Q.P. Code :11708

[Time: 3 Hours] **[Marks:80]**

Please check whether you have got the right question paper.

N.B:
1. Question No.ONE is compulsory
2. Solve any THREE Questions out of remaining FIVE
3. Figures to the right indicate full marks.
4. Write the sub – Questions of main question collectively together.

Q.1 a) Solve the PDE $\frac{\partial z}{\partial z} = 2\frac{\partial z}{\partial z} + 2$ GIven $z(x,0) = 6e^{-3x}$ **05**

b) Evalute $\int_A^B (y^2\,dx + xydy)$ along $y = 2t, x = t^2$ where $A(1,-2)$ to $B(0,0)$ **05**

c) If $f(x) = C_1 g_1(x) + C_2(x) + C_3(x)$, where C_1, C_2, C_3 are constants and $g_1\ g_2\ g_3$ are orthomormal sets on (a,b)then show that $\int_A^B f(x^2)dx = c_1^2 + c_2^2 + c_3^2$ **05**

d) Find the fourier transform of $f(x) = x \quad 0 < x < 1$ **05**

$= 2 - x, \quad 1 < x < 2$

$= 0, \quad x \geq 2$

Q.2 a) Obtain the fourier expansaion of **06**

$f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 < x < 2\pi$

b) A tightly stretched string with fixed end points x = 0, & x = L in the shape defined by y = Kx (L-x) where k is a constant is released from position of rest. Find y. **06**

c) Prove that $\bar{F} = (y^2\cos + z^3)I + (2y\sin x - 4)j + (3xz^2 + 2)k$ is a conservaive field. Find (i) scaler potentiel for $\vec{F}$ (ii)the work done in moving an object in this field from (0,1,-1) to $(\frac{\pi}{2}, -1, 2)$. **08**

Q.3 a) Obtain the complex form of fourier series for **06**

$f(x) = \cosh x + \sinh x$ in $(-1,1)$.

b) Obtain half range sine series for f(x) when **06**

$F(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$ Hence find the sum of $\sum_{(2n-1)}^{\infty} \frac{1}{n4}$.

c) Verify Green's Theorem in the plane for $\int_c (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ & $y = x^2$. **08**

Q.4 a) Find the fourier interier representation of **06**

$f(x) = e^{ax} \; x \leq 0$

$\quad = e^{-ax} \; x \geq 0$

b) Show thet the set of functions $\sin x, \sin 2x, \sin 3x \ldots\ldots$ is orthogonal over $(0,\pi)$. **06**

c) A rod of length 30cm has its ends A and B kept at 20^0V & 80^0C respectively until steady state condition prevail. The temperature at each end is then suddenly reduced to 0^0 C & kept so. Find the resulting temperature function $u(x,t)$ taking $x = 0$ at A. **08**

Q.5 a) Find the fourier expansion of $f(x) = 4 - x^2, \; -2 \leq x \leq 2$ **06**

b) A rectangular metal plate with insulated surfaces is of width a and so long as compared to its breadth that it can be considered infinite in length without introducing an appreciable error if the temperature along short edge is y=o given by $u(x, 0) = u \sin\frac{\pi x}{a}$ for $0 < x < a$ & other long edges x=0 & x=a & the short edges are kept at zero degree temperature, find the function u(x,y) describing the steady **06**

c) Find Fourier series for f(x)= $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$ **08**

Q.6 a) Find fourier sin Transform oF $f(x) = \frac{e^{-ax}}{x}$ **06**

b) Solve $\int_c \overline{F}.\, d\overline{r}$ by stockes thoerem for $\overline{F} = yi + zj + xk$ over the surface **06**

$$x^2 + y^2 = 1 - z > 0.$$

c) Dinf the Fourier sreries for f(x)= $\begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ **08**
