(3 Hours) **[Total marks : 80**

Note : -
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q 1.A) Show that u = $y^3 - 3x^2y$ is a harmonic function. Also find its harmonic conjugate. **(5)**

B) Find half range Fourier sine series for f(x) = x^3, $-\pi < x < \pi$. **(5)**

C) If $\bar{F} = xye^{2z}i + xy^2cosz j + x^2cosxy k$ find div$\bar{F}$ and curl$\bar{F}$ **(5)**

D) Evaluate $\int_0^\infty e^{-2t} sin^3 t \, dt$. **(5)**

Q.2) A) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ cosx$ **(6)**

B) Find an analytic function f(z) whose imaginary part is $e^{-x}(ysiny + xcosy)$ **(6)**

C) Obtain Fourier series for f(x) = $1 + \frac{2x}{\pi}$ $\quad -\pi \le x \le 0$

$= 1 - \frac{2x}{\pi} \quad 0 \le x \le \pi$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$ **(8)**

Q.3) A) Show that $\bar{F} = (2xyz^2)i + (x^2z^2 + zcosyz)j + (2x^2yz + ycosyz)k$, is a conservative field. Find its scalar potential φ such that $\bar{F} = \nabla\varphi$and hence, find the work done by $\bar{F}$ in displacing a particle from A(0,0,1) to B(1,$\pi/4$,2) along straight line AB **(6)**

B) Show that the set of functions $f_1(x) = 1, f_2(x) = x$ are orthogonal over (-1, 1). Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and $f_2$on that interval **(6)**

TURN OVER

C) Find (i) $L^{-1}\left\{log\left[\frac{s^2+a^2}{\sqrt{s+b}}\right]\right\}$

(ii) $L\{(e^{-t}cost.H(t-\pi)\}$ **(8)**

Q.4) A) Prove that $\int J_5(x)\,dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x)$ **(6)**

B) Find inverse Laplace of $\frac{s}{(s^2-a^2)^2}$ using Convolution theorem. **(6)**

C) Expand $f(x) = \frac{3x^2-6x\pi+2\pi^2}{12}$ in the interval $0 \le x \le 2\pi$ as a Fourier series.

Hence, deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ **(8)**

Q.5) A) Using Gauss Divergence theorem, prove that $\iint_s (y^2z^2i + z^2x^2j + z^2y^2k).\bar{N}ds = \frac{\pi}{12}$

where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ and above the xy-plane. **(6)**

B) Prove that $J_3(x) + 3J_0(x) + 4J_0'''(x) = 0$ **(6)**

C) Solve $(D^3-2D^2+5D)y = 0$, with y(0)= 0, y'(0)=0 and y''(0)= 1, **(8)**

Q.6) A) Evaluate by Green's theorem for $\int_c (\frac{1}{y}\,dx + \frac{1}{x}dy)$ where C is the

the boundary of the region define by x = 1, x = 4, y = 1 and y = $\sqrt{x}$ **(6)**

B) Find the bilinear transformation which maps the points z = 1, i, -1

onto points w = i, 0 –i **(6)**

C) Find Fourier cosine integral representation for $f(x) = e^{-ax}, x > 0$

Hence, show that $\int_0^\infty \frac{cos\omega s}{1+\omega^2} d\omega = \frac{\pi}{2}e^{-x}$, x≥ 0 **(8)**

---X---X---X---