KINEMATICS OF MACHINERY SOLUTION

SEM4 (CBCGS-DEC 2019)

BRANCH-MECHANICAL ENGINEERING

Q 1) A) What is Kutzback's criterion for degrees of freedom of plane mechanism? In what way is Gruebler's criterion different from it?. (05)

Solution:

i) Kutzback's criterion:

Let us consider a plane mechanism with l number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be $(l - 1)$ and thus the total number of degrees of freedom will be $3\ (l - 1)$ before they are connected to any other link. In general, a mechanism with l number of links connected by j number of binary joints or lower pairs (i.e. single degree of freedom pairs) and h number of higher pairs (i.e. two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$$n = 3\ (l - 1) - 2\ j - h \qquad \text{... (i)}$$

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.

If there are no two degree of freedom pairs (i.e. higher pairs), then $h = 0$. Substituting $h = 0$ in equation (i), we have

$$n = 3\ (l - 1) - 2\ j \qquad \text{... (ii)}$$

ii) Gruebler's criterion:

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting $n = 1$ and $h = 0$ in Kutzbach equation, we have

$$1 = 3\ (l - 1) - 2\ j \text{ or } 3l - 2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

Q 1) B) Differentiate between lower pair and higher pair. (05)

Solution:

i) Lower pair

When two elements of a pair are joined together with the surface contact between them the joint is called lower pair. Area of two elements comes together when relative motion occurs between the elements to form a lower pair.

The elements has sliding motion mutually in a lower pair because of surface contact. Basic kinematic pair like prismatic pair, revolute pair, screw pair, are some examples of lower pair.

ii) Higher pair

In The higher pair, only one point or line are responsible to form a joint between two links. The elements of higher pair must have curve in its shape. These joints are found in the cylinders or spheres of equal or different radius which have their axis parallel to each other. A cylinder or sphere lying on a flat surface has a point Contact and makes a higher pair. The relative motion of cam and follower makes a point Contact between them. The point of contact between two involute gear meshing makes a higher pair. These are some basic examples to explain the higher pair better.

Q 1) C) Define with respect to cam i) Base circle ii) pitch circle iii) trace point iv) pressure angle. (05)

Solution:

i) Base circle:

It is the smallest circle that can be drawn to the cam profile.

ii) Pitch circle:

It is a circle drawn from the centre of the cam through the pitch points.

iii) Trace point:

It is a reference point on the follower and is used to generate the pitch curve.

iv) Pressure angle:

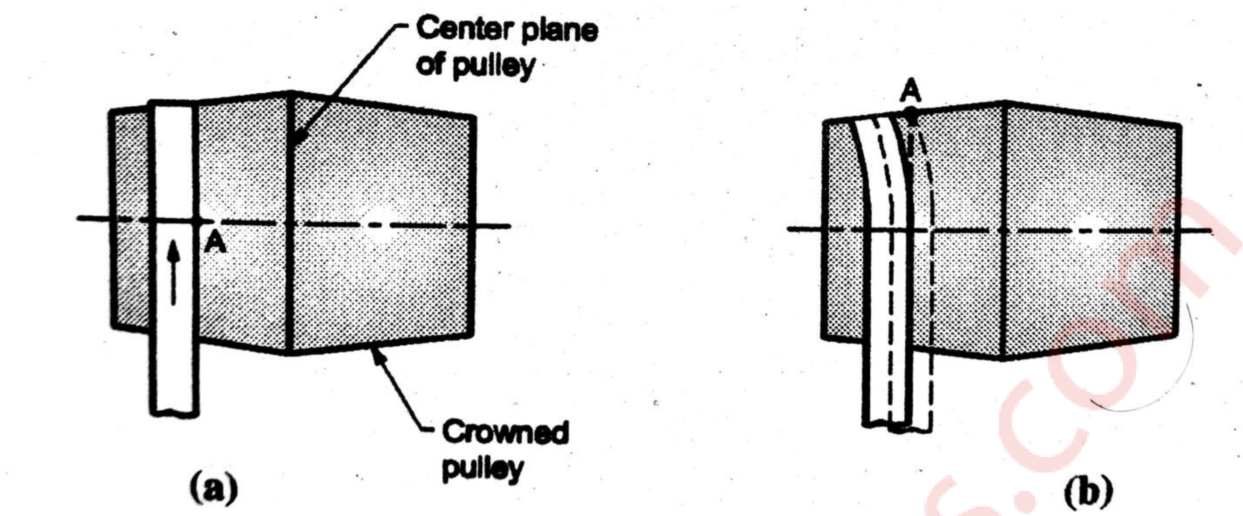
It is the angle between the direction of the follower motion and a normal to the pitch curve.

Q 1) D) What is crowning of pulley in flat drives? What is its use. (05)

Solution:

1) The pulley of flat belt drive is made tapered or rounded off slightly to prevent the slipping of the belt from the pulley. This is called crowning of pulley which is shown in fig.

2) Consider fig (a), the belt will touch the rim surface at its one edge only. This is impractical. The belt always tends to stick to the rim surface.



3) Thus a belt has to bend as shown in fig (b).

4) When point 'A' will move on the pulley in the direction shown in ¼ turn, the belt will moved towards the center plane of pulley shown by dotted line.

5) But the belt is still bend hence pull is applied towards center plane and it will come at exactly center plane of pulley action repeats for another ¾ turn.

6) Thus if pulley is made up of tapered or converse surface, the belt will remain at the center plane of the pulley.

Q 1) E) Explain the self locking and self energizing in brakes. (05)

Solution:

i) Self energizing brake:

1) The frictional force helps to apply the brake. Such type of brakes are said to be self energizing brakes.

2) $P = \frac{R_n(x-\mu.a)}{l}$ $\quad$ $R_n x - \mu.R_n.a = P.l$

$R_n.x = P.l + \mu.R_n.a$

From the above equation we can say that moment of frictional force $(\mu.R_n.a)$ adds to the moment force $(p.l)$. In other word frictional force helps to apply brake. Such types of brakes are called self energizing brakes.

ii) Self locking brakes :

1) When the frictional force is great enough to apply the brake with no external force, then the brake is said to be self-locking brake

2) $R_n = \frac{P.l}{x-\mu.a}$ $P = \frac{R_n(x-\mu.a)}{l}$

In above equation if $x \leq \mu.a$, the effort P become negative or zero. Thus no effort is required to apply brakes and hence brake is self locking.

Therefore, condition for self-locking is,

$$x \leq \mu.a$$

Q 2) A) The mechanism, as shown in Fig.1, has the dimensions of various links as follows : $AB = DE = 150\ mm$; $BC = CD = 450\ mm$; $EF = 375\ mm$. The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC. The block F moves in the horizontal guides, being driven by the link EF. Determine velocity of the block F and angular velocity of DC (14)

1. By instantaneous centre method

2. By relative velocity method

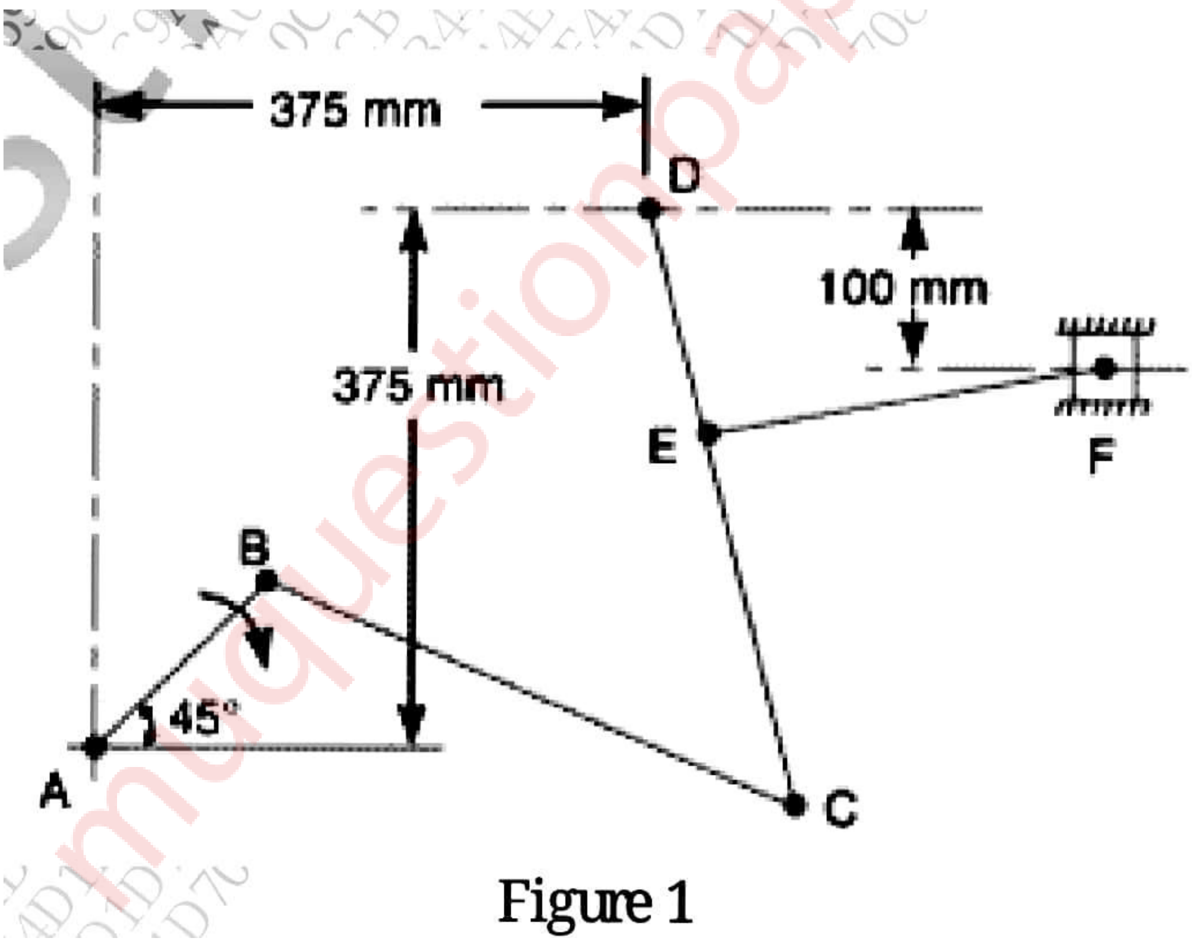


Figure 1

Solution:

1. By Instantaneous centre method :

Speed of crank OA,

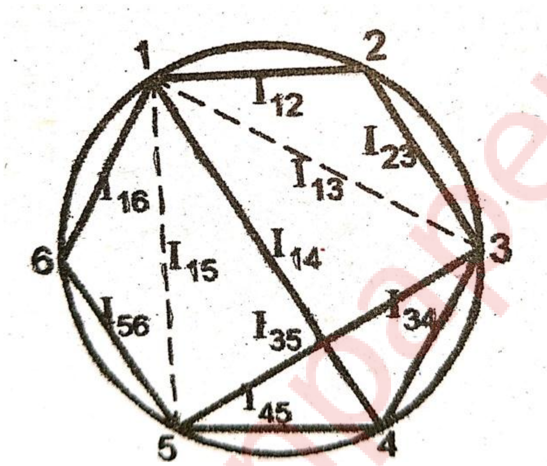
$$N_{AO} = 180 \text{ r.p.m. or } \omega_{AO} = 2\,\pi \times 180/60 = 18.85 \text{ rad/s}$$

Velocity of link AO is,

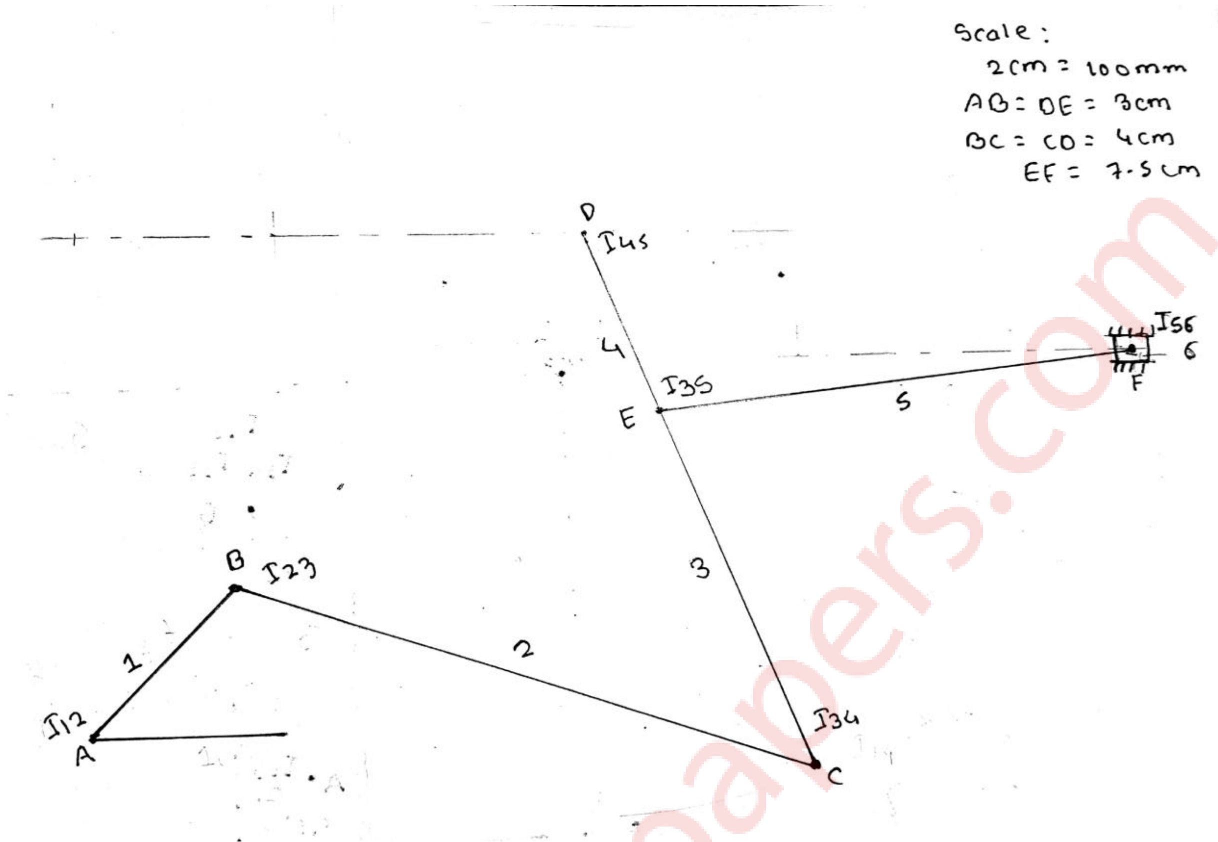
$$v_{AO} = v_A = \omega_{AO} \times OA = 5.23 \times 0.03 = \mathbf{1569} \text{ m/s}$$

The No. of instantaneous centres are,

$$n = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$



Link	1	2	3	4	5	6
ICR's	(12)	(23)	(34)	(45)	(56)	
	13	24	35	46		
	(14)	25	36			
	15	26				
	(16)					



By measurement:

$I_{13}I_{23} = 100 \text{ mm}$

$I_{13}I_{34} = 90 \text{ mm}$

$I_{14}I_{45} = 50 \text{ mm}$

$I_{14}I_{34} = 50 \text{ mm}$

$I_{15}I_{45} = 50 \text{ mm}$

$I_{15}I_{56} = 38 \text{ mm}$

Velocity of links

$v_2 = \omega_3 \times (I_{13}I_{23})$

$$\omega_3 = \frac{v_{AO}}{I_{13}I_{23}} = \frac{157.0}{100} = 1.57 \text{ mm/s}$$

Angular Velocity ratio for link 3 and 4 is

$$\frac{\omega_3}{\omega_4} = \frac{I_{14}I_{34}}{I_{13}I_{34}}$$

$$\frac{1.57}{\omega_4} = \frac{50}{90}$$

$$\omega_4 = 2.82 \text{ cm/s}$$

Angular Velocity ratio for link 4 and 5 is

$$\frac{\omega_4}{\omega_5} = \frac{I_{15}I_{45}}{I_{14}I_{45}}$$

$$\frac{2.82}{\omega_5} = \frac{50}{50}$$

$$\omega_5 = 2.82 \text{ mm/s}$$

Velocity of slider C

$$v_C = \omega_5 \times (I_{15}I_{56}) = 2.82 \times 50 = 141 \text{ mm/s} = 0.141\ m/s$$

2. By relative velocity method :

$N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 4\pi$ rad/s

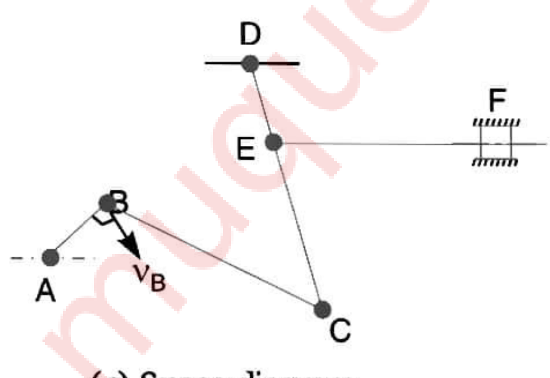
Since the crank length $AB = 150$ mm $= 0.15$ m, therefore velocity of B with respect to A or velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\pi \times 0.15 = 1.885 \text{ m/s}$$

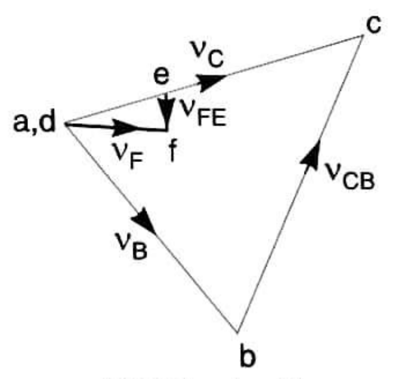
… (Perpendicular to AB)

1. Velocity of the block F

First of all draw the space diagram, to some suitable scale, as shown in Fig (a). Now the velocity diagram, as shown in Fig (b), is drawn as discussed below:



(a) Space diagram



(b) Velocity diagram

1. Since the points A and D are fixed, therefore these points are marked as one point* as

shown in Fig (b). Now from point a, draw vector ab perpendicular to AB, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

$$\text{vector ab} = v_{BA} = v_B = 1.885 \text{ m/s}$$

2. The point C moves relative to B and D, therefore draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. v_{CB}), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.

3. Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD in Fig (a). In other words

$$ce/cd = CE/CD$$

4. From point e, draw vector ef perpendicular to EF to represent the velocity of F with respect to E (i.e. v_{FE}) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. v_F. The vectors ef and df intersect at f. By measurement, we find that velocity of the block F,

$$v_F = \text{vector df} = 0.7 \text{ m/s}$$

2. Angular velocity of DC

By measurement from velocity diagram, we find that velocity of C with respect to D,

$$v_{CD} = \text{vector dc} = 2.25 \text{ m/s}$$

Since the length of link DC = 450 mm = 0.45 m, therefore angular velocity of DC,

$$\omega_{DC} = \frac{v_{CD}}{DC} = \frac{2.25}{0.45} = 5 \text{ rad/s(Anticlockwise about D)}$$

Q 2) B) State and prove law of gearing. (06)

Solution:

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure.

Let T T be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2 , draw O_1M and O_2N perpendicular to MN. A little

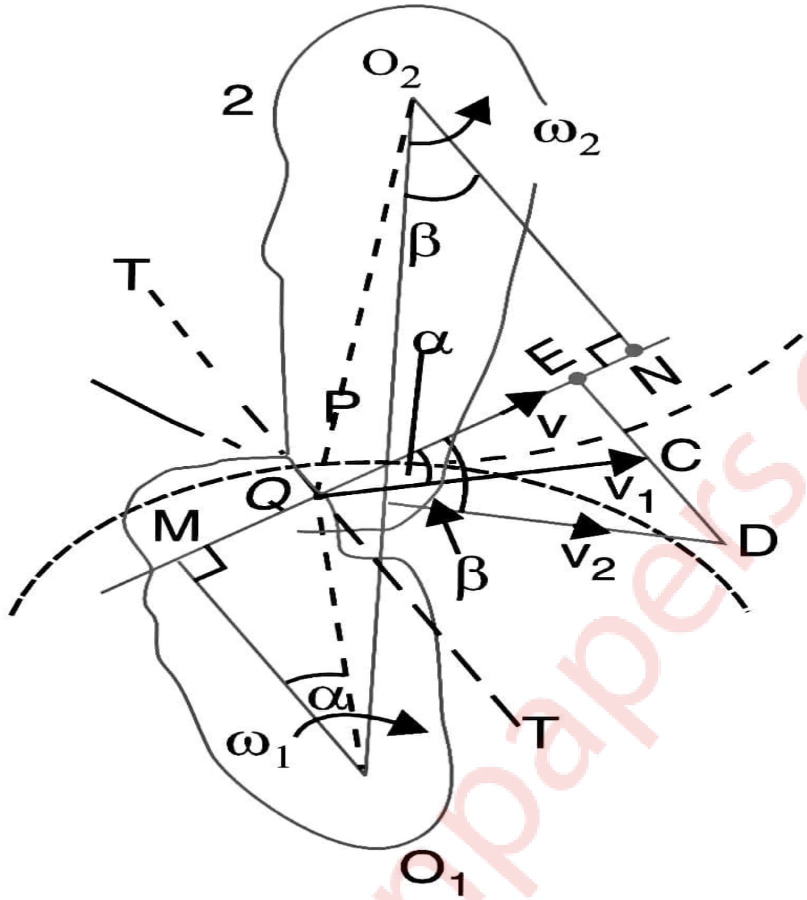


FIG: LAW OF GEARING

consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v1 and v2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

$\therefore\ v1 \cos\alpha = v2 \cos\beta$

$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$

$(\omega_1 \times O_1Q)\ \frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\ \frac{O_2N}{O_2Q}$ or $\omega_1 \times O_1M = \omega_2 \times O_2N$

$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

Combining equations (i) and (ii), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres $O1_1$ and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

Q 3) A) A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m. Determine the velocity of sliding between the gear teeth 10 faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20° involute form, addendum length is 5 mm and the module is 5 mm. Also find the angle through which the pinion turns while any pairs of teeth are in contact. (10)

Solution:

Given : $T = 40$; $t = 20$; $N_1 = 2000$ r.p.m. ; $\varphi = 20°$; addendum $= 5$ mm ;

$m = 5$ mm

We know that angular velocity of the smaller gear,

$$\omega_1 = \frac{2\pi \times N_1}{60} = \frac{2\pi \times 0.3 \times 2000}{60} = 209.5 \text{ rad/s}$$

and angular velocity of the larger gear,

$$\omega_2 = \omega_1 \times \frac{t}{T} = 209.5 \times \frac{20}{40} = 104.75 \text{ rad/s}$$

Pitch circle radius of the smaller gear,

$$r = m.t / 2 = 5 \times 20/2 = 50 \text{ mm}$$

and pitch circle radius of the larger gear,

$$R = m.t / 2 = 5 \times 40/2 = 100 \text{ mm}$$

∴ Radius of addendum circle of smaller gear,

$$r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}$$

and radius of addendum circle of larger gear,

$$R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}$$

The engagement and disengagement of the gear teeth is shown in fig . The point K is the point of engagement, P is the pitch point and L is the point of disengagement. MN is the common tangent at the points of contact.

We know that the distance of point of engagement K from the pitch point P or the length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\varphi} - r\sin\varphi$$

$$= \sqrt{(105)^2 - (100)^2\cos^2 20°} - 100\sin 20°$$

$$= 46.85 - 34.2 = 12.65 \text{ mm}$$

and the distance of the pitch point P from the point of disengagement L or the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\varphi} - r\sin\varphi$$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20°} - 50\sin 20°$$

$$= 28.6 - 17.1 = 11.5 \text{ mm}$$

Velocity of sliding at the point of engagement

We know that velocity of sliding at the point of engagement K,

$$v_{SK} = (\omega_1 + \omega_2)\, KP = (209.5 + 104.75)\, 12.65 = 3975\, mm/s$$

Velocity of sliding at the pitch point

Since the velocity of sliding is proportional to the distance of the contact point from the pitch point, therefore the velocity of sliding at the pitch point is zero. Ans.

Velocity of sliding at the point of disengagement

We know that velocity of sliding at the point of disengagement L,

$$v_{SK} = (\omega_1 + \omega_2)\, PL = (209.5 + 104.75)\, 11.5 = 3614 \text{ mm/s}$$

Angle through which the pinion turns

We know that length of the path of contact,

$\mathrm{KL} = \mathrm{KP} + \mathrm{PL} = 12.65 + 11.5 = 24.15\ \mathrm{mm}$

and length of arc of contact $= \frac{\mathrm{KL}}{\cos\varphi} = \frac{24.15}{\cos 20°} = 25.7\ \mathrm{mm}$

Circumference of the smaller gear or pinion

$$= 2\,\pi\,\mathrm{r} = 2\pi \times 50 = 314.2\ \mathrm{mm}$$

∴ Angle through which the pinion turns

$$= \text{length of arc of contact} \times \frac{360°}{\text{circumference of pinion}}$$

$$= 25.7 \times \frac{360°}{314.2} = 29.45°$$

Q 3) B) An open belt drive is required to transmit 10KW of power from a motor running at 600rpm. Diameter of the driving pulley is 250mm. The speed of the driven pulley is 220rpm. The belt is 12mm thick and has a mass density of $.001g/mm^3$. Safe stress in the belt is not to exceed $2.5N/mm^2$. The two shafts are 1.25 m apart. The coefficient of friction is 0.25. Determine the width of the belt. (10)

Solution:

Speed of the driving pulley, $\mathrm{N_1} = 600\ \mathrm{rpm}$

Speed of the driven pulley, $\mathrm{N_2} = 220\ \mathrm{rpm}$

Thus, smaller pulley is the driver and

$\mathrm{d} = 250\ \mathrm{mm}$

$\mathrm{P} = 10\ \mathrm{KW}\ ;\ \mathrm{t} = 12\ \mathrm{mm}$

$\rho = 0.001\ \mathrm{g/mm^3} = 1000\ \mathrm{kg/m^2}\ ;\ \mathrm{r} = 125\ \mathrm{mm};$

$\mathrm{C} = 1.25\ \mathrm{m};\ \mu = 0.25;$

$\sigma_t = 2.5\mathrm{N/\,mm^2} = 2.5 \times 10^6 2.5\mathrm{N/\,m^2}$

To calculate the width of belt, we need to know maximum tension in the belt which is the sum of the tight side tension and the centrifugal tension.

i.e., $\mathrm{T} = \mathrm{T_1} + \mathrm{T_2}$

calculation of $\mathrm{T_1}$

$$\mathrm{P} = (\mathrm{T_1} - \mathrm{T_2})\mathrm{v}$$

Where $\mathrm{v} = \omega\left(\mathrm{r} + \frac{\mathrm{t}}{2}\right) = \frac{2\pi\mathrm{N}}{60}\left(\mathrm{r} + \frac{\mathrm{t}}{2}\right)$

$= \frac{2\pi \times 600}{60}\left(125 + \frac{12}{2}\right) = 8230 \text{ mm/s or } 8.23 \text{ m/s}$

$\therefore \quad 10000 = (T_1 - T_2) \times 8.23$

Or $T_1 - T_2 = 1215$

$\frac{T_1}{T_2} = e^{\mu.\theta}$

Where $\theta = \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{r_1 - r_2}{C}\right)$

$\theta = \pi - 2\beta = \pi - 2\sin^{-1}\left(\frac{125 \times 600/220 - 125}{1250}\right)$

$\theta = \pi - 19.9° = \pi - 0.347 = 2.79$

$\frac{T_1}{T_2} = e^{0.25 \times 2.79} = 2.01 \text{ or } T_1 = 2.01T_2$ (ii)

From i) and ii),

$2.01T_2 - T_1 = 1215$

$T_2 = 1203 \text{ N}$

$T_1 = 2418 \text{ N}$

Calculation of T_c,

$T_c = mv^2$

$= \text{mass per unit length } \times v^2$

$= \text{volume per unit length } \times \text{density} \times v^2$

$= (\text{x} - \text{sectional area} \times \text{length } \times \text{density}) \times v^2$

$= b \times 0.012 \times 1 \times 1000 \times (8.23)^2$

$= 812.8b \text{ N}$

$$T = T_1 + T_c = \sigma_t \times (b \times t)$$

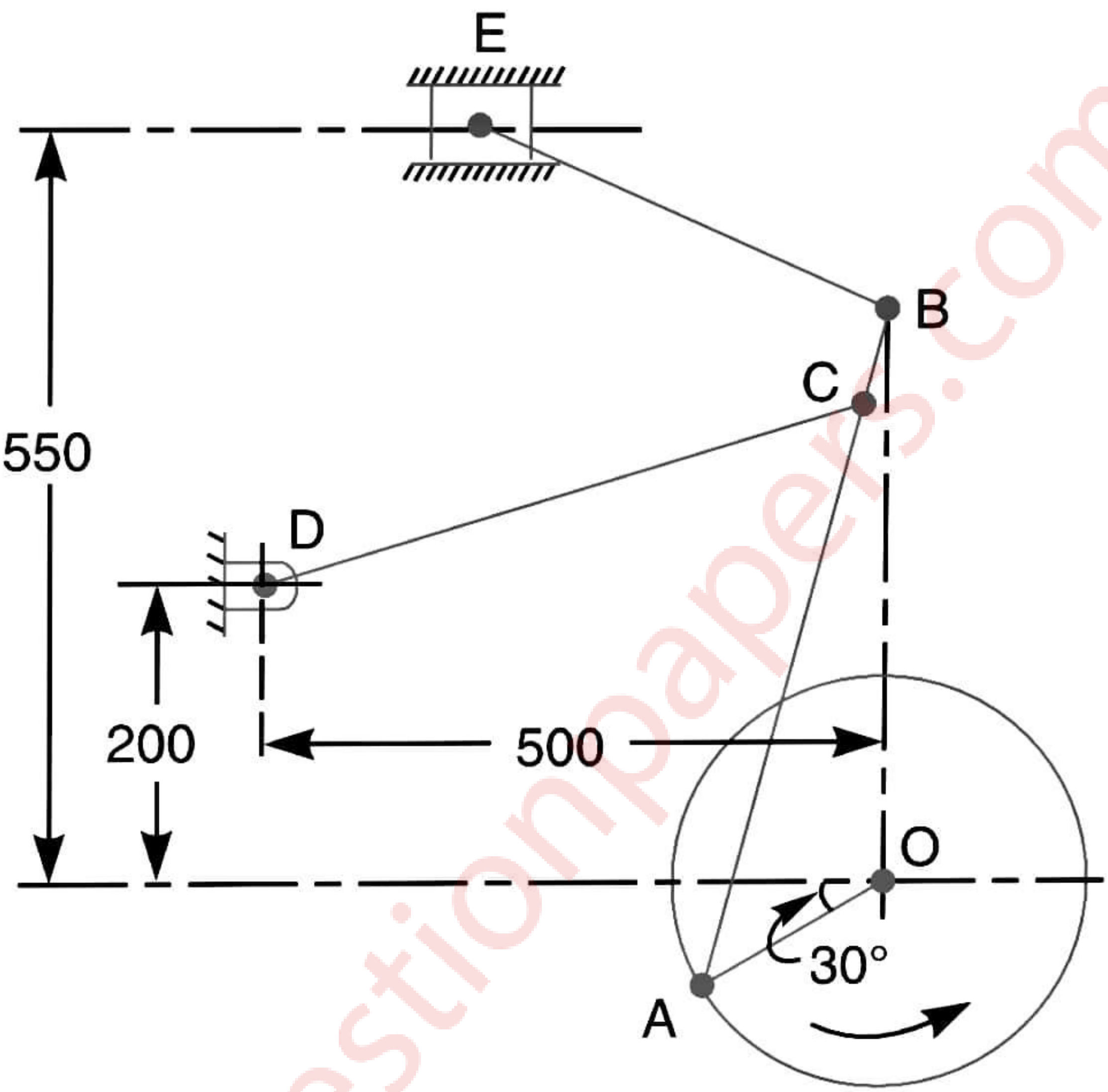
$2418 + 812.8b = 2.5 \times 10^2 \times b \times 0.012$

$29187b = 2418$

$b = 0.0828 \text{ m or } 82.8 \text{ mm}$

Q 4) A) The mechanism as shown in fig. 2 of a radial valve gear. The crank OA turns uniformly at 150 r.p.m and is pinned at A to rod AB. The point C in the rod is guided in the circular path with D as centre and DC as radius. The dimensions of various links are: OA = 150

mm ; AB = 550 mm ; AC = 450 mm ; DC = 500 mm ; BE = 350 mm. Determine the velocity and acceleration of the ram E for the given position of the mechanism. (14)



All dimensions in mm.

Solution:

Given : $N_{AO} = 150$ r.p.m or $\omega_{AO} = 2\pi \times 150/60 = 15.71$ rad/s; OA = 150 mm = 0.15 m; AB = 550 mm = 0.55 m ; AC = 450 mm = 0.45 m ; DC = 500 mm = 0.5 m ; BE = 350 mm = 0.35 m

We know that linear velocity of A with respect to O or velocity of A,

$$V_{AO} = \omega_{AO} \times OA$$

$$= 15.71 \times 0.15 = 2.36 \text{ m/s}$$

...(Perpendicular to OA)

Velocity of the ram E

First of all draw the space diagram, as shown in Fig. (a), to some suitable scale. Now

the velocity diagram, as shown in Fig. (b), is drawn as discussed below:

1) Since O and D are fixed points, therefore these points are marked as one point in the velocity diagram. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A, such that

vector oa $= v_{AO} = v_A = 2.36$ m/s

2) From point a, draw vector ac perpendicular to AC to represent the velocity of C with respect to A (i.e. v_{CA}), and from point d draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors ac and dc intersect at c.

3) Since the point B lies on AC produced, therefore divide vector ac at b in the same ratio as B divides AC in the space diagram. In other words $ac : cb = AC : CB$. Join ob. The vector ob represents the velocity of B (i.e. v_B)

4) From point b, draw vector be perpendicular to be to represent the velocity of E with respect to B (i.e. v_{EB}), and from point o draw vector oe parallel to the path of motion of the ram E (which is horizontal) to represent the velocity of the ram E. The vectors be and oe intersect at e.

By measurement, we find that velocity of C with respect to A,

$$v_{CA} = \text{vector ac} = 0.53 \text{ m/s}$$

Velocity of C with respect to D,

$$v_{CD} = v_C = \text{vector dc} = 1.7 \text{ m/s}$$

Velocity of E with respect to B,

$$v_{EB} = \text{vector be} = 1.93 \text{ m/s}$$

and velocity of the ram E, $v_E = \text{vector oe} = 1.05$ m/s

Acceleration of the ram E

We know that the radial component of the acceleration of A with respect to O or the

acceleration of A,

$$a^r_{AO} = \frac{V^2_{AO}}{AO} = \frac{(2.36)^2}{0.15} = 37.13 \text{ m/s}^2$$

Radial component of the acceleration of C with respect to A,

$$a_{AC}^{r} = \frac{V_{CA}^{2}}{OA} = \frac{(0.53)^2}{0.45} = 0.624\ m/s^2$$

Radial component of the acceleration of C with respect to D,

$$a_{CD}^{r} = \frac{V_{CD}^{2}}{CD} = \frac{(1.7)^2}{0.5} = 5.78\ m/s^2$$

Radial component of the acceleration of E with respect to B,

$$a_{EB}^{r} = \frac{V_{EB}^{2}}{BE} = \frac{(1.93)^2}{0.35} = 10.64\ m/s^2$$

The acceleration diagram, as shown in Fig (c), is drawn as discussed below:

1. Since O and D are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector o'a' parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

$$o'a' = a_{AO}^{r} = a_A = 37.13\ m/s^2$$

2. From point d', draw vector d'x parallel to DC to represent the radial component of the acceleration of C with respect to D, such that

$$d'x' = a_{CD}^{r} = a_A = 5.78\ m/s^2$$

3. From point x, draw vector xc' perpendicular to DC to represent the tangential component of the acceleration of C with respect to D (i.e. a_{CD}^{t}) whose magnitude is yet unknown.

4. Now from point a', draw vector a'y parallel to AC to represent the radial component of the acceleration of C with respect to A, such that

$$a'y = a_{CA}^{r} = 0.624\ m/s^2$$

5. From point y, draw vector yc' perpendicular to AC to represent the tangential component of acceleration of C with respect to A (i.e. a_{CA}^{t}). The vectors xc' and yc' intersect at c'.

6. Join a'c'. The vector a'c' represents the acceleration of C with respect to A (i.e. a_{CA}).

7. Since the point B lies on AC produced, therefore divide vector a'c' at b' in the same ratio as B divides AC in the space diagram. In other words, $a'c' : c'b' = AC : CB$.

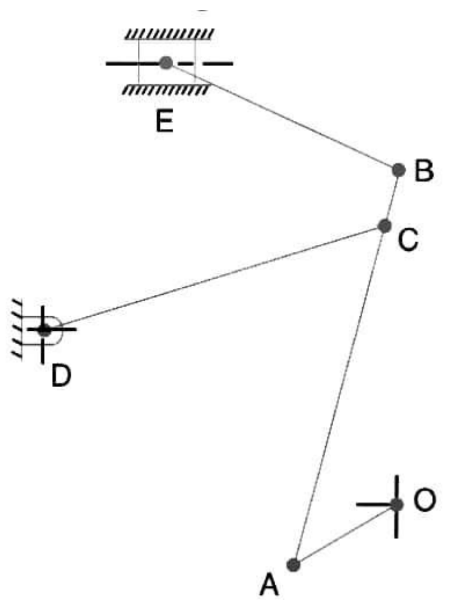
8. From point b', draw vector b' z parallel to BE to represent the radial component of the acceleration of E with respect to B, such that

$$b'z = a_{EB}^{r} = 10.64 \text{ m/s}^2$$

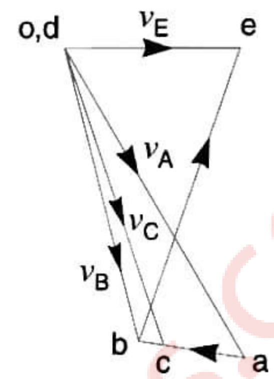
9. From point z, draw vector ze' perpendicular to BE to represent the tangential component of the acceleration of E with respect to B (i.e. a_{EB}^{t}) whose magnitude is yet unknown.

10. From point o', draw vector o'e' parallel to the path of motion of E (which is horizontal) to represent the acceleration of the ram E. The vectors ze' and o'e' intersect at e'. By measurement, we find that the acceleration of the ram E,

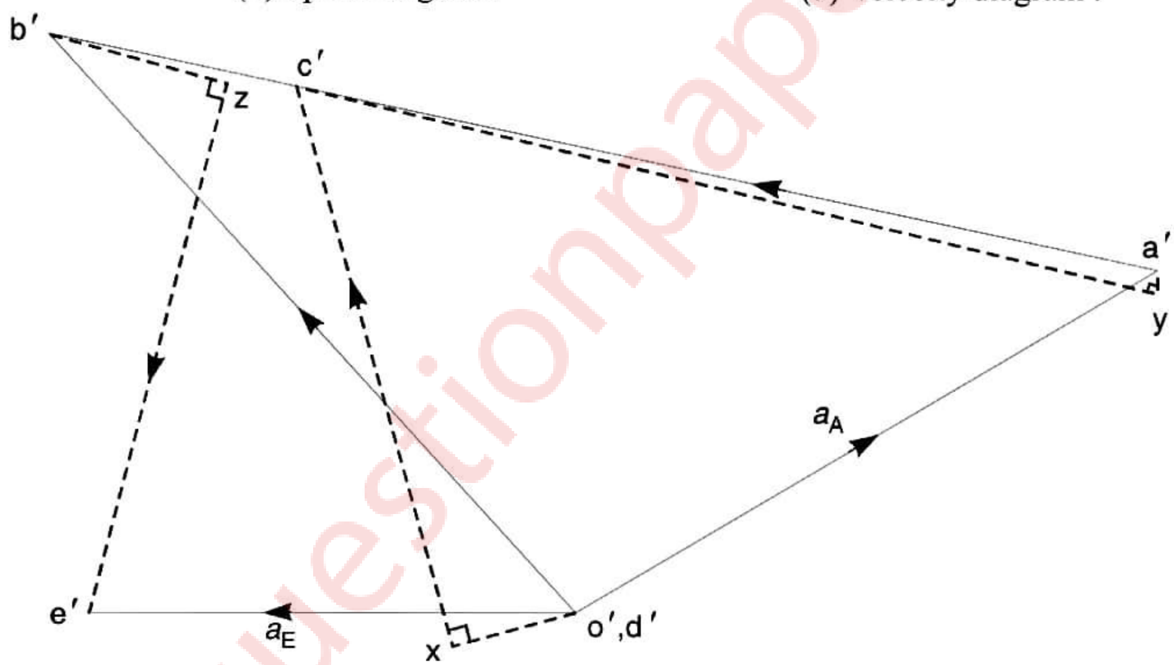
$$a_E = \text{vecor } o'e' = 3.1 \text{ m/s}^2$$



(*a*) Space diagram.



(*b*) Velocity diagram .

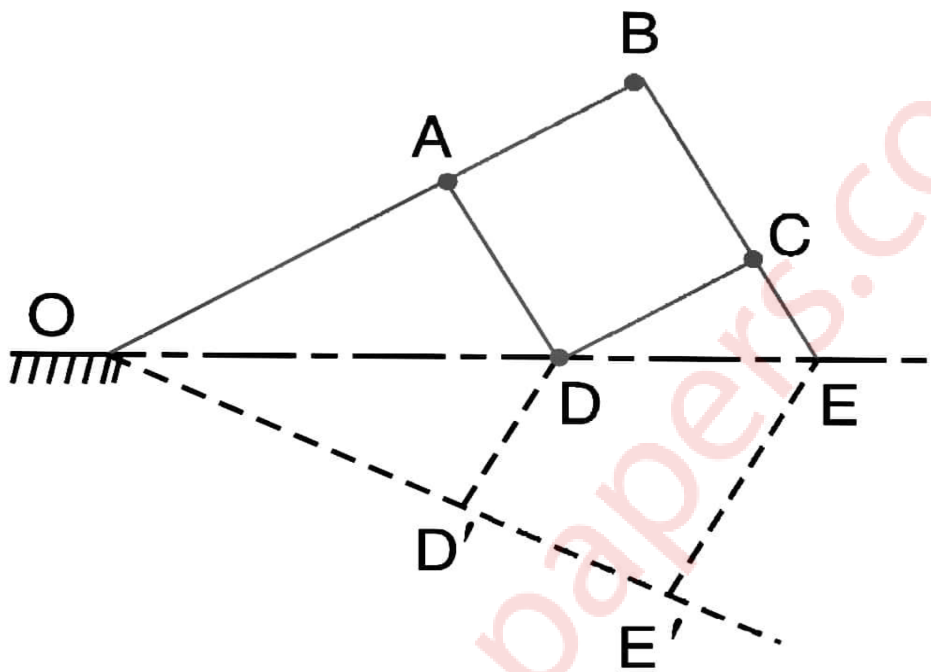


(*c*) Acceleration diagram.

Q 4) B) What is pantograph? Show that it can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or reduced scale. (06)

Solution:

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.



It consists of a jointed parallelogram *ABCD* as shown in Fig. 9.1. It is made up of bars connected by turning pairs. The bars *BA* and *BC* are extended to *O* and *E* respectively, such that

$$OA/OB = AD/BE$$

Thus, for all relative positions of the bars, the triangles *OAD* and *OBE* are similar and the points *O*, *D* and *E* are in one straight line. It may be proved that point *E* traces out the same path as described by point *D*.

From similar triangles *OAD* and *OBE*, we find that

$$OD/OE = AD/BE$$

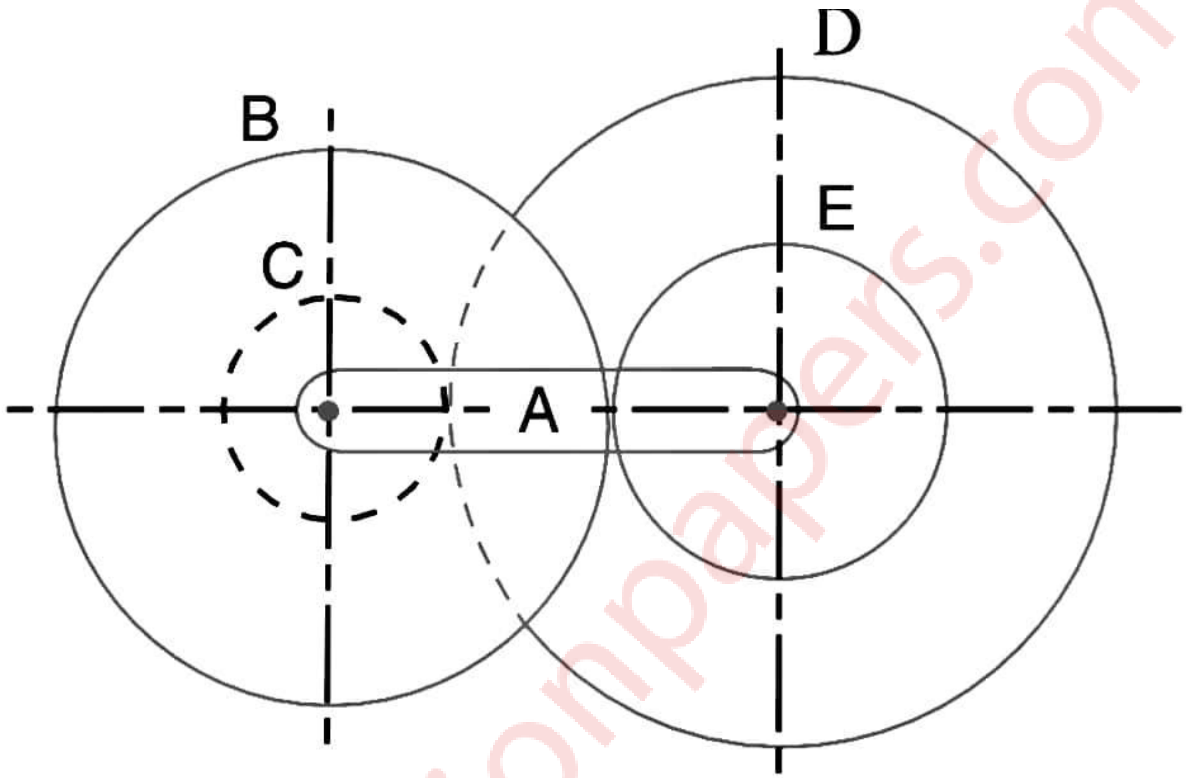
Let point *O* be fixed and the points *D* and *E* move to some new positions *D*′ and *E*′. Then

$$OD/OE = OD'/OE'$$

A little consideration will show that the straight line *DD*′ is parallel to the straight line *EE*′. Hence, if *O* is fixed to the frame of a machine by means of a turning pair and *D* is attached to a point in the machine which has rectilinear motion relative to the frame, then *E* will also trace out a straight line path. Similarly, if *E* is constrained to move in a straight line, then *D* will trace out a straight line parallel to the former.

Q 5) A) In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise. (08)

Solution.



Given: $T_B = 75$; $T_C = 30$; $T_D = 90$;

$N_A = 100$ r.p.m. (clockwise)

The reverted epicyclic gear train is shown in Fig.. First of all, let us find the

number of teeth on gear E (T_E). Let d_B , d_C , d_D

and d_E be the pitch circle diameters of gears B, C, D and E respectively. From the geometry of the figure,

$$d_B + d_E = d_C + d_D$$

Since the number of teeth on each gear, for the same module, are proportional to their pitch circle diameters, therefore

$$T_B + T_E = T_C + T_D$$

$$\therefore \; T_E = T_C + T_D - T_B$$

$$= 30 + 90 - 75 = 45$$

The table of motions is drawn as follows :

Steps	Operation	Revolutions of elements (N)			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear D-E rotated through + 1 revolution (i.e. 1 rev. anticlockwise)	0	$+1$	$-\frac{t_E}{t_B}$	$-\frac{t_D}{t_C}$
2.	Arm fixed-compound gear D-E rotated through + x revolutions	0	+x	$-x.\frac{t_E}{t_B}$	$-x.\frac{t_D}{t_C}$
3.	Add 'y' revolutions to all elements	+y	+y	+y	+y
4.	Total motion	+y	$x + y$	$y - x.\frac{t_E}{t_B}$	$y - x.\frac{t_D}{t_C}$

Since the gear B is fixed, therefore from the fourth row of the table,

$y - x.\frac{t_E}{t_B} = 0$ or $y - x.\frac{45}{75} = 0$

$y - 0.6 = 0$ ...(i)

Also the arm A makes 100 r.p.m. clockwise, therefore

$$y = -100 \quad ...(ii)$$

Substituting $y = -100$ in equation (i), we get

$$-100 - 0.6\,x = 0 \text{ or } x = -100 / 0.6 = -166.67$$

From the fourth row of the table, speed of gear C,

$$N_C = y - x.\frac{t_D}{t_C} = -100 - 166.67 \times \frac{90}{30} = +400 \text{ rpm}$$

$= 400 \text{ r.p.m (anticlockwise)}$

Q 5) B) A sphere of radius 0.2m starts rolling without slip up an inclined at an angle of 300 wit the horizontal. If the initial velocity of sphere 10rad/s. Determine how far sphere will travel before it reverse its motion. (06)

Solution:

Consider a sphere of mass m and radius r rolled down from position 1 to position 2.

Calculation of work done

1) W.D bygravity force $= \ mgh \ = \ mg.S\sin\theta$

2) No W.D by frictional force as body rolls without sliding

Energy calculation : $KE_1 = 0$

$KE_2 = \frac{1}{2} I_e.\omega^2$

Where I_e = Mass M.I about point of contact

By work-enegy principle

$$U_{1\to 2} = KE_1 - KE_2$$

$mg.S\sin\theta = \frac{1}{2} I_e.\omega^2 - 0$

$\omega^2 = \frac{2\ mg.S\sin\theta}{I_e}$

but $\frac{v^2}{r^2} = \frac{2\ mg.S\sin\theta}{I_e}$

$v = \sqrt{\frac{2\ mg.S\sin\theta.r^2}{I_e}}$

For sphere $I_e = I_G + mk^2 = \frac{1}{2}\ mr^2 + mr^2 = \frac{7}{5}\ mr^2$

velocity of center $V = \sqrt{\frac{2\ mg.S\sin\theta.r^2}{7/5\ mr^2}}$

$v = 0.845\sqrt{2\ gS\sin\theta}$

$\omega.r = 0.845\sqrt{2\ gS\sin\theta}$

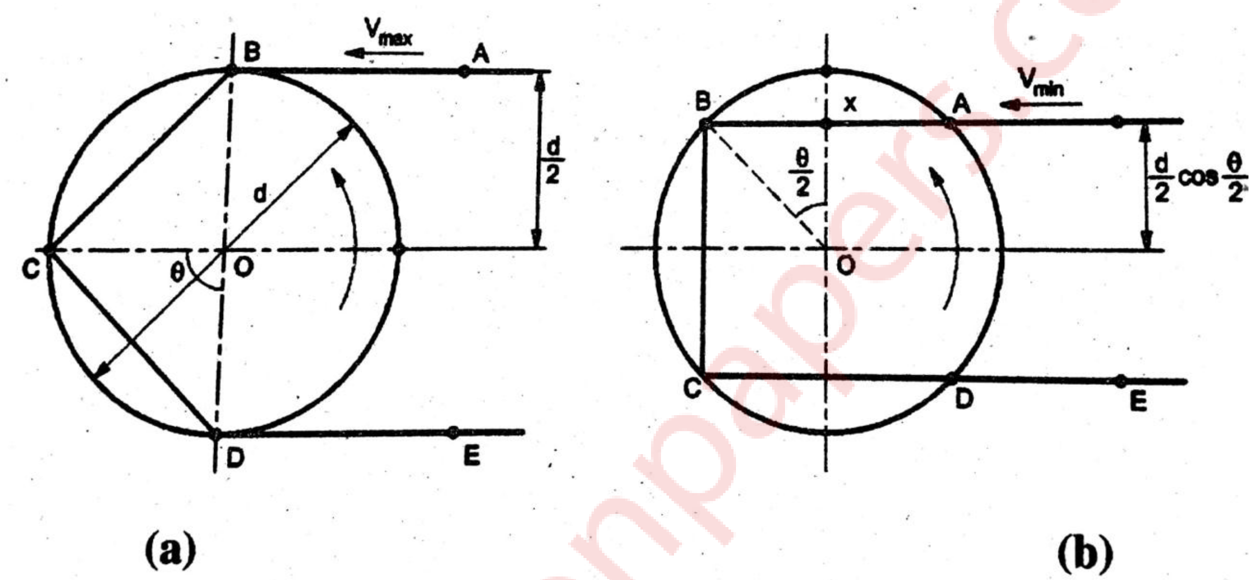
$10 \times 0.2 = 0.845\sqrt{2 \times 9.81 \times S\sin 30°}$

$S = 0.5710\ m$

Q 5) C) Explain chordal action in chain drive. (06)

Solution:

The chain passes around a sprocket as a series of chordal links. This action is similar to that of non-slipping belt wrapped around a rotating polygon. The chordal action is shown in fig. (a) and (b) where the sprockets has only four teeth. It is assumed that the sprocket is rotating at a constant speed of N rpm, the chain AB is at a distance of $\left(\frac{d}{2}\right)$ from the center of the sprocket as shown in fig (a).



$(V_{max} - V_{min}) \infty \left[1 - \cos\left(\frac{180°}{T}\right)\right]$

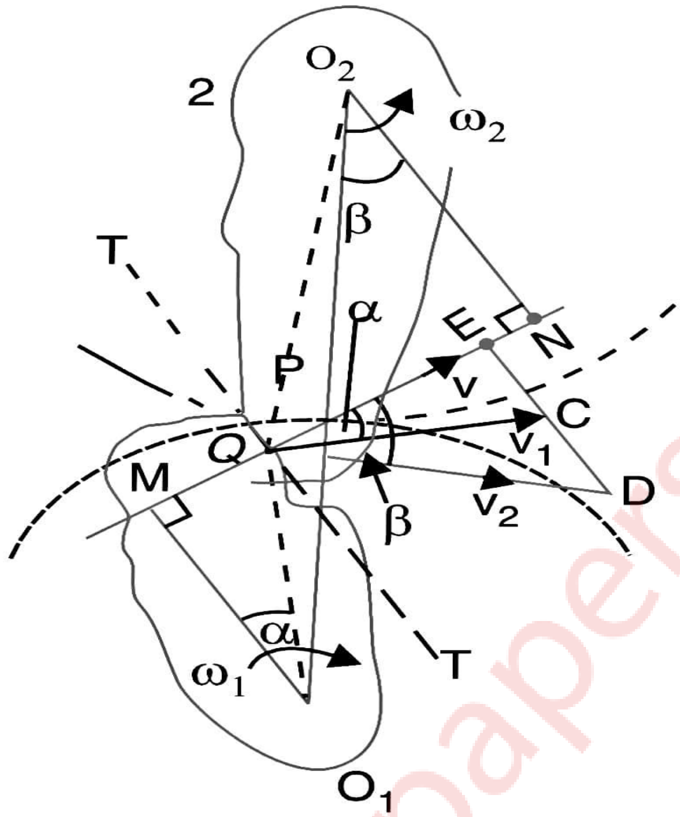
From the above equation it is clear that as number of teeth increases, the variation in linear velocity will be goes on decreasing.

Q 6) A) Prove that the velocity of sliding in gears is proportional to the distance of the point of contact from the pitch point . (08)

Solution:

The sliding between a pair of teeth in contact at Q occurs along the common tangent T T to the tooth curves as shown in Fig. The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact.

The velocity of point Q, considered as a point on wheel 1, along the common tangent T T is represented by EC. From similar triangles QEC and Q_1MQ,



$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_2 \quad \text{or} \quad EC = \omega_1 . MQ$$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent T T is represented by ED. From similar triangles QCD and O_2NQ,

$$\frac{ED}{QN} = \frac{v}{O_2Q} = \omega_2 \quad \text{or} \quad ED = \omega_2 . QN$$

Let v_S = Velocity of sliding at Q.

$$v_S = ED - EC = \omega_1 . MQ - \omega_2 . QN$$

$$= \omega_2(QP + PN) - \omega_1(MP - QP)$$

$$= (\omega_1 + \omega_2)QP + \omega_2 . PN - \omega_1 MP \quad \text{...(i)}$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{PM}$ or $\omega_1 MP = \omega_2 . PN$ therefore equation (i) becomes

$$v_s = (\omega_1 + \omega_2)QP \quad \text{...(ii)}$$

We see from equation (ii), that the velocity of sliding is proportional to the distance of the point of contact from the pitch point.

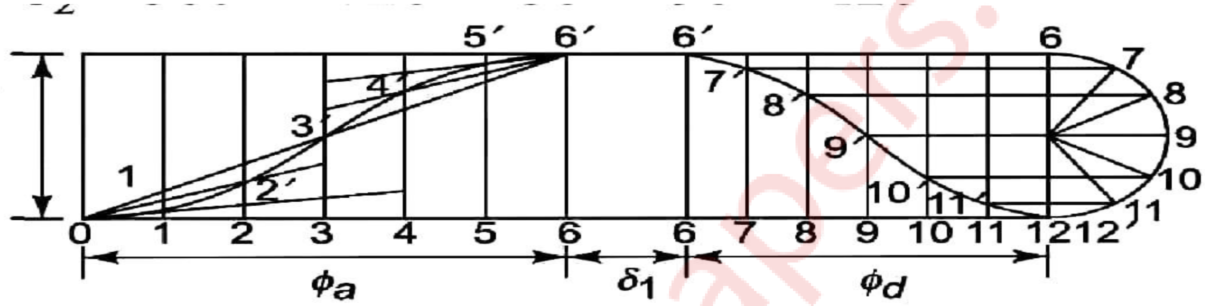
Q 6) B) A cam is rotating at 800 rpm operate a reciprocating knife edge follower. The least radius of cam is 30mm , stroke of follower is 30mm. Ascent takes place by uniform acclereation and deceleration and descent by simple harmonic motion. Ascent take place by 120° and descent during 90° of cam rotation. Dwell between ascent and descent 30°. Sketch displacement, velocity and acceleration. (12)

Solution:

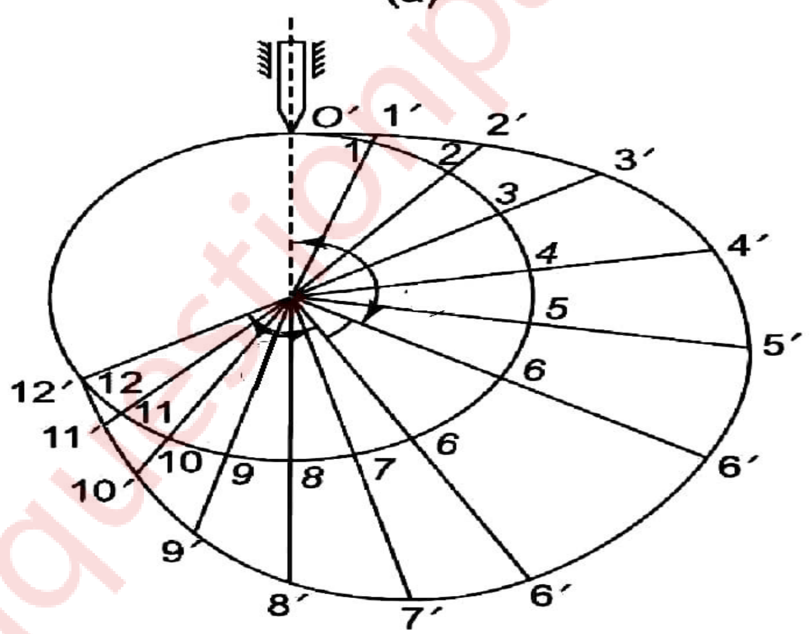
Cam speed $N = 180$ rpm; $h = 30$ mm; $r_c = 30$ mm

$\emptyset_a = 120°$; $\emptyset_d = 90°$; $\delta_1 = 30°$

$\delta_2 = 360° - 120° - 30° - 90° = 120°$



(a)



(b)

Draw the displacement diagram of the follower as shown in fig. (a). As the rotation of the cam shaft is counter-clockwise, the cam profile is to be drawn assuming the cam to be stationary and the follower rotating clockwise about the cam. Construct the cam profile as described below (fig. b):

i) Draw a circle with radius r_c.

ii) From the vertical position, mark angles $\emptyset_a$, δ_1, $\emptyset_d$, and δ_2 in the clockwise direction.

iii) Divide the angles $\emptyset_a$ and $\emptyset_d$ into same number of parts as is done in the displacement diagram. In this case, $\emptyset_a$ as well as $\emptyset_d$ have been divided into 6 equal parts.

iv) On the radial lines produced, mark the distances from the displacement diagram.

v) Draw a smooth curve tangential to end points of all radial lines to obtain the required cam profile.

The displacement diagram is reproduced in fig. (a). The velocity and aceleration diagrams are to be drawn below this figure.

$$\therefore\ \omega = \frac{2\pi N}{60} = 88 \text{ rad/sec}$$

i) During ascent,

During the ascent period, the acceleration and the deceleration are uniform. Thus, the velocity is linear and is given by

$$v = \frac{4h\omega}{{\emptyset_a}^2}.\theta$$

The maximum velocity is at the end of the acceleration period, i.e., When $\theta = \varphi_{a/2}$

$$v_{max} = 2h\frac{\omega}{\emptyset_a} = 2 \times 0.03 \times \frac{18.85}{60\pi/180} = 2.52\ m/s$$

The plot of velocity variation during the ascent period is shown in fig. (b)

$$f_{uniform} = \frac{4\ h\ \omega^2}{(\emptyset_a)^2} = \frac{4\times0.03\times\ (88)^2}{\left(\frac{120\pi}{180}\right)^2}$$

$$= 211.9\ m/s^2$$

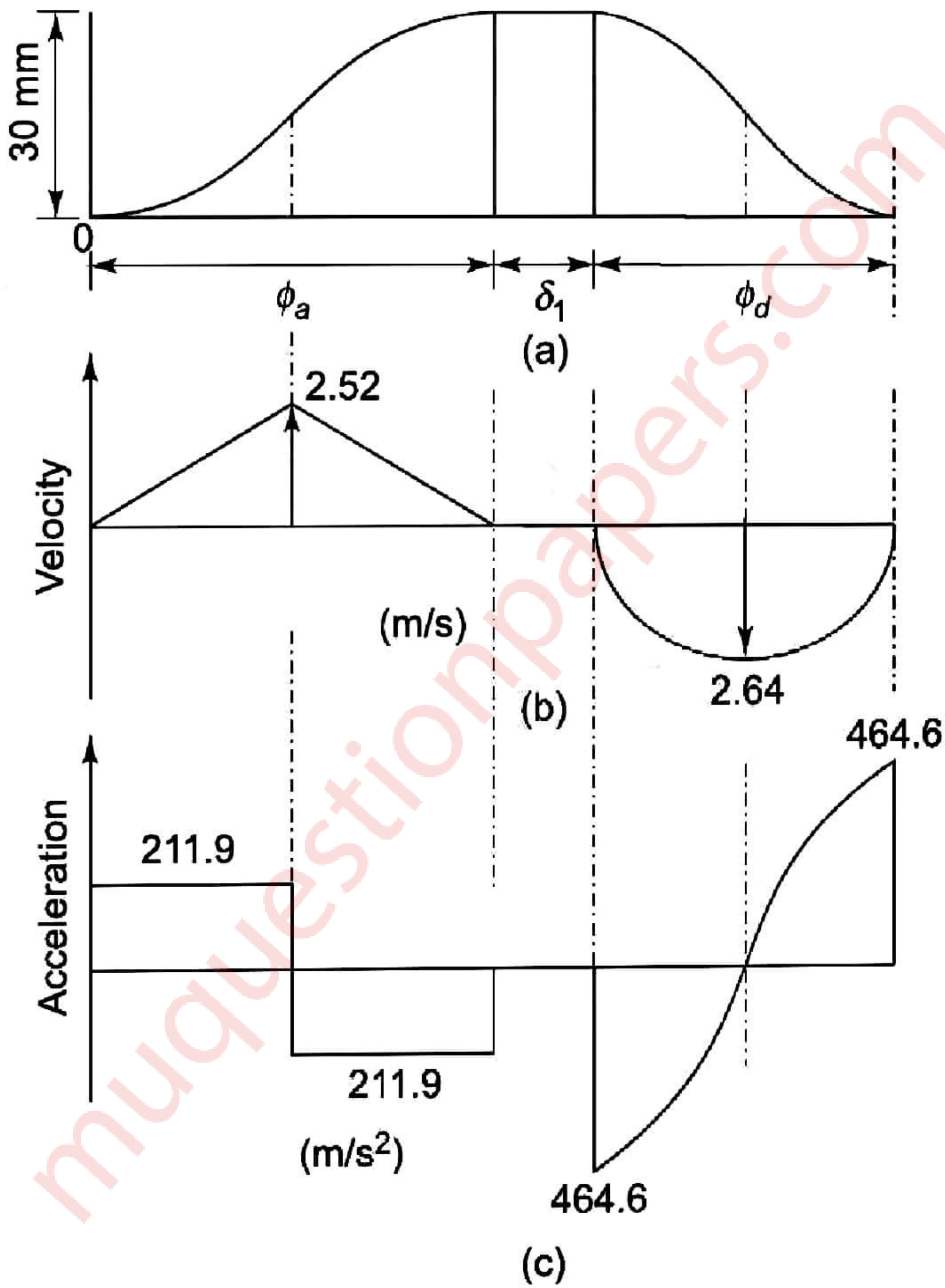
This has been shown in fig .(c)

During descent

ii) During descent it is a simple harmonic motion

The variation of velocity is given by

This variation is shown in fig



$$\mathrm{v} = \frac{h}{2}\frac{\pi\omega}{\emptyset_{\mathrm{d}}}\, sin\, \frac{\pi\theta}{\emptyset_{\mathrm{d}}}$$

maximum value is at $\theta = \emptyset_{\mathrm{d}}/2$

$$\mathrm{v}_{\max} = \frac{h}{2}\frac{\pi\omega}{\emptyset_{\mathrm{d}}} = \frac{0.03}{2} \times \frac{\pi\times 88}{90\pi/180}$$

$$= 2.64\ m/s$$

The plot of velocity variation during the descent period is shown in fig (b)

Acceleration variation is given by

$$\mathrm{f} = \frac{h}{2}\left(\frac{\pi\ \omega}{(\emptyset)}\right)^2 cos\, \frac{\pi\theta}{\varphi}$$

it is max at $\theta = 0\ i.e.$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\ \omega}{(\emptyset)}\right)^2 = \frac{0.03}{2} \times \left(\frac{\pi\times 88}{90\pi/180}\right)^2$$

$$= 464.6\ \ \mathrm{m/s^2}$$