N.B. (1) Question No.1 is compulsory.

(2) Answer any three questions from remaining.

(3) Figures to the right indicate full marks.

Q1.  
   a) Evaluate \( \int_{0}^{\infty} \frac{\sin 3t + \sin 2t}{te^t} \, dt \).  

   b) Find the directional derivative of the function \( \phi = 4xz^2 + x^2yz \) at \((1, -2, 1)\) in the direction of \( 2\hat{i} - \hat{j} - 2\hat{k} \).

   c) Expand \( f(x) = \pi x - x^2 \) in a half range sine series in the interval \((0, \pi)\).

   d) Show that the function \( u(x, y) = x^3 - 3xy^2 + 3x^2y - 3y^2 + 1 \) is harmonic. Find the corresponding analytic function \( f(z) \).

Q2.  
   a) Prove that \( J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[ \frac{\sin x}{x} - \cos x \right] \).

   b) Find Fourier series to represent \( f(x) = 4 - x^2 \) in the interval \((0, 2)\).

   c) Solve the following differential equation using Laplace transform \( \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + y = 3te^{-t} \), given \( y(0) = 4 \), \( y'(0) = 2 \).

Q3.  
   a) Show that \( \vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k \) is conservative. Find the scalar potential for \( \vec{F} \) and also find the work done by \( \vec{F} \) in moving a particle from \((1,0,1)\) to \((2,1,3)\).

   b) Obtain the complex form of the Fourier series for \( f(x) = e^{3x} \) in \((0,3)\).

   c) Find the Inverse Laplace Transform of
      
      i) \( \frac{8s + 20}{s^3 - 12s + 32} \)
      ii) \( \tan^{-1} \left( \frac{s + a}{b} \right) \).
Q.4
a) Prove that \( \int J_1(x)dx + 2 \frac{J_1(x)}{x} + J_2(x) = 0 \)  

b) Evaluate \( \int_c \left(x^2ydx + x^2dy\right) \) where C is the boundary described in the anti clockwise direction of the triangle with vertices (0,0), (1,0) and (1,1).

c) Find Fourier series expansion of
\[
f(x) = \begin{cases} 
2 & -2 < x < 0 \\
0 & 0 < x < 2 
\end{cases}
\]

Q.5
a) Show that the map of the real axis of the z plane is a circle under the transformation \( w = \frac{2}{z + i} \). Find the centre and radius of the circle.

b) Find the Fourier Integral representation of \( f(x) = 1 \) \(|x| < 1 = 0 \) \(|x| > 1 \) hence evaluate \( \int_0^\infty \frac{\sin \omega \cos \omega x}{\omega} d\omega \)

c) i) Find the Laplace Transform of \( (1 + 2t - t^2 + t^3) H(t - 4) \)

ii) If \( \vec{F} = x^2 z \hat{i} - 2y^3 z^3 \hat{j} + xy^2 z^2 \hat{k} \) find \( \text{div} \vec{F} \) and \( \text{curl} \vec{F} \)

Q.6
a) Use Convolution theorem to find \( L^{-1} \left( \frac{s^2}{(s^2 + 4)^2} \right) \)

b) Use Gauss Divergence Theorem to evaluate \( \iint_S \vec{N} \cdot \vec{F} ds \) where \( \vec{F} = 4x \hat{i} + 3y \hat{j} - 2z \hat{k} \) and S is the surface bounded by \( x=0, y=0, z=0 \) and \( 2x+2y+z=4 \).

c) If \( f(z) = u + iv \) is an analytic function of \( z = x + iy \) and \( u + v = \cos x \cosh y - \sin x \sinh y \) find \( f(z) \) in terms of \( z \)