

(3 Hours)

[Total Marks : 80]

- N.B. :** (1) Question No. 1 is **Compulsory**.  
 (2) **Answer** any **Three** from remaining.  
 (3) **Figures** to the **right** indicate **marks**.

1. (a) Find laplace of  $\sin \sqrt{t}$  5
- (b) Show that the set of functions  $\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right)$  is orthogonal over  $(0, L)$ . 5
- (c) Show that  $u = \sin x \cos hy + 2 \cos x \sin hy + x^2 - y^2 + 4xy$  Satisfies laplace equation and find its corresponding analytic function  $f(z) = u + iv$ . 5
- (d) Determine constants  $a, b, c, d$  if  $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$  is analytic. 5
2. (a) Find complex form of fourier series  $f(x) = e^{3x}$  in  $0 < x < 3$ . 6
- (b) Using Crank Nicholson Method solve  $u_t = u_{xx}$  subject to  $u(x, 0) = 0$   $u(0, t) = 0$  and  $u(l, t) = t$  for two time steps. 6
- (c) Solve using laplace transforms  $\frac{d^2 y}{dt^2} + y = t, y(0) = 1, y^1(0) = 0$ . 8
3. (a) Find bilinear transformation that maps the points  $0, 1 - \infty$  of the  $z$  plane into  $-5, -1, 3$  of  $w$  plane. 6
- (b) By using Convolution Theorem find inverse laplace transform of  $\frac{1}{(S^2 + 4S + 13)^2}$  6
- (c) Find fourier series of  $f(x) = x^2 - \pi \leq x \leq \pi$  and prove that 8
- (i)  $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
- (ii)  $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$
- (iii)  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
4. (a) Evaluate  $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$  6
- (b) Solve  $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$  by 6

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Bender schmidt method subject to conditions  $u(0, t) = 0$   $u(x, 0) = 0$   
 $u(l, t) = t$  taking  $h = 0.25$   $0 < x < 1$

- (c) Obtain two distinct Laurent's Series for  $f(z) = \frac{2z-3}{z^2-4z-3}$  in Powers of  $(z-4)$  indicating Region of Convergence. 8
5. (a) Evaluate  $\int_0^{1+i} Z^2 dZ$  along 6
- (i) line  $y = x$   
(ii) Parabola  $x = y^2$   
Is line independent of path? Explain.
- (b) Find half range Cosine Series for  $f(x) = e^x$   $0 < x < 1$ . 6
- (c) Find analytic function 8  
 $f(z) = u + iv$  such that
- $$u-v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{-y}}$$
- when  $f(\pi/2) = 0$
6. (a) A tightly stretched string with fixed end points  $x = 0$  and  $x = \ell$  in the shape defined by  $y = Kx(l-x)$  where  $K$  is a Constant is released from this position of rest. Find  $y(x,t)$  The vertical displacement 6
- if  $\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$
- (b) Find image of region bounded by  $x = 0$ ,  $x = 2$   $y = 0$   $y = 2$  in the  $z$  plane under the transformation  $w = (1+i)Z$  6
- (c) Evaluate  $\int_0^{2\pi} \frac{d\theta}{25-16\cos^2\theta}$  8
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