(Revised course)

Total marks: 80

N.B.: (1) Question No.1 is compulsory.
(2) Answer any three questions from remaining.
(3) Assume suitable data if necessary.

Evaluate

1. (a) \[ \int_0^1 e^{-t} \left( \frac{\sinh t \sin t}{t} \right) dt \]

(b) Obtain the Fourier Series expression for
\[ f(x) = 9 - x^2 \] in (-3, 3)

(c) Find the value of \( p \) such that the function \( f(z) \) expressed in polar co-ordinates as
\[ f(z) = r^p \cos p\theta + ir^p \sin 3\theta \] is analytic.

(d) If \( \vec{F} = (y^3 - x + 3yz - 2x) \hat{i} + (3xz + 2xy) \hat{j} + (3x - 2xz + 2z) \hat{k} \),
Show that \( \vec{F} \) is irrotational and solenoidal.

2. (a) Solve the differential equation using Laplace Transform
\[ \frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 8y = 1 , \text{ given } y(0) = 0 \text{ and } y'(0) = 1 \]

(b) Prove that
\[ J_n(x) = \left( \frac{4}{x^2} \right) J_1(x) - \left( \frac{24}{x^8 - 1} \right) J_3(x) \]

(c) i) Find the directional derivative of
\[ f = 4x^2 - 3xy^2 \] at \((2, -1, 2)\) in the direction of \(2\hat{i} + 3\hat{j} + 6\hat{k} \).

ii) If \( \vec{r} = xi + yj + zk \)
Prove that \( \nabla \log r = \frac{\vec{r}}{r^2} \)

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[TURN OVER]
3. (a) Show that \( \{\cos x, \cos 2x, \cos 3x, \ldots\} \) is a set of orthogonal functions over \((-\pi, \pi)\). Hence construct an orthonormal set.

(b) Find an analytic function \( f(z) = u + iv \) where,

\[
u = \frac{x}{2} \log(x^2 + y^2) - y \tan^{-1}\left(\frac{y}{x}\right) + \sin x \cosh y
\]

(c) Find Laplace transform of

i) \( \int_0^\infty e^{-ax} \cos^2 2udu \)

ii) \( \sqrt{1 + \sin t} \)

4. (a) Find the Fourier Series for

\[
f(x) = \frac{3x^2 - 6\pi x + 2\pi^2}{12}
\]

in \((0, 2\pi)\).

Hence deduce that

\[
\frac{1}{p^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}
\]

(b) Prove that

\[
\int_0^b x J_a(\alpha x) \, dx = \frac{b}{a} J_{a+1}(ab)
\]

c) Find

i) \( L^+ \left[ \log \left( \frac{x^2 + 1}{\pi(x + 1)} \right) \right] \)

ii) \( L^+ \left[ \frac{x^2 + 1}{(x^2 - 2x + 17)} \right] \)
5. (a) Obtain the half range cosine series for

\[ f(x) = \begin{cases} 
 0, & 0 < x < \frac{\pi}{2} \\
 \pi - x, & \frac{\pi}{2} < x < \pi 
\end{cases} \]

(b) Find the Bi-linear Transformation which maps the points 1, i, -1 of z plane onto i, 0, -i of w-plane

(c) Verify Green's Theorem for \( \int_C \vec{F} \cdot d\vec{r} \) where \( \vec{F} = (x^2 - xy)\hat{i} + (x^3 - y^2)\hat{j} \) and C is the curve bounded by \( x^2 = 2y \) and \( x = y \)

6. (a) Show that the transformation \( w = \frac{1 + z}{1 - z} \) maps the unit circle \(|z| = 1\) into real axis of w plane.

(b) Using Convolution theorem, find

\[
L^1\left[\frac{s}{(s^2+1)(s^2+4)}\right]
\]

(c) i) Use Gauss Divergence Theorem to evaluate \( \iiint_S \vec{F} \cdot n \, ds \) where \( \vec{F} = xi + yj + zk \) and \( S \) is the sphere \( x^2 + y^2 + z^2 = 9 \) and \( n \) is the outward normal to \( S \)

ii) Use Stoke's Theorem to evaluate \( \int_C \vec{F} \cdot d\vec{r} \) where \( \vec{F} = x^2 i - xy j \) and C is the square in the plane \( z = 0 \) and bounded by \( x = 0, y = 0, x = a \) and \( y = a \).

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