N.B. 1) Question No. 1 is compulsory.
   2) Answer any Three from remaining
   3) Figures to the right indicate full marks

1. a) Find Laplace transform of \( f(t) = \int_0^t u e^{-3u} \sin u \, du \).

   b) Show that the set of functions \( \{ \cos nx, n = 1, 2, 3 \ldots \} \) is orthogonal on \((0, 2\pi)\).

   c) Does there exist an analytic function whose real part is \( u = k(1 + \cos \theta) \)? Give justification.

   d) The equations of lines of regression are \( x + 6y = 6 \) and \( 3x + 2y = 10 \). Find
   i) means of \( x \) and \( y \), ii) coefficient of correlation between \( x \) and \( y \).

2. a) Evaluate \( \int_0^\infty e^{-t} \frac{\sin^2 t}{t} \, dt \).

   b) Find the image of the triangle bounded by lines \( x = 0, y = 0, x + y = 1 \) in the \( z \)-plane under the transformation \( w = e^{i\pi/4}z \).

   c) Obtain Fourier series of \( f(x) = x^2 \) in \((0, 2\pi)\). Hence, deduce that
   \[ \frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \]

3. a) Find the inverse Laplace transform of \( F(s) = \frac{s}{(s^2 + 4)^2} \).

   b) Solve \( \frac{\partial^2 u}{\partial x^2} - 100 \frac{\partial u}{\partial t} = 0 \), with \( u(0, t) = 0, u(1, t) = 0, u(x, 0) = x(1 - x) \)
   taking \( h = 0.1 \) for three time steps up to \( t = 1.5 \) by Bender –Schmidt method.

   c) Using Residue theorem, evaluate
   \[ \iint \frac{d\theta}{5 - 4\cos \theta} \quad \int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^2} \]
4. a) Solve by Crank –Nicholson simplified formula \( \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0 \),

\[ u(0, t) = 0, \ u(5, t) = 100, \ u(x, 0) = 20 \text{ taking } h = 1 \text{ for one-time step.} \]

b) Obtain the Taylor’s and Laurent series which represent the function

\[ f(z) = \frac{z-1}{z^2-2z-3} \text{ in the regions, i) } |z| < 1 \text{ ii) } 1 < |z| < 3 \]

5. a) Find an analytic function \( f(z) = u + iv \), if

\[ u = e^{-x} \{(x^2 - y^2) \cos y + 2xy \sin y\} \]

b) Find the Laplace transform of

\[ f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases} \text{ and } f(t + 2) = f(t) \text{ for } t > 0. \]

6. a) If \( f(a) = \int_C \frac{4z^2 + z + 4}{z-a} \, dz \) where \( C \) is the ellipse \( 4x^2 + 9y^2 = 36 \).

Find, i) \( f(4) \) ii) \( f'(-1) \) and iii) \( f''(-i) \)

b) Use least square regression to fit a straight line to the following data,

<table>
<thead>
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<th>x</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>24</td>
<td>31</td>
<td>33</td>
<td>37</td>
<td>37</td>
<td>40</td>
<td>40</td>
<td>42</td>
<td>41</td>
</tr>
</tbody>
</table>

c) A string is stretched and fastened to two points distance \( l \) apart. Motion is started by displacing the string in form \( y = asin(\pi x / l) \) from which it is released at a time \( t = 0 \). If the vibrations of a string is given by \( \frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \), show that the displacement of a point at a distance \( x \) from one end at time \( t \) is given by

\[ y(x, t) = a \sin(\pi x / l) \cos(\pi ct / l). \]