Evaluate

1. (a) \[ \int_{0}^{\pi} e^{i \left( \frac{\cos 3t - \cos 2t}{t} \right)} dt \]

(b) Obtain the Fourier Series expression for \( f(x) = 2x - 1 \) in \((0,3)\)

(c) Find the value of ‘p’ such that the function \( f(x) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \left( \frac{y}{x} \right) \) is analytic.

(d) If \( \vec{F} = (y \sin z - \sin x)i + (x \sin z + 2yz)j + (xy \cos z + y^2)k \), show that \( \vec{F} \) is irrotational. Also find its scalar potential.

2. (a) Solve the differential equation using Laplace Transform \[ \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + y = 3te^{-t} \], given \( y(0) = 4 \) and \( y'(0) = 2 \)

(b) Prove that \( J_1(x) = \left( \frac{48}{x^2} - \frac{8}{x} \right) J_0(x) - \left( \frac{24}{x^2} - 1 \right) J_2(x) \)

(c) i) In what direction is the directional derivative of \( \phi = x^2 + y^2 \) at \((3,-1,-2)\) maximum? Find its magnitude.

ii) If \( \vec{r} = xi + yj + zk \), prove that \( \nabla r^n = nr^{n-2}r \)

N.B: (1) Question No.1 is compulsory.

(2) Answer any three questions from remaining.

(3) Assume suitable data if necessary.
3. (a) Obtain the Fourier Series expansion for the function
\[ f(x) = 1 + \frac{2x}{\pi}, -\pi \leq x \leq 0 \]
\[ = 1 - \frac{2x}{\pi}, 0 \leq x \leq \pi \]

(b) Find an analytic function \( f(z) = u + iv \) where.
\[ u - v = \frac{x - y}{x^2 + 4xy + y^2} \]

(c) Find Laplace transform of
i) \( \cosh \frac{t}{2} e^{i\sinh t} \)
ii) \( \sqrt{1 + \sin t} \)

4. (a) Obtain the complex form of Fourier series for \( f(x) = e^x \) in \((-L, L)\)

(b) Prove that
\[ \int x^2 J_n(x) dx = x^2 J_n(x) - 2x J_1(x) + c \]

(c) Find
i) \( L^1 \left[ \frac{2x-1}{x^2 + 4x + 29} \right] \)
ii) \( L^1 \left[ \cot \left( \frac{x + 3}{2} \right) \right] \)

5. (a) Find the Bi-linear Transformation which maps the points 1, i, -1 of \( z \)-plane onto 0, 1, \( \infty \) of \( w \)-plane

(b) Using Convolution theorem find
\[ L^1 \left[ \frac{s^3}{(s^2 + 4)^2} \right] \]

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(c) Verify Green's Theorem for \( \int_C \vec{F} \cdot d\vec{r} \) where
\[
\vec{F} = (x^2 - y^2)\hat{i} + (x + y)\hat{j}
\]
and \( C \) is the triangle with vertices \((0,0)\), \((1,1)\) and \((2,1)\).

6. (a) Obtain half range sine series for \( f(x) = x, 0 \leq x \leq 2 \)
\[
f = 4 - x, 2 \leq x \leq 4
\]

(b) Prove that the transformation \( w = \frac{1}{z+i} \) transforms the real axis of the \( z \)-plane into a circle in the \( w \)-plane.

(c) i) Use Stoke's Theorem to evaluate \( \int_C \vec{F} \cdot d\vec{r} \) where
\[
\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}
\]
and \( C \) is the rectangle in the plane \( z=0 \), bounded by \( x=0, y=0, x=a \) and \( y=b \).

ii) Use Gauss Divergence Theorem to evaluate \( \iint_S \vec{F} \cdot d\vec{a} \) where \( \vec{F} = 4xi + 3yj - 2zk \) and \( S \) is the surface bounded by \( x=0, y=0, z=0 \) and \( 2x + 2y + z = 4 \).

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