CIRCULAR:-

A reference is invited to the syllabi relating to the B.A./B.Sc Degree Course vide this office Circular No.UG/05 of 2012, dated 15\textsuperscript{th} February, 2012 and the Principals of the affiliated Colleges in Arts & Science and The Professor-cum-Director, Institute of Distance and Open Learning (IDOL) are hereby informed that the recommendation made by Board of Studies in Mathematics, at its meeting held on 27\textsuperscript{th} March, 2017 has been accepted by the Academic Council at its meeting held on 11\textsuperscript{th} May, 2017 vide item 4.194 and that in accordance therewith, the revised syllabus as per the (CBCS) for the T.Y.B.A./T.Y.B.Sc Mathematics (Sem-V & VI) which is available on the University's website (www.mu.ac.in) and that the same has been brought into force with effect from the academic year 2017-18.

MUMBAI– 400032
27\textsuperscript{th} July, 2017

To

The Principals of the affiliated Colleges in Arts & Science.

A.C/4.194/11/05/2017

MUMBAI-400 032
26\textsuperscript{th} July, 2017

Copy forwarded with Compliments for information to:-
1) The Co-ordinator, Faculty of Arts & Science.
2) The Offg. Director, Board of Examinations and Evaluation.
3) The Chairman/Chairperson, Board of Studies in Mathematics.
4) The Director, Board of Studies Development.
5) The Co-Ordinator, University Computerization Centre.

REGISTRAR

....PTO
UNIVERSITY OF MUMBAI

Syllabus
for
T.Y.B.A./B.Sc. (CBCS)
Program: B.A/B.Sc.
Course: Mathematics
with effect from the academic year 2018-2019
### T.Y.B.A./T.Y.B.Sc. (CBCS)
#### Semester V

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# T.Y.B.A./T.Y.B.Sc. (CBCS)
## Semester VI

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### Algebra V

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### Metric Topology

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<td>Complete Metric Spaces</td>
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### Numerical Analysis-II (Elective A)

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### Number Theory and its Applications-II (Elective B)

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### Graph Theory and Combinatorics (Elective C)

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<td>Unit II</td>
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### Operations Research (Elective D)

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<td>Unit II</td>
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<tr>
<td>USMT6PJ6, UAMTPJ6</td>
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Note:
1. USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.
2. Candidate has to opt one Elective Course from USMT5A4/UAMT5A4, USMT5B4/UAMT5B4, USMT5C4/UAMT5C4 and USMT5D4/UAMT5D4 for Semester V.
3. USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.
4. Candidate has to opt one Elective Course from USMT6A4/UAMT6A4, USMT6B4/UAMT6B4, USMT6C4/UAMT6C4 and USMT6D4/UAMT6D4 for Semester VI.
5. Passing in theory and practical shall be separate in the compulsory courses.
6. Candidate has to do a project in the courses USMT5PR/UAMT5PR of Semester V and USMT6PR/UAMT6PR of Semester VI.

Teaching Pattern for SY B.A./B.Sc:
1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).
3. Each project for the courses USMT5PR/UAMT5PR in Semester V and USMT6PR/UAMT6PR in Semester VI shall have at most 08 (eight) students and the workload for each project is 1L/W. However a teacher guiding more than one project gets 1L/W workload, irrespective of the number of projects he/she guides.

Syllabus for Semester V & VI

SEMESTER V

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

USMT501/UAMT501 Discrete Mathematics

Unit I: Preliminary Counting (15 Lectures)
1. Finite and infinite sets, Countable and uncountable sets, examples such as $\mathbb{N}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$ and open interval $(0, 1)$.
2. Addition and multiplication principle, Counting sets of pairs, two ways counting.
3. Stirling numbers of second kind, Simple recursion formulae satisfied by $S(n, k)$
and direct formulae for \( S(n, k) \) for \( k = 0, 1, \ldots, n - 1 \).

4. Pigeon hole principle and its strong form, its applications to geometry, monotonic sequences.


Unit II: Advanced Counting (15 Lectures)

Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs: \[ \sum_{i=0}^{n} \binom{n}{i} = 2^n \] and \[ \sum_{k=0}^{r} \binom{m}{k} \binom{m+n}{r-k} = \binom{m+n}{r}, \sum_{k=r}^{n} \binom{k}{r} = \binom{n+1}{r+1}, \sum_{i=0}^{k} \binom{k}{i} = \binom{n+1}{r+1}. \]

Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems. Non-negative and positive integral solutions of equation \( x_1 + x_2 + \cdots + x_k = n \).

Principle of Inclusion and Exclusion and its applications, derangements, explicit formula for \( d_n \), various identities involving \( d_n \).

Unit III: Permutations (15 Lectures)

1. Permutation of objects, composition of permutations, results such as every permutation is product of disjoint cycles, every cycle is product of transpositions, even and odd permutations, \( S_n, A_n \), rank and signature of permutation, results such as\[ \epsilon(\sigma \circ \eta) = \epsilon(\sigma)\epsilon(\eta) \ (\sigma, \eta \in S_n), \ \epsilon(\sigma) = \prod_{1 \leq i < j \leq n} \frac{\sigma(i) - \sigma(j)}{i - j}. \]

2. Partially ordered sets, Mobius Inversion Formula with application to deriving the formula for Eulers phi-function \( \varphi(n) \).

3. Recurrence relation, definition of homogeneous, non-homogeneous, linear and non linear recurrence relation, obtaining recurrence relation in counting problems, solving (homogeneous as well as non-homogeneous) recurrence relation by using iterative method, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.


Recommended Text Book:


(Sections 2.1, 2.2, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 5.1, 5.2, 5.3, 6.1, 6.2, 6.3, 6.6, 7.1, 7.2, 7.3, 7.4, 7.5, 7.6, 8.2.)

Additional Reference Books:

USMT502/UAMT502  Algebra IV

Unit I: Quotient Spaces and Orthogonal Linear Transformations (15 Lectures)
Review of vector spaces over \( \mathbb{R} \), subspaces and linear transformations.

Quotient Spaces: For a real vector space \( V \) and a subspace \( W \), the cosets \( v + W \) and the quotient space \( V/W \). First Isomorphism theorem for real vector spaces (Fundamental theorem of homomorphism of vector spaces), dimension and basis of the quotient space \( V/W \) when \( V \) is finite dimensional.

Inner product spaces: Examples of inner product including the inner product \( \langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)\,dx \) on \( \mathcal{C}[-\pi, \pi] \), the space of continuous real valued functions on \( [-\pi, \pi] \). Orthogonal sets and orthonormal sets in an inner product space. Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process and simple examples in \( \mathbb{R}^3, \mathbb{R}^4 \).

Real Orthogonal transformations and isometries of \( \mathbb{R}^n \). Translations and Reflections with respect to a hyperplane. Orthogonal matrices over \( \mathbb{R} \).

Equivalence of orthogonal transformations and isometries of \( \mathbb{R}^n \) fixing the origin. Characterization of isometries as composites of orthogonal transformations and translations.

Orthogonal transformation of \( \mathbb{R}^2 \). Any orthogonal transformation in \( \mathbb{R}^2 \) is a reflection or a rotation.

Unit II: Eigenvalues, Eigenvectors (15 Lectures)
Eigenvalues and eigenvectors of a linear transformation \( T : V \longrightarrow V \) where \( V \) is a finite dimensional real vector space and examples, eigenvalues and eigenvectors of \( n \times n \)-real matrices, linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation / Matrix.

Characteristic polynomial of an \( n \times n \)-real matrix. Result: A real number \( \lambda \) is an eigenvalue of an \( n \times n \) matrix \( A \) if and only if \( \lambda \) is a root of the characteristic polynomial of \( A \). Cayley-Hamilton Theorem (statement only), Characteristic roots. Similar matrices and relation with a change of basis. Invariance of the characteristic polynomial and (hence of the) eigenvalues of similar matrices.

Reference for Unit II: Sections 1, 2, 3 of Chapter VIII of Introduction to Linear Algebra (Second Edition) by SERGE LANG.

Recommended Text Books:


**Unit III: Diagonalisation (15 Lectures)**
Diagonalizability of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself. Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an $n \times n$ real matrix and of a linear transformation. Examples of non-diagonalisable matrices over $\mathbb{R}$.

An $n \times n$ real matrix $A$ is diagonalisable if and only if $\mathbb{R}^n$ has a basis of eigenvectors of $A$ if and only if the sum of dimension of eigen spaces of $A$ is $n$ if and only if the algebraic and geometric multiplicities of eigenvalues of $A$ coincide.

Diagonalisation of real Symmetric matrices and applications to real quadratic forms, rank and signature of a real quadratic form, classification of conics in $\mathbb{R}^2$ and quadric surfaces in $\mathbb{R}^3$.

**Recommended Text Books for Unit I & unit II:**

**Additional Reference books:**

**USMT503/UAMT503 Topology of Metric Spaces**
Note: In this course, definitions of *closed set* in a metric space, *limit point* and *closure* of a subset of metric space shall be used as indicated below in Unit II.

**Unit I: Metric spaces (15 Lectures)**
Definition of metric space, Euclidean space $\mathbb{R}^n$ with its Euclidean norm function and the distance metric induced by it, sup metric and sum metric on $\mathbb{R}^n$ and $\mathbb{C}$ (complex numbers). Discrete metric spaces and examples such as $\mathbb{Z}$.

Normed linear spaces: Definition, the distance (metric) induced by the norm, translation invariance of the metric induced by the norm. Examples of normed linear spaces including
1. \( \mathbb{R}^n \) with sum norm \( \| \cdot \|_1 \), the Euclidean norm \( \| \cdot \|_2 \), and the sup norm \( \| \cdot \|_\infty \).

2. \( C[a,b] \), the space of continuous real valued functions on \([a,b]\) with norms \( \| \cdot \|_1 \), \( \| \cdot \|_2 \), and \( \| \cdot \|_\infty \) where \( \| f \|_1 = \int_a^b |f(t)| \, dt \), \( \| f \|_2 = \left( \int_a^b |f(t)|^2 \, dt \right)^{1/2} \), \( \| f \|_\infty = \sup \{ |f(t)| : t \in [a,b] \} \).

Open balls, open subsets of a metric space. Verification of the result: any open ball of a metric space is an open subset of the metric space. Examples of open sets in various metric spaces, structure of an open set in \( \mathbb{R} \).

Properties of open subsets of a metric space: the intersection of finitely many open subsets of a metric space is an open subset of the metric space, the union of arbitrary collection of open subsets of a metric space is an open subset of the metric space. Interior of a subset of a Metric space. Hausdorff property of a metric space. Subspaces of a Metric space. Product of two metric spaces. Equivalent metrics.

Distance of a point from a set, distance between two sets, diameter of a set in a metric space.

Unit II: Sequences, closed sets, limit Points (15 Lectures)

Sequences in a metric space, convergent sequences and Cauchy sequences in a metric space, subsequence of a sequence, examples.

Closed set in a metric space (as complement of an open set), limit point of a set (If \( A \) is subset of a metric space \( X \), \( x \in X \) is a limit point of \( A \) if each open ball of \( X \) with center at \( x \) contains a point of \( A \) other than \( x \)), isolated point. A closed set contains all its limit points. Closed balls. Closure of a subset of a metric space (closure \( \overline{E} \) of a subset \( E \) of a metric space is \( E \cup E' \) where \( E' \) denotes the set of all limit points of \( E \) in \( X \)) and properties: If \( X \) is a metric space and \( E \subseteq X \). Then

1. \( \overline{E} \) is closed in \( X \).
2. \( E = \overline{E} \) if and only if \( E \) is closed in \( X \).
3. \( \overline{E} \subseteq F \) for every closed subset \( F \) of \( X \) such that \( E \subseteq F \).
4. \( \overline{E} \) equals the intersection of all the closed superset of \( E \) in \( X \).

Boundary of a set in a metric space. Complete metric spaces.

Unit III: Continuity (15 Lectures)

\( \varepsilon, \delta \) definition of continuity at a point for a function from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets.

Continuity of a function on a metric space. Characterization of continuity of a function in terms of inverse image of open sets and closed sets. Algebra of continuous real valued functions. Uniform continuity of a function defined on a metric space: definition and examples (emphasis on \( \mathbb{R} \)).

Recommended Text Books:


**Additional Reference Books:**

5. D. Somasundaram, B. Choudhary, *A first Course in Mathematical Analysis*.

**USMT5A4/UAMT5A4 Numerical Analysis I (Elective A)**

Note: Derivations and geometrical interpretation of all numerical methods have to be covered.

**Unit I: Errors Analysis, Transcendental and Polynomial Equations**

15 Lectures:


**Unit II: Transcendental and Polynomial Equations**

15 Lectures


**Unit III: Linear System of Equations**

15 Lectures


**Recommended Text Books:**

USMT5B4/UAMT5B4
Number Theory and its applications I (Elective B)

Unit I. Congruences and Factorization (15 Lectures)
Review of Divisibility, Primes and The fundamental theorem of Arithmetic.

Congruences : Definition and elementary properties, Complete residue system modulo \( m \), Reduced residue system modulo \( m \), Euler’s function \( \phi(n) \) and its properties, Fermat’s little Theorem, Euler’s generalization of Fermat’s little Theorem, Wilson’s theorem, Linear congruence, The Chinese remainder Theorem, Congruences of higher degree, The Fermat-Kraitchik Factorization Method.


Unit II: Diophantine equations and their solutions (15 Lectures)
The linear equations \( ax + by = c \). The equations \( x^2 + y^2 = p \) where \( p \) is a prime. The equation \( x^2 + y^2 = z^2 \), Pythagorean triples, primitive solutions, the equations \( x^4 + y^4 = z^2 \) and \( x^4 + y^4 = z^4 \) have no solutions \((x; y; z)\) with \( xyz \neq 0 \). Every positive integer \( n \) can be expressed as sum of squares of four integers, Universal quadratic form \( x_1^2 + x_2^2 + x_3^2 + x_4^2 \).


Unit III: Primitive Roots and Cryptography (15 Lectures)
Order of an integer and Primitive Roots. Basic notions such as encryption (en-ciphering) and decryption (deciphering), Crypto-systems, symmetric key cryptography, simple examples such as shift cipher, Affine cipher, Hill’s cipher, Vigenere cipher. Concept of Public Key Crypto-system; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference for Unit III: Elementary number theory, David M. Burton, Chapter 8 sections 8.1, 8.2 and 8.3, Chapter 10, sections 10.1, 10.2 and 10.3.

Additional Reference Books:
2. NEVILLE ROBINS, Beginning Number Theory, Narosa Publications.

**USMT5C4/UAMT5C4**  Graph Theory (Elective C)

**Unit I: Basics of Graphs** (15 Lectures)
Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs, Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problems.

**Unit II: Trees** (15 Lectures)
Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of $K_n$, Binary and $m$—ary tree, Weighted graphs and minimal spanning trees.

**Unit III. Eulerian and Hamiltonian graphs** (15 Lectures)
Eulerian graph and its characterization Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G - S$ where $S$ is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs- Ore’s theorem and Dirac’s theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.

**Recommended Text Book:**
J.A. BONDY AND U.S.R. MURTY, Graph Theory with Applications, Elsevier.

**Additional Reference books:**
1. R. BALAKRISHNAN AND K. RANGANATHAN, A Textbook of Graph Theory, Springer.
2. BEHZAD AND CHARTLAND, Graph Theory.
3. CHOUDAM S.A., Introduction to Graph Theory.
4. WEST D.G., Graph Theory. Allyn and Bacon.
USMT5D4/UAMT5D4

Basic Concepts of Probability and Random Variables
(Elective D)

Unit I: Basic Concepts of Probability and Random Variables

Basic Concepts: Algebra of events including countable unions and intersections, Sigma field \( \mathcal{F} \), Probability measure \( P \) on \( \mathcal{F} \), Probability Space as a triple \( (\Omega, \mathcal{F}, P) \), Properties of \( P \) including Sub-additivity.

Discrete Probability Space, Independence and Conditional Probability, Theorem of Total Probability. Random Variable on \( (\Omega, \mathcal{F}, P) \) definition as a measurable function, Classification of random variables - Discrete Random variable, Probability function, Distribution function, Density function and Probability measure on Borel subsets of \( \mathbb{R} \), Absolutely continuous random variable. Function of a random variable; Result on a random variable \( R \) with distribution function \( F \) to be absolutely continuous, Assume \( F \) is continuous everywhere and has a continuous derivative at all points except possibly at finite number of points, Result on density function \( f_2 \) of \( R_2 \) where \( R_2 = g(R_1) \), \( h_j \) is inverse of \( g \) over a suitable subinterval \( f_2(y) = \sum_{i=1}^{n} f_1(h_j(y))|h_j'(y)| \) under suitable conditions.

Reference for Unit 1, sections 1.1-1.6, 2.1-2.5 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit II: Properties of Distribution function, Joint Density function (15 lectures)

Properties of distribution function \( F \), \( F \) is non-decreasing, \( \lim_{x \to -\infty} F(x) = 0 \), \( \lim_{x \to \infty} F(x) = 1 \), Right continuity of \( F \), \( \lim_{x \to x_0} F(x) = P(\{R < x_0\}) \), \( P(\{R = x_0\}) = F(x_0) - F(x_0^-) \).

Joint distribution, Joint Density, Results on Relationship between Joint and Individual densities, Related result for Independent random variables. Examples of distributions like Binomial, Poisson and Normal distribution. Expectation and \( k \)-th moments of a random variable with properties.


Unit III: Weak Law of Large Numbers (15 lectures)

Joint Moments, Joint Central Moments, Schwarz Inequality, Bounds on Correlation Coefficient \( \rho \), Result on \( \rho \) as a measure of linear dependence, \( \text{Var}(\sum_{i=1}^{n} R_i) = \sum_{i=1}^{n} \text{var}(R_i) + 2 \sum_{1 \leq i < j \leq n} \text{Cov}(R_i, R_j) \), Method of Indicators to find expectation of a random variable, Chebyshevs Inequality, Weak law of Large numbers.


Additional Reference Books:
A. Practicals for USMT501/UAMT501:

1. Problems based on counting principles, Two way counting.
2. Stirling numbers of second kind, Pigeon hole principle.
5. Derangement and rank signature of permutation.
6. Recurrence relation.
7. Miscellaneous Theoretical Questions based on full paper.

B. Practicals for USMT502/UAMT502:

1. Quotient spaces.
2. Orthogonal transformations, Isometries.
3. Eigenvalues, eigenvectors of $n \times n$ matrices over $\mathbb{R}, \mathbb{C}$ ($n = 2, 3$).
4. Diagonalization.
5. Orthogonal diagonalization.
6. Miscellaneous Theoretical Questions based on full paper.

C. Practicals for USMT503/UAMT503:

2. Open balls, open sets in metric spaces, subspaces and normed linear spaces.
3. Limit points and closure points, closed balls, closed sets, closure of a set, boundary of a set, distance between two sets.
5. Continuity.
6. Uniform continuity in a metric space.
7. Miscellaneous Theoretical Questions based on full paper.
USMTPJ5, UAMTPJ5: Projects
A student can submit a project which shall have 20-30 typed pages, on one of the following topics:

1. Computer implementation of rational numbers in python or C++:
   R.G. Dromey, *How to Solve it by Compute*, Pearson Education.

2. Various Sorting Algorithms like merge sort, insertion sort, quick sort, heap sort, bucket sort, radix sort:
   R.G. Dromey, *How to Solve it by Computer*, Pearson Education.

3. Algorithms: Integer knapsack problem, fractional knapsack problem, backtracking algorithm for the n-queens problem:
   - R.G. Dromey, *How to Solve it by Computer*, Pearson Education.

4. Normalization in databases:

5. Vector Fields, Integral curves, Phase flows in the plane:
   V.I. Arnold, *Ordinary Differential Equations*, PHI.

6. Eigenvalues, Eigenfunctions of the vibrating string and Applications to the Heat Equation, Dirichlet problem for the circle:
   G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.

7. Bessel Functions and the vibrating membrane:
   G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.

8. Sturm-Liouville Boundary value problems, Eigenvalues, Eigenfunctions:
   G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.

9. Continued Fractions and applications to irrational numbers:

10. Distribution of primes:


12. Transcendental and algebraic numbers, Transcendence of $e$, Irrationality of $\pi$ and $e$:

13. The real numbers-a survey of constructions:
    https://arxiv.org/pdf/1506.03467
14. Fourier series of circular functions and applications to Series of real numbers: 

15. Fourier series, Orthogonal Functions, Dirichlet’s problem: 
   G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGRAW-Hill International.

16. Pointwise convergence of Fourier series, the Gibbs Phenomenon: 

17. Cesaro summability and Fejer’s theorem: 

18. Construction of everywhere continuous but no-where differentiable functions: 

19. Study of uniform convergence of various sequences of real valued continuous functions and plotting the functions of the sequences. 

20. Henstock Kurzweil integration: 

21. Symmetric matrices, Spectral theorem, quadratic forms in \( n \)-variables: 

22. Classification of Isometries of \( \mathbb{R}^2 \): 

23. Discrete subgroups of isometries of the plane: 

24. Surface integrals, Line integrals, Theorem on Curl, Divergence theorem of Gauss: 

25. Parametrised regular surfaces in \( \mathbb{R}^3 \), tangent spaces, Orientable surfaces: 

26. Applications of WX-Maxima plot graphs of surfaces, tangent vectors, level sets of real valued functions \( f(x, y, z) \).

27. Circular Permutations, Study of Sterling numbers of First Kind: 


29. Recurrence relations and applications: 
30. Forbidden position problems:

31. Applications of Pigeon Hole Principle:

32. Basic Logic, Poset and Lattices:

33. Boolean algebra (Lattices and Algebraic Systems):

34. Algorithms in Cryptography:

35. Berge Vieta and Bairstow Method, proofs and programming implementation:

36. Jordan Rational Form, Algorithmic proofs and computations:

37. Homogeneous coordinates, transformations and computer geometry:

38. Bezier curves, B-splines implementation and definition:

39. Number systems in various bases:

40. Financial Mathematics (Theory of interest rates and Discounted cash flow):

41. Financial Mathematics (Valuation of securities, Cumulative Sinking Funds):

42. Mathematical Economics (Demand and Supply Analysis, Cost and Revenue Functions, Theory of Consumer Behaviour):

43. Basic Statistics (Correlation and Regression):

44. Implementing Statistical methods using R:

45. Social Network Analysis:

46. Basics of R programming:

47. Topics in Data Sciences:

**SEMESTER VI**

**USMT601/UAMT601 Real and Complex Analysis**

**Unit I: Sequence and series of functions (15 Lectures)**
Sequence of real valued functions, pointwise and uniform convergence of sequences of real-valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true.


Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence
of functions converge to the integral and derivative of uniform limit on a closed and bounded interval, examples. Consequences of these properties for series of functions, term by term differentiation and integration.

\[ \liminf_{n \to \infty} x_n \quad \text{and} \quad \limsup_{n \to \infty} x_n \text{ for a bounded sequence } (x_n)_{n \in \mathbb{N}} \text{ of } \mathbb{R}. \]

Properties of \( \limsup_{n \to \infty} x_n =: x^* \):

1. \( \exists \) a subsequence \( (x_{n_k})_{k \in \mathbb{N}} \) of the sequence \( (x_n) \) such that \( x_{n_k} \to x^* \).
2. If \( x > x^* \), then \( \exists n_0 \in \mathbb{N} \) such that \( x_n \leq x \forall n \geq n_0 \).

Power series in \( \mathbb{R} \) centered at origin and at some point \( x_0 \) in \( \mathbb{R} \), radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

**Reference for Unit I:**


**Unit II: Introduction to Complex Analysis (15 Lectures)**

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivres formula, \( \mathbb{C} \) as a metric space, bounded and unbounded subsets of \( \mathbb{C} \), point at infinity and the extended complex plane, sketching of set in complex plane. (No question be asked).

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions \( f: \mathbb{C} \to \mathbb{C} \), real and imaginary part of functions, continuity at a point and algebra of continuous functions.

Derivative of \( f: \mathbb{C} \to \mathbb{C} \), comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic functions. If \( f, g \) are complex analytic then \( f + g, f - g, fg \) and \( f/g \) are analytic.

Theorem: If \( f'(z) = 0 \) everywhere in a domain \( D \), then \( f(z) \) must be constant throughout \( D \). Harmonic functions and harmonic conjugate.

**Reference for Unit II:**


**Unit III: Complex power series (15 Lectures)**

Contour integral \( \int_C f(z)dz \) over a contour \( C \), the contour integral \( \int_C f(z)dz \) where \( C \) is the circle \( |z - z_0| = r \) in \( \mathbb{C} \).

Cauchy-Gursat theorem (statement only).

Principle of deformation of paths (statement only): Let \( C_1, C_2 \) denote positively
oriented circles where $C_2$ is interior to $C_1$. If a function $f$ is analytic in the closed region consisting of those contours and all points between them, then $\int_{C_1} f \, dz = \int_{C_2} f \, dz$.

Cauchy integral formula (with proof): If $f : \mathbb{C} \rightarrow \mathbb{C}$ is an analytic function, then $f(w) = \frac{1}{2\pi i} \int_{C} \frac{f(z) \, dz}{z-w}$ ($w \in B(z_0, r)$ where $C$ is the circle $|z-z_0|=r$ taken in the positive sense.

Taylors theorem (with proof) for an analytic function.

Mobius transformations, examples.

Exponential function and its properties (without proof), trigonometric functions, hyperbolic functions.

Power series of complex numbers and related results following from Unit I, radius of convergence of a power series, disc of convergence of a power series, uniqueness of series representation, examples.

Definition of Laurent series, definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, Cauchy’s residue theorem (statement only), calculation of residue.

Reference for Unit III:

Additional Reference Books:
3. T. W. Gamelin, *Complex analysis*.
5. W. Fleming, *Functions of Several Variables*.

**USMT602/UAMT602 Algebra VI**

Unit I: Normal Subgroups (15 Lectures)
Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite
groups, Cyclic groups, The Center $Z(G)$ of a group $G$, Cosets, Lagranges theorem, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms.

Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group $A_n$, Cycles. List of all normal subgroups of $A_4, S_3$.

First Isomorphism theorem (Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem.

Cayley’s theorem (statement only). External direct product of a group, properties of external direct products, order of an element in a direct product, criterion for direct product to be cyclic. The classification of groups of order upto 7.

References for unit I:

Unit II: Ring Theory (15 Lectures)
Definition of a ring (the definition should include the existence of a unity element). Properties and examples of rings including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, M_n(\mathbb{R}), \mathbb{Q}[X], \mathbb{R}[X], \mathbb{C}[X], \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{5}], \mathbb{Z}_n$.

Commutative rings. Units in a ring. The multiplicative group of units of a ring.

Characteristic of a ring.

Ring homomorphisms. First Isomorphism theorem of rings.

Ideals in a ring, sum and product of ideals in a commutative ring.

Quotient rings. Integral domains and fields. Definition and examples. A finite integral domain is a field. Characteristic of an integral domain, and of a finite field. Construction of quotient field of an integral domain (emphasis on $\mathbb{Z}, \mathbb{Q}$). A field contains a subfield isomorphic to $\mathbb{Z}_p$ or $\mathbb{Q}$.

References for Unit II:

Unit III: Factorisation (15 Lectures)
Prime ideals and maximal ideals. Definition and examples. Characterization in terms of quotient rings.

Polynomial rings. Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field (statement only). Divisibility in an integral domain, irreducible and prime elements, ideals generated by prime and irreducible elements.

Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique
Factor-ization Domain (UFD). Examples of ED including \( \mathbb{Z}, F[X] \) where \( F \) is a field, and \( \mathbb{Z}[i] \). An ED is a PID, a PID is a UFD.

Prime (irreducible) elements in \( \mathbb{R}[X], \mathbb{Q}[X], \mathbb{Z}_p[X] \). Prime and maximal ideals in polynomial rings. \( \mathbb{Z}[X] \) is not a PID. \( \mathbb{Z}[X] \) is a UFD (Statement only).

Reference for Unit III:


Additional Reference Books:


USMT603/UAMT603 Metric Topology

All concepts have to be taught with plenty of examples and worked out in special case of Euclidean space, Complex plane and other metric spaces.

Unit I. Complete metric spaces (15 Lectures)

Convergent sequences, Cauchy’s principle of convergence, convergent Cauchy sequences, Complete metric spaces. Completeness property in subspaces of a complete metric space: Any closed subset of a complete metric space is complete.

Cantor’s intersection theorem. Examples of Complete metric spaces: \( \mathbb{R}, \mathbb{R}^n, \mathbb{C}[a, b] \). If \( X, Y \) are complete metric spaces with metrics \( d_1, d_2 \) respectively, then \( X \times Y \) is complete with metric \( d((x_1, y_1), (x_2, y_2)) = \sqrt{d_1(x_1, x_2)^2 + d_2(x_2, y_2)^2} \).

Reference for unit I:


Unit II: Compact metric spaces:

(a) Definition of a compact set in a metric space (as a set for which every open cover has a finite subcover), examples. Properties such as: i) Continuous image of a compact set is compact, ii) Compact subsets of a metric space are closed and bounded, iii) A continuous function on a compact set is uniformly continuous.

Compactness and finite intersection property: A metric space \( X \) is compact if and only if for every infinite family \( \{F_\alpha : \alpha \in S\} \) of closed subsets of \( X \) with finite intersection property, \( \bigcap_{\alpha \in S} F_\alpha \) is not empty. Every infinite, bounded subset of a compact metric space has an accumulation point (cluster point). A compact metric space is complete.

Characterization of compact sets in \( \mathbb{R}^n \):

The following are equivalent statements for a subset of \( \mathbb{R}^n \) to compact:

1. Heine-Borel property.
2. Closed and boundedness property.
4. Sequentially compactness property.

Reference for Unit II:

Unit III. Connected sets (15 lectures)
Connected metric spaces (a metric space which cannot be represented as the union of two disjoint non-empty open subsets). Characterization of a connected space, namely a metric space $X$ is connected if and only if every continuous function from $X$ to the discrete metric space $\{1, 1\}$ is a constant function. Connected subsets of a metric space, connected subsets of $\mathbb{R}$ are intervals. A continuous image of a connected set is connected, applications such as: i) $GL(2, \mathbb{R}), O(n, \mathbb{R})$ are not connected, ii) graph of a real valued continuous function defined on an interval is a connected subset of $\mathbb{R}^2$.

For $A, B$ be two connected subsets of a metric space $X$, i) $A \cap B \neq \emptyset$ implies $A \cup B$ is connected, ii) $A \subset B \subset \overline{A}$ implies $B$ is connected. Circle $S^1$ is a connected subset of $\mathbb{R}^2$.

Definition of a path connected metric space, examples including $\mathbb{R}^n, S^n (n \in \mathbb{N})$. A path connected metric space is connected and applications including connectedness of $\mathbb{R}^n, C^n$. An example of a connected subset of $\mathbb{R}^2$ which is not path connected (proof not required). An open subset of $\mathbb{R}^n$ is connected if and only if it is path-connected (proof not required).

Reference for Unit III:

Recommended Text Books:

Additional Reference Books:
USMT6A4/UAMT6A4  Numerical Analysis II (Elective A)

N.B. Derivations and geometrical interpretation of all numerical methods with the-orem mentioned have to be covered.

Unit I: Interpolation (15 Lectures)
Interpolating polynomials, uniqueness of interpolating polynomials. Linear, Quadratic and higher order interpolation. Lagranges Interpolation.

Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them. Difference table, Relation between difference and derivatives. Interpolating polynomials using finite differences: Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirlings Interpolation. Results on interpolation error.

Unit II: Polynomial Approximations and Numerical Differentiation (15 Lectures)
Piecewise Interpolation: Linear, Quadratic and Cubic. Bivariate Interpolation: La-granges Bivariate Interpolation, Newtons Bivariate Interpolation.

Numerical differentiation: Numerical differentiation based on Interpolation, Numerical differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

Unit III: Numerical Integration (15 Lectures)

Recommended Text Books:

USMT6B4/UAMT6B4

Number Theory and its applications II (Elective B)

Unit I: Quadratic Reciprocity (15 Lectures)
Quadratic residues and Legendre symbol, Gauss Lemma, Theorem on Legendre symbol \( \left( \frac{2}{p} \right) \), the result: If \( p \) is an odd prime and \( a \) is an odd integer, then
\[
\left( \frac{a}{p} \right) = (-1)^t, \quad \text{where} \quad t = \sum_{k=1}^{(p-1)/2} \left\lfloor \frac{ka}{p} \right\rfloor,
\]
Quadratic Reciprocity law. Theorem on Legendre Symbol \( \left( \frac{2}{p} \right) \). The Jacobi symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit II: Continued Fractions (15 Lectures)
Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit III. Pells equation, Arithmetic function and Special numbers (15 Lectures)
Pell’s equation \( x^2 - dy^2 = n \), where \( d \) is not a square of an integer. Solutions of Pell’s equation (The proofs of convergence theorems to be omitted).

Arithmetic functions of number theory: \( d(n) \) or \( \tau(n) \), \( \sigma(n) \), \( \sigma_k(n) \), \( \omega(n) \) and their properties, \( \mu(n) \) and the Mbius inversion formula.

Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Recommended Text Books:

Additional Reference Books:
7. W. Stallings, Cryptology and network security.
USMT6C4/UAMT6C4

Graph Theory and Combinatorics (Elective C)

Unit I. Colorings of graph (15 Lectures)
Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge coloring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem.


Unit II. Planar graphs (15 Lectures)
Definition of a planar graph. Euler formula and its consequences. Non planarity of $K_5$, $K(3; 3)$. Dual of a graph. Polyhedrons in $\mathbb{R}^3$ and existence of exactly five regular polyhedra- (Platonic solids).

Colorability of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem.


Unit III: Combinatorics (15 Lectures)
Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems.

Introduction to partial fractions and using Newton’s binomial theorem for real power find series expansion of some standard functions.

Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall’s theorem of SDR.

Introduction to matching, $M$ alternating and $M$ augmenting path, Berge theorem. Bipartite graphs.

Recommended Text Books:

2. R. BALKRISHNAN AND K. RANGANATHAN, Graph theory and applications, North Holland, 1982.
3. D.G. WEST, Introduction to Graph theory, Pearson Modern Classics.

**Additional Reference Books:**

2. S.A. Choudam, *A First course in Graph Theory*, Macmillam India Ltd.

**USMT6D4/UAMT6D4 Operations Research (Elective D)**

**Unit I: Linear Programming-I** (15 Lectures)

Prerequisites: Vector Space, Linear independence and dependence, Basis, Convex sets, Dimension of polyhedron, Faces.


**Reference for Unit-I:**


**Unit II: Linear programming-II** (15 Lectures)


**Reference for Unit-II:**


**Unit III: Queuing Systems** (15 Lectures)


**Reference for Unit III:**


**Additional Reference Books:**


**USMT07, UAMTP07**

Practicals for USMT601/UAMT601, USMT602/UAMT602 & USMT603/UAMT603

A. Practical for USMT601/UAMT601:
1. Pointwise and uniform convergence of sequence functions, properties.
2. Pointwise and uniform convergence of series of functions and properties.
3. Analytic function, finding harmonic conjugate, Mobius transformations.
5. Limit continuity and derivatives of functions of complex variables.
7. Miscellaneous theory questions based on full paper (3 theory questions from each unit).

B. Practicals for USMT602/UAMT602:
1. Normal Subgroups and quotient groups.
2. Cayley’s Theorem and external direct product of groups.
3. Rings, Ring Homomorphism and Isomorphism.
4. Ideals, Prime Ideals and Maximal Ideals.
5. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain.
6. Fields.
7. Miscellaneous theory questions based on full paper.

C. Practicals for USMT603/UAMT603:
1. Completeness of \( \mathbb{R}, \mathbb{R}^n \).
2. A metric space \( X \) is complete if and only if every closed ball of \( X \) is complete.
3. Compact sets in a metric space, Compactness in \( \mathbb{R}^n \) (emphasis on \( \mathbb{R}, \mathbb{R}^2 \)), properties.
5. Example of a closed and bounded subset of a metric space which is not compact.
6. Connectedness, Path connectedness.
7. Continuous image of a connected set.
8. Miscellaneous Theoretical Questions based on full paper.
USMTPJ6, UAMTPJ6: Projects
A student can submit a project which shall have 20-30 typed pages, on one of the following topics:

1. Apps for small devices using Python:
   Chapter 7 of Head First Python by Paul Barry, O’Reilly Media, second edition.

2. Apps for small devices using Java:
   Java How to Program (early objects) by Paul Deitel and Harvey Deitel, Pearson 9th edition (2012).

3. Elliptic Curves and their uses in Cryptography, Pollard’s Algorithm:

4. Runge-Kutta methods, principle and proofs of second and fourth order computer programs:
   S.S. Sastry, Introductory Methods of Numerical Analysis, Prentice hall India.

5. The matrix exponential and applications to system of Differential equations
   \[ X' = AX \]
   M. Artin, Algebra, Pearson India Education.

6. Iterated solutions of Picard’s theorem and solutions of second order linear ODE:

7. The Qualitative properties of the solutions of \[ y'' + P(x)y' + Q(x)y = 0, \]
   Sturm Separation theorem:
   G. F. Simmons, Differential Equations with Applications and Historical Notes, McGRAW-Hill International.


9. Algebraic numbers, algebraic integers:

10. Quadratic fields, units, primes, UFD:

11. Structure of finite Abelian groups:
    S. Lang, Algebra, Springer.
12. The Class Equation, Application to $p$--groups:
13. The Class Equation of Icosahedral group.:
14. The Class Equation, classification of groups of order 12:
15. Construction of numbers by Ruler & Compass:
16. Field Extensions, Cubic equations, Cardano’s method:
17. Character groups of small Order:
18. Finite Division ring is a field and sum of two squares:
19. Study of Polya theory of Counting:
20. Hall’s Marriage Theorem, Graph theory & Applications:
21. Ramsey numbers:
22. Axiom of choice, Zorn’s Lemma:
   Set theory related topics, Schaum series. See also S. Lang, Analysis II.
23. Introduction to Cryptography:
24. Separable metric spaces, study of completions of $C[a,b]$ under norms such as sup-norm, $L^1$--norm, $L^2$--norm:
25. Study of Baire spaces and application to limit of a sequence of real valued continuous functions defined on $\mathbb{R}$:
26. Completion of Metric spaces:
27. Maximum principle for analytic functions and applications:
28. Exponential function $e^z$, Epimorphism Theorem, $e^{ix} = c(x) + is(x)$, study of circular functions $c(x), s(x)$ and identification with Trigonometric functions: R. Remmert, *Classical Topics in Complex Function Theory*, Springer.


38. Alternating $k$-tensors on a finite dimensional real vector space, orientation and volume elements and theory of differential forms: M. Spivak, *Calculus on manifolds*, W.A. Benjamin Inc.

39. Differential forms, Stokes’ theorem and applications to Green’s theorem, Gauss’ Divergence theorem: M. Spivak, *Calculus on manifolds*, W.A. Benjamin Inc.

40. Locally compact metric spaces, One-Point Compactification, One-Point compactification of $\mathbb{C}$ is homeomorphic to the unit sphere $S^2 \subset \mathbb{R}^3$: J. R. Munkres, *Topology*, Pearson Education India.

41. First Fundamental Groups of a metric space, computation of $\pi_1(S^1, 1)$: J. R. Munkres, *Topology*, Pearson Education India.


44. Simplicial complexes in $\mathbb{R}^2, \mathbb{R}^3$, singular chains, Homology groups $H_0, H_1$.

45. Homology groups:

46. A non-computational proof of Cayley-Hamilton theorem, canonical isomorphism with double dual Tensor products:
   S. Lang, *Introduction to Linear Algebra*, Springer Verlag.

47. Topics in Projective Geometry:
   R. Artzy, *Linear geometry*, Addison-Wesley.

48. Topics in Non-Euclidean Geometries:

49. Special relativity:

50. LU factorization using Gaussian elimination:

51. Error estimates of proofs and implementation of Trapezoidal rule, Simpson rule, Romberg method adaptive integration:

52. Discrete Fourier transform, Fast Fourier transform:

53. Topics in Automata Theory:

54. Operations Research (Game Theory and Quality Control):

55. Operations Research (Integer L.P.P. and Inventory models):

56. Operations Research and Markov Chains:

57. Discrete and Continuous probability distributions:

**Scheme of Examination**

I. Semester End Theory Examinations:

There shall be a Semester-end external Theory examination of 100 marks for all the courses of Semester V and VI- except for the two project courses USMTPJ5/UAMTPJ5, USMTPJ6/UAMTPJ6- to be conducted by the University.

1. Duration: The examinations shall be of 3 Hours duration.
2. Theory Question Paper Pattern:
   a) There shall be FIVE questions. All the questions shall be compulsory.
      The first question Q1 shall be of objective type for 20 marks based on the entire syllabus.
      The next four questions Q2, Q3, Q4, Q5 shall be of 20 marks each.
      The questions Q2, Q3, Q4 shall be based on the units I, II , III respectively.
      The question Q5 shall be based on the entire syllabus.
   b) The questions Q2,Q3,Q4,Q5 shall have internal choices within each question. Including the choices, the marks for each question shall be 30-32.
   c) The questions Q2,Q3,Q4,Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
   d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. Semester End Examinations Practicals:

There shall be a Semester-end practical examinations of three hours duration and 150 marks for each of the courses USMTP05/UAMTP05 of Semester V and USMTP06/UAMTP06 of semester VI.

In semester V, the Practical examinations for USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are conducted together by the college.

Similarly in semester VI, the Practical examinations for USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are conducted together by the college.

Question Paper pattern: The question paper shall have three parts A,B, C. Every part shall have three questions of 20 marks each. Students to attempt any two questions from each part.

For each course USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503, USMT601/UAMT601, USMT602/UAMT602, and USMT603/UAMT603 marks for
journal and viva are as follows:

<table>
<thead>
<tr>
<th>Journals</th>
<th>Viva</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 marks</td>
<td>5 marks</td>
</tr>
</tbody>
</table>

Each Practical of every course of Semester V & VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal.

<table>
<thead>
<tr>
<th>Practical Course</th>
<th>Part A</th>
<th>Part B</th>
<th>Part C</th>
<th>Marks out of</th>
<th>duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>USMTP05</td>
<td>Questions from USMT501</td>
<td>Questions from USMT502</td>
<td>Questions from USMT503</td>
<td>120</td>
<td>3 hours</td>
</tr>
<tr>
<td>USMTP06</td>
<td>Questions from USMT601</td>
<td>Questions from USMT602</td>
<td>Questions from USMT603</td>
<td>120</td>
<td>3 hours</td>
</tr>
</tbody>
</table>

III. Evaluation of Project work
( courses: USMTPJ5/UAMTPJ5 & USMTPJ6/UAMTPJ6):

The evaluation of the Project submitted by a student shall be made by a Committee appointed by the Head of the Department of Mathematics of the respective college.

The presentation of the project is to be made by the student in front of the committee appointed by the Head of the Department of Mathematics of the respective college. This committee shall have two members, possibly with one external referee.

The Marks for the project are detailed below:

- Contents of the project : 40 marks
- Presentation of the project : 30 marks
- Viva of the project : 30 marks.

Total Marks= 100 per project per student.

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