**Q1**

A) Find Laplace transform of \( f(t) = \sin^2 t \)

B) Prove that \( u = x^2 - y^2 \) is harmonic function also find corresponding analytic function \( f(z) \)

C) Find the half range sine series of \( f(x) = 2x \) in \((0, \pi)\)

D) Find the Unit normal vector to the surfaces \( x^2 + y^2 + z^2 = 9 \) and \( z = x^2 + y^2 - 3 \) at \((2, -1, 2)\) hence find angle between them

**Q2**

A) Prove that \( J_{\nu} (x) = \left[ \frac{2}{\pi x} \right] (\frac{\cos x}{x} + \sin x) \)

B) Find the Bilinear transformation which maps the points \( w = 0, 1, \infty \)

C) Obtain the fourier series for \( f(x) = x \cos x \) in \((-\pi, \pi)\)

**Q3**

A) Find inverse Laplace transform of

(i) \( \log \left( \frac{1 + s^2}{4 + s^2} \right) \)

(ii) \( \frac{s + 5}{(s+4)^3} \)

B) Show that the set of functions \( \{ \cos x, \cos 3x, \cos 5x, \ldots \} \) is an orthogonal over \([0, \pi/2]\). Hence construct orthonormal set of functions.

C) Prove that \( y = \sqrt{x} J_n(x) \) is a solution of the equation,

\[ x^2 \frac{d^2 y}{dx^2} + (x^2 - n^2 + \frac{1}{4})y = 0 \]
Q4
A) Prove that \( \int x^4 J_1(x) \, dx = x^4 J_2(x) - 2x^3 J_3(x) \)

B) Use Gauss’s Divergence theorem to evaluate \( \oint_S \vec{F} \cdot d\vec{S} \) where \( \vec{F} = 4x\hat{i} + 3y\hat{j} + 3z\hat{k} \) and \( S \) is the surface bounded by \( x=0, y=0, z=0 \) and \( 2x + 2y + z = 4 \)

C) Solve using Laplace transform( \( D^2 + 2D + 1)y = 3te^t \) given \( y(0)=4 \) and \( y'(0)=2 \)

Q5
A) Find Fourier series for \( f(x) = \begin{cases} \pi + x, & 0 < x < \pi \\ \pi - x, & -\pi < x < 0 \end{cases} \)

B) Find the image of the region bounded by \( x+y=0, x=y, x+y=1, x-y=1 \) under the bilinear transformation \( w = 2z + 2i \)

C) Prove that \( \vec{F} = (y^2\cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k \) is a conservative field. Find (i) Scalar Potential for \( \vec{F} \) (ii) The work done in moving an object in this field from \( (0, 1, -1) \) to \( (\frac{\pi}{2}, -1, 2) \).

Q6
A) Find the Laplace Transform of \( e^t \int_0^t \sin 3u \cos 2u \, du \)

B) Find Complex form of Fourier Series of \( \sinh 2x \) in \((-2, 2)\)

C) Express the function \( f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases} \) as Fourier integral. Hence evaluate \( \int_0^\infty \frac{\sin w \cos wx}{w} \, dw \)