

- N.B. (1) Question No.1 is compulsory.
 (2) Attempt any three questions out of the remaining five questions.
 (3) Figures to right indicate full marks.

- 1 a) Evaluate $\int_C (\bar{z} + 2z) dz$ along the circle $x^2 + y^2 = 1$ (5)
- b) Evaluate the integral using Laplace Transform $\int_0^{\infty} e^{-t} (t \sqrt{1 - \sin t}) dt$ (5)
- c) Determine the analytic function whose real part is $u = -r^3 \sin 3\theta$. (5)
- d) A rod of length l has its ends A and B kept at $0^\circ C$ and $100^\circ C$ respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to $0^\circ C$ and kept so while that of A is maintained. Find the temperature $u(x, t)$ at a distance from A and at time t . (5)
- 2 a) Find complex form of Fourier series of $f(x) = e^{2x}$ in $(0, 2)$ (6)
- b) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$ (6)
- c) Using Bender Schmidt method solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ subject to the conditions $u(0, t) = 0$, $u(1, t) = 0$, $u(x, 0) = \sin \pi x$, $0 \leq x \leq 1$. Assume $h=0.2$ (8)
- 3 a) Find k such that $\frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}(\frac{kx}{y})$ is analytic (6)
- b) Evaluate $\int_C \frac{1}{(z^3 - 1)^2} dz$ where C is the circle $|z - 1| = 1$ (6)
- c) Show that the set of functions $\left\{ \sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), \dots \right\}$ forms an orthogonal set over the interval $[0, L]$. Construct corresponding orthonormal set. (8)

4 a) Find Laplace Transform of the periodic function (6)

$$f(t) = \begin{cases} \sin 2t, & 0 < t < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < t < \pi \end{cases} \quad f(t) = (t + \pi)$$

b) Find half range sine series for $x \sin x$ in $(0, \pi)$ (6)

c) Expand $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ around $z = 1$ (8)

5 a) Using residue theorem evaluate $\oint_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ where C is $|z| = 4$ (6)

b) Find Fourier expansion of $f(x) = x + x^2$ in $(-\pi, \pi)$ and $f(x + 2\pi) = f(x)$ (6)

c) Find i) $L(e^{-4t} \int_0^t u \sin 3u du)$ ii) $L^{-1}\left(\frac{1}{s} \log\left(1 + \frac{1}{s^2}\right)\right)$ (8)

6 a) Show that the function $w = \frac{4}{z}$ transform the straight lines $x = c$ in the z -plane into circles in the w -plane. (6)

b) Solve using Laplace Transform $R \frac{dQ}{dt} + \frac{Q}{c} = V$, $Q = 0$ when $t = 0$ (6)

c) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for the following data by successive iterations. (Calculate first two iterations) (8)

	0	8.7	12.1	12.8	9.0	
0	u_7	u_8	u_9			17.0
0	u_4	u_5	u_6			21.0
0	u_1	u_2	u_3			21.9
0	11.1	17.0	19.7	18.6		