

SE (SEM - III) (REV 2012)
(CBSGS) (MECHANICAL ENGG)

20 NOV
3:00 PM
to
6:00 PM

SEM - III (CBSGS)

APPLIED MATHEMATICS - III
QP Code: 5052

Mechanical/Automobile

(3 Hours)

[Total Marks :80

- N.B. : (1) Question no. 1 is compulsory.
(2) Answer any **three** from remaining.
(3) **Figures** to the right indicate full marks.

1. (a) Find Laplace transform of $t \sin^3 t$. 5
(b) Find half range sine series in $(0, \pi)$ for $x(\pi - x)$ 5
(c) Find the image of the rectangular region bounded by $x = 0, x = 3, y = 0, y = 2$ under the transformation $w = z + (1+i)$ 5
(d) Evaluate $\int f(z) dz$ along the parabola $y = 2x^2, z = 0$ to $z = 3 + 18i$ 5
where $f(z) = x^2 - 2iy$

2. (a) Find two Laurent's series of $f(z) = \frac{1}{z^2(z-1)(z+2)}$ about $z = 0$ for 8
(i) $|z| < 1$ (ii) $1 < |z| < 2$
(b) Find complex form of Fourier series for $f(x) = \cos h2x + \sin h2x$ in $(-2, 2)$ 6
(c) Find bilinear transformation that maps $0, 1, \infty$ of the z plane into $-5, -1, 3$ of w plane. 6

3. (a) Solve by using Laplace transform 8
 $(D^2 + 2D + 5)y = e^{-t} \sin t$ when $y(0) = 0$ and $y'(0) = 1$
(b) Solve $\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial t} = 0$ by Bender schmidt method given 6
 $u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 - x)$ 6
(c) Expand $f(x) = \ell x - x^2, 0 < x < 1$ in a half range cosine series. 6

4. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{(2 + \cos \theta)^2}$ 8
(b) Evaluate $\int_0^{\infty} e^{-2t} \frac{\cos 2t \sin 3t}{t} dt$ 6
(c) Using Crank Nicholason method solve 6

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$$

$$u(0, t) = 0, u(4, t) = 0$$

$$u(x, 0) = \frac{x}{3} (16 - x^2)$$

Find u_{ij} for $i = 0, 1, 2, 3, 4$ and $j = 0, 1, 2$.

[TURN OVER

5. (a) Find analytic function whose real part is

$$\frac{\sin 2x}{\cosh 2y + \cos 2x}$$

8

(b) Find (i) $L^{-1} \left[\frac{e^{-\pi s}}{s^2 - 2s + 2} \right]$

6

(ii) $L^{-1} \left[\tan^{-1} \left(\frac{s+a}{b} \right) \right]$

- (c) Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

6

under the boundary conditions $u(0, t) = 0$

$u(1, t) = 0$ and $u(x, 0) = x$

$0 < x < \ell$, ℓ being length of the rod.

8

6. (a) A string is stretched and fastened to two points distance ℓ apart. Motion is started by displacing the string in the form $y = a \sin \left(\frac{\pi x}{\ell} \right)$ which it is released at time $t = 0$. Show that the displacement of a point at a distance x from one

end at time t is given by $y_{(x,t)} = a \sin \left(\frac{\pi x}{\ell} \right) \cos \left(\frac{\pi ct}{\ell} \right)$

6

6

- (b) Find the residue of $\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)^2}$ at its poles.

- (c) Find Fourier series of $x \cos x$ in $(-\pi, \pi)$