurent's expansion of \( f(z) = \frac{z-1}{z^2 - 2z - 3} \)
indicating the region of convergence.
3. (a) Verify Cayley-Hamilton Theorem for
\[
A = \begin{bmatrix}
2 & -1 & 1 \\
-1 & 2 & -1 \\
1 & -1 & 2 \\
\end{bmatrix}
\]
and hence find \( A^{-1} \)

(b) Using Cauchy Residue Theorem, evaluate
\[
\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} \, dx
\]

(c) Show that a closed curve 'C' of given fixed length (perimeter) which encloses maximum area is a circle.

4. (a) Find an orthonormal basis for the subspace of \( \mathbb{R}^3 \) by applying Gram-Schmidt process where \( S \) \{\((1,1,1), (0,1,1), (0,0,1)\)\}.

(b) Find \( A^{10} \), where
\[
A = \begin{bmatrix}
2 & 3 \\
-3 & -4 \\
\end{bmatrix}
\]

(c) Reduce the following Quadratic form into canonical form & hence find its rank, index, signature and value class where,
\[
Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1
\]

5. (a) Using the Rayleigh-Ritz method, find an approximate solution for the extremal of the functional
\[
\int_{0}^{1} \{xy + \frac{1}{2}y^2\} \, dx \quad \text{subject to} \quad y(0) = y(1) = 0 .
\]

(b) Prove that \( W = \{(x,y) | x = 3y\} \) subspace of \( \mathbb{R}^2 \). Is \( W_1 = \{(a,1,1) | a \in \mathbb{R}\} \) subspace of \( \mathbb{R}^3 \)?

[ TURN OVER ]
(c) Prove that $A$ is diagonalizable matrix. Also find diagonal form and transforming matrix where:

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

6. (a) By using Cauchy Residue Theorem, evaluate $\frac{2\pi \cos^2 \theta}{5 + 4 \cos \theta} \, d\theta$.  

(b) Evaluate $\int_{C} \frac{z+4}{z^2 + 2z + 5} \, dz$ where $C : |z+1+i| = 2$.  

(c) (i) Determine the function that gives shortest distance between two given points.  
(ii) Express any vector $(a,b,c)$ in $\mathbb{R}^3$ as a linear combination of $v_1, v_2, v_3$ where $v_1, v_2, v_3$ are in $\mathbb{R}^3$. 

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