Duration: 3 Hours

N.B: 1) Q1 is compulsory.
2) Attempt any three from the remaining.

Q1: 
a) Find the extremal of 
\[ \int_{x_1}^{x_2} (y^2 - y'^2 - 2ycoshx)\,dx \] (5)
b) Find an orthonormal basis for the subspaces of \( \mathbb{R}^3 \) by applying Gram-Schmidt process where \( S = \{(1, 2, 0), (0, 3, 1)\} \) (5)
c) Show that eigenvalues of unitary matrix are of unit modulus. (5)
d) Evaluate \( \int \frac{dx}{x^2(z+4)} \) where \( |z| = 4 \). (5)

Q2: 
a) Find the complete solution of 
\[ \int_{x_0}^{x_2} (2xy - y'^2)\,dx \] (6)

Q3: 
a) Verify Cayley Hamilton Theorem and find the value of \( A^4 \) for the matrix \( A = \begin{bmatrix} 1 & 2 & 6 \\ 4 & 6 & 2 \\ 1 & 3 & 2 \end{bmatrix} \) (6)
b) Using Cauchy's Residue Theorem evaluate 
\[ \int_{C} \frac{z^2}{z^2+1}\,dz \] (6)
c) Show that a closed curve 'C' of given fixed length (perimeter) which encloses maximum area is a circle. (8)

Q4: 
a) State and prove Cauchy-Schwarz inequality. Verify the inequality for vectors \( u = (-4, 2, 1) \) and \( v = (8, -4, -2) \) (6)
b) Reduce the quadratic form \( xy + yz + zx \) to diagonal form through congruent transformation. (6)
c) If \( A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix} \) then find \( e^A \) and \( 4^A \) with the help of Modal matrix. (8)

Q5: 
a) Solve the boundary value problem 
\[ \int_{0}^{1} (2xy + y^2 - y'^2)\,dx , \quad 0 \leq x \leq 1, \quad y(0) = 0, y(1) = 0 \] by Rayleigh-Ritz Method. (6)
b) If \( W = \{ \alpha \in \mathbb{R}^n \text{ and } a_1 \geq 0 \} \) a subset of \( V = \mathbb{R}^n \) with \( \alpha = (a_1, a_2, \ldots, a_n) \) in \( \mathbb{R}^n \), show that \( W \) is not a subspace of \( V \) by giving suitable counter example. (6)

c) Show that the matrix \( A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} \) is similar to diagonal matrix. Find the diagnosing matrix and diagonal form. (8)

Q6:

a) State and prove Cauchy's Integral Formula for the simply connected region and hence evaluate
\[
\int \frac{z + 5}{z - 2} \, dz, \quad |z - 2| = 5
\]
(6)

b) Show that
\[
\int_0^{2\pi} \frac{\sin^2 \theta}{a + b \cos \theta} \, d\theta = \frac{\pi}{b^2} (a - \sqrt{a^2 - b^2}), \quad 0 < b < a.
\]
(6)

c) Find the Singular value decomposition of the following matrix \( A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \) (8)