

THERMODYNAMICS SOLUTION

NOV 2019 / SEM3 / MECHANICAL

Q.1(a) State and prove Carnot's theorem.

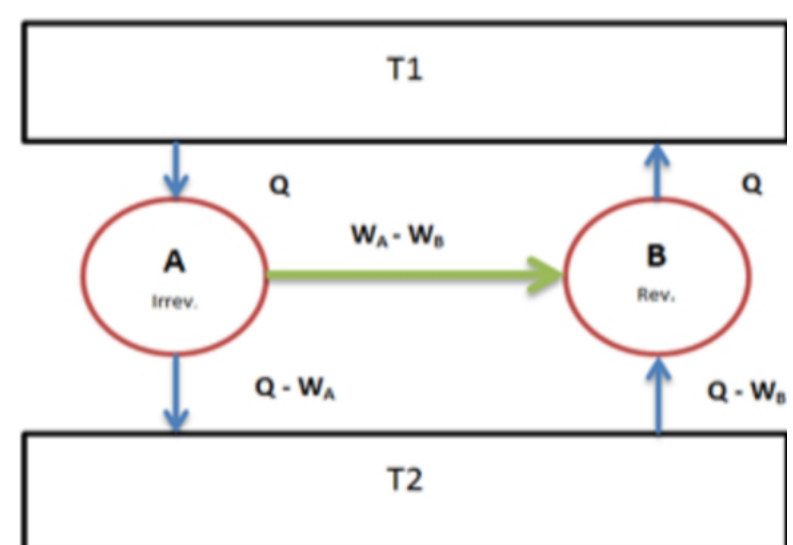
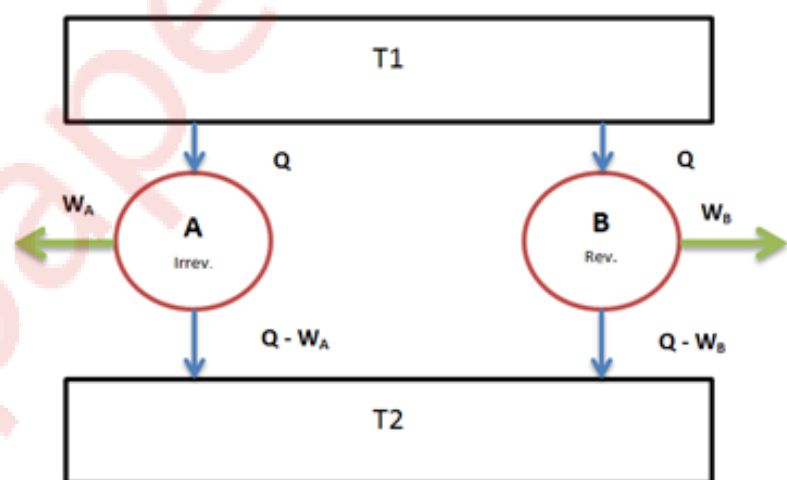
(5M)

Ans: Carnot theorem states that no heat engine working in a cycle between two constant reservoirs can be more efficient than a reversible engine working between the same reservoirs. In other words it means that all the engines operating between a given constant temperature source and a given constant temperature sink, none, has a higher efficiency than a reversible engine.

Proof: Suppose there are two engines E_A and E_B operating between the given source at temperature T_1 and the given sink at temperature T_2 . Let E_A be any irreversible heat engine and E_B be any reversible heat engine. We have to prove that efficiency of heat engine E_B is more than that of heat engine E_A . Suppose both the heat engines

receive same quantity of heat Q from the source at temperature T_1 . W_A and W_B be the work output from the engines and their corresponding heat rejections be $(Q - W_A)$ and $(Q - W_B)$ respectively. Assume that the efficiency of the irreversible engine be more than the reversible engine i.e. $\eta_A > \eta_B$. Hence, $W_A Q > W_B Q$ i.e. $W_A > W_B$.

Now let us couple both the engines and E_B is reversed which will act as a heat pump. It receives $(Q - W_B)$ from sink and W_A from irreversible engine E_A and pumps heat Q to the source at temperature T_1 . The net result is that heat $W_A - W_B$ is taken from sink and equal amount of work is produced. This violates second law of thermodynamics. Hence the assumption we made that irreversible engine having higher efficiency than the reversible engine is wrong.

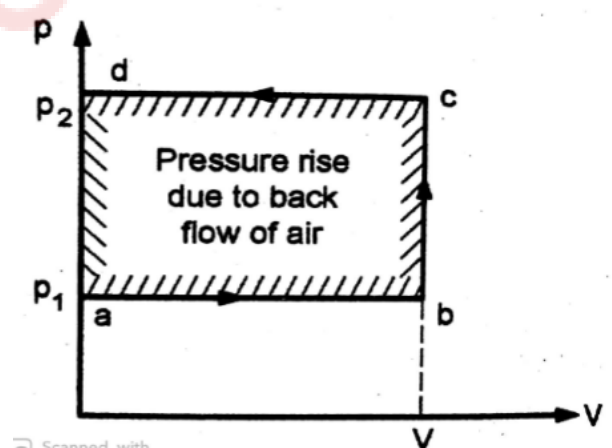
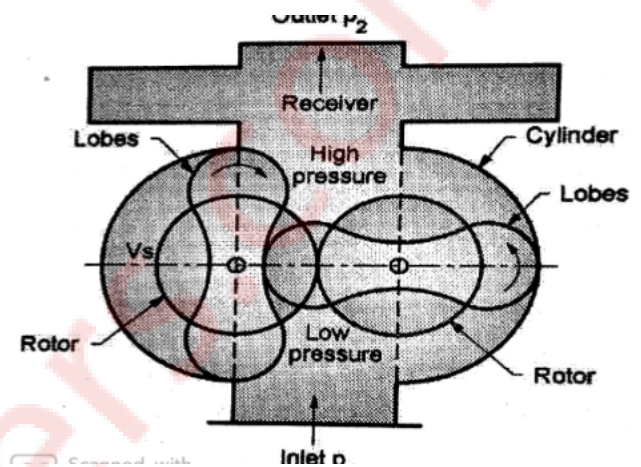


Hence it is concluded that reversible engine working between same temperature limits is more efficient than irreversible engine thereby proving Carnot's theorem.

(b) Explain working principle of roots blower and P-V diagram for it. (5M)

Ans: The root blower shown in Figure consists of two lobes mounted on different rotors of epicycloidal or involute profiles rotating in opposite directions.

Construction : These profiles help in keeping the smooth motion and correct mating between the lobes. One of the rotors is driven externally and the other is driven through the gears from the first. The rotors with more than two lobes are sometimes used when an increase in pressure ratio is required. A small clearance between the lobes and the cylinder surfaces is maintained to prevent wear. However, due to the clearance between the surfaces, there are internal leaks and it reduces its volumetric efficiency.



The working of root blower : Gas is taken at intake at pressure and as the rotor rotates, a mass of gas equal to volume V corresponding to volume of each face of the lobe transferred to the receiver side. At this instant some high pressure gas will rush back from the receiver and it mixes with the volume V until the pressure is equalised. This process is known as back low process. The air is now delivered to the receiver by the rotation of the lobes at pressure P_2 . The process is represented on (p-V) diagram in Fig. It should be noted that the air delivered will be 4 times the volume V for two kobe rotors and 0 times of volume V in case of three lobe rotors. The work required to drive the root blower with two lobes per revolution is $W = 4V (P_2 - P_1)$

(c) What is the difference between heat and internal energy. (5M)

Ans.: Internal energy is the energy stored in a body. It increases when the temperature of the body rises, or when the body changes from solid to liquid or from liquid to gas. Internal energy is the sum of kinetic and potential energy of all particles in the body. Unit of internal energy: joule (J)

Heat is the energy transferred from one body to another as a result of a temperature difference. When two bodies of different temperatures touch each other, energy is transferred from the hot body to the cold body. Typically the two bodies reach the same temperature. The bodies are then said to be in thermal equilibrium.

(d) Why is Carnot cycle not practicable for steam power plant. (5M)

Ans: 1. Firstly, Carnot cycle involves transition from an isothermal process to an adiabatic process. This is impossible to attain practically because isothermal is very slow and adiabatic is very fast.

2. The heat exchange in Carnot cycle is isothermal which needs the working fluid to either phase change or the process be extremely slow.

Thus constant pressure process is used in Rankine cycle.

3. In Rankine cycle, the pumping losses are less because the pumping starts at saturated water condition which is easy to pump. In Carnot, the pumping work, from diagram, is comparable to the turbine work.

4. Also, in Carnot cycle wet fluid is at the turbine exit and pump inlet. This increases the chances of blade corrosion in the open flow devices.

(e) Calculate the state of steam (i.e. whether it is dry, wet or superheated). when steam has a pressure of 15 bar and specific volume of $0.12 \text{ m}^3/\text{kg}$. (5M)

Ans.: Given that $P=15\text{bar}$, $V_{\text{specific}} = 0.12 \text{ m}^3/\text{kg}$.

For finding nature of the steam we have to check dryness fraction

So, from steam table we get $V_g = 0.132 \text{ m}^3/\text{kg}$, $T_s = 198.3^\circ\text{C}$

By formula , $V_{\text{specific}} = x \times V_g$

$$0.12 = x \times 0.132$$

$$x = 0.9 \gg \text{Nature of steam is wet steam}$$

Note : x is dryness fraction

x < 1 (wet steam), x = 1 (dry steam) , x > 1 (saturated steam)

Q.2 (a) In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are 50 m/s and 110 m/s respectively. Calculate:(i) The rate at which heat is rejected to the turbine, and ii) The area of the inlet pipe, given that the specific volume of the gases at the inlet is 0.45 m³/kg. (10M)

Ans. Mass of gas, m = 15kg/s , Power P = 12000KW ,

Enthalpy at inlet, h₁ = 1260KJ/kg, Enthalpy at outlet, h₂ = 400KJ/kg,

Velocity at inlet, C₁ = 50m/s, Velocity at outlet, C₂ = 110m/s,

Specific Volume, V = 0.45m³/kg

1) By Steady Flow Energy Equation,

$$Q - W = \Delta h + \Delta PE + \Delta KE \quad \dots (1)$$

$$\Delta KE = \frac{C_2^2 - C_1^2}{2} = 4.8 \text{KJ/Kg}$$

$$\Delta h = h_2 - h_1 = -860 \text{KJ/kg}$$

$$\Delta PE = 0$$

Now , P = m/W

$$\gg W = P/m = 12000/15$$

$$\gg W = 800 \text{KJ/kg}$$

So , Equation (1) will become

$$Q - 800 = -860 + 0 + 4.8 \gg Q = - 55.2 \text{KJ/kg}$$

i.e Heat rejected = 55.2 × 15 = 828KW ...Ans

2) Also we have , mass Flow rate , $m = \rho_1 A_1 C_1 = A_1 C_1 / V$

$$15 = A_1 \times 50 / 0.45$$

Area of inlet pipe , $A_1 = 0.132\text{m}^2 \dots \text{Ans}$

(b) Show that heat transferred through finite temperature difference is irreversible. (5M)

Ans. Whenever heat is transferred through a finite temperature difference, there is a decrease in the availability of energy so transferred.

Let us consider a reversible heat engine operating between T_1 and T_0 (Fig 1)

$$Q_1 = T_1 \cdot \Delta S \quad \& \quad Q_0 = T_0 \cdot \Delta S \quad \text{and} \quad W = A.E = (T_1 - T_0) \cdot \Delta s \dots (1)$$

Let us now assume that heat Q_1 is transferred through a finite temperature difference from the reservoir or source at T_1 to the engine absorbing heat Q_1 at lower than T_1' (Fig 2).

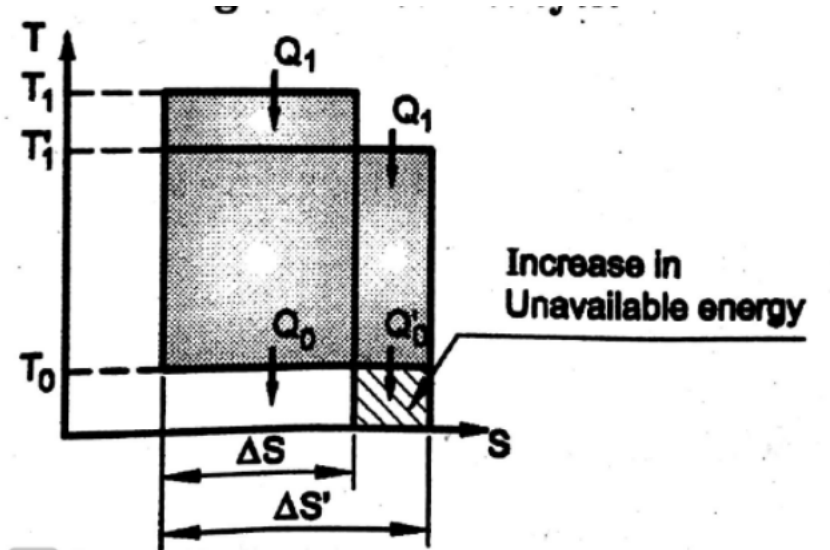
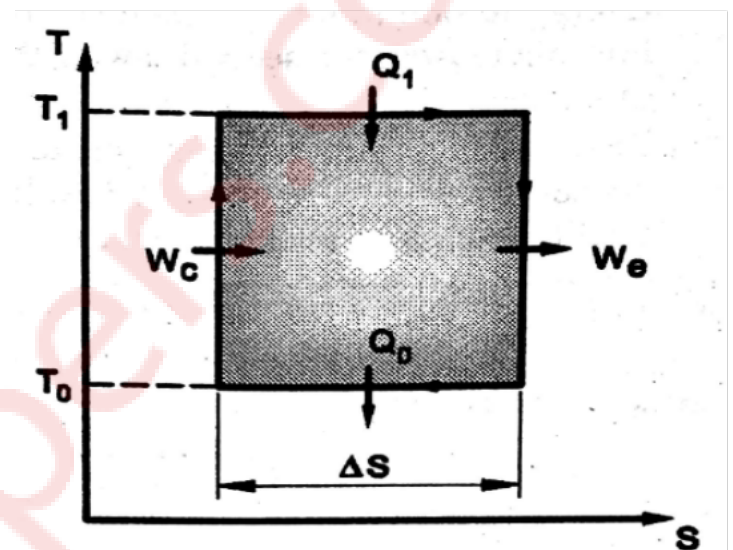
The availability of Q_1 as received by the engine

at T_1' can be found by allowing the engine to operate reversibly in a cycle between T_1' and T_0' as shown in figure 2

During the cycle it rejects heat Q_0' to the surrounding.

$$\text{Now } Q_1 = T_1 \cdot \Delta S = T_1' \cdot \Delta S' \dots (2)$$

$$\text{Since , } T_1 > T_1' \text{ , it implies that , } \Delta S' > \Delta S \dots (3)$$



$$Q_o = T_o \cdot \Delta S \text{ and } Q_o' = T_o \cdot \Delta S' \dots(4)$$

It is obvious from equations (3) and (4) that $Q_o' > Q_o$

Also, Q_o' being greater than Q_o , therefore $W' < W$

$$\text{Loss of available energy} = W - W' = (Q_o' - Q_o)$$

$$\text{Loss of available energy} = T_o(\Delta S' - \Delta S)$$

Available energy or exergy lost due to irreversible heat transfer through finite temperature difference between the source and the working fluid of the engine. Greater is the temperature difference greater is heat rejection and greater will be unavailable energy.

(c) A system at 500 K receives 7200 kJ min from a source at 1000 K. The temperature of atmosphere is 300 K. Assuming that the temperatures of system and source remain constant during heat transfer find out: (i) The entropy produced during heat transfer; (ii) The decrease in available energy after heat transfer. (5M)

Ans. Temperature of source, $T_1 = 1000 \text{ K}$

Temperature of system, $T_2 = 500 \text{ K}$

Temperature of surroundings, $T_o = 300 \text{ K}$

Heat given by source, $Q = -120 \text{ KJ/s}$

Heat received by the system, $Q = 120 \text{ KJ/S}$

$$\begin{aligned} \text{Entropy change of heat source, } (\Delta S)_{\text{source}} &= Q/T_1 = -120/1000 \\ &= -0.12 \text{ KJ/sK} \end{aligned}$$

$$\begin{aligned} \text{Entropy change of system, } (\Delta S)_{\text{system}} &= Q/T_2 = 120/500 \\ &= 0.24 \text{ KJ/sK} \end{aligned}$$

(i) Net change in entropy

$$(\Delta S)_{\text{system}} = (\Delta S)_{\text{source}} + (\Delta S)_{\text{system}} = -0.12 + 0.24 = 0.12 \text{ KJ/sK}$$

(ii) Available energy of heat source

$$A1 = (T1 - T_o) (\Delta S)_{\text{fluid}} = (1000 - 300)(0.12) = 84 \text{ KJ/s}$$

(iii) Available energy of system

$$A2 = (T2 - T_o) (\Delta S)_{\text{system}} = (500 - 300)(0.24) = 48 \text{ KJ/s}$$

(iv) Decrease in available energy

$$A1 - A2 = 84 - 48 = 36 \text{ KJ/s}$$

Q 3 (a) Three reversible engines of Carnot type are operating in series between the limiting temperatures of 1100 K and 300 K. Determine the intermediate temperatures if the work output from engines is in proportion of 3:2:1. (10M)

Ans. Work outputs are in proportion $W1:W2:W3 = 3:2:1$... (1)

For engine R1

$$\text{Efficiency, } \eta_1 = \frac{T1 - T2}{T1} = \frac{W1}{Q1}$$

$$\gg W1 = Q1(T1 - T2)/T1 \quad \dots(2)$$

Similarly,

For engine R2 $W2 = Q2(T2 - T3)/T2 \quad \dots(3)$

For engine R3 $W3 = Q3(T3 - T4)/T3 \quad \dots(4)$

Assume,

$$Q1/T1 = Q2/T2 = Q3/T3 = \text{any constant}(A)$$

Now, Divide (2) by (3), we get

$$W1/W2 = \frac{T1 - T2}{T2 - T3} = 3/2$$

$$\gg 5T2 - 3T3 = 2200 \quad \dots(5)$$

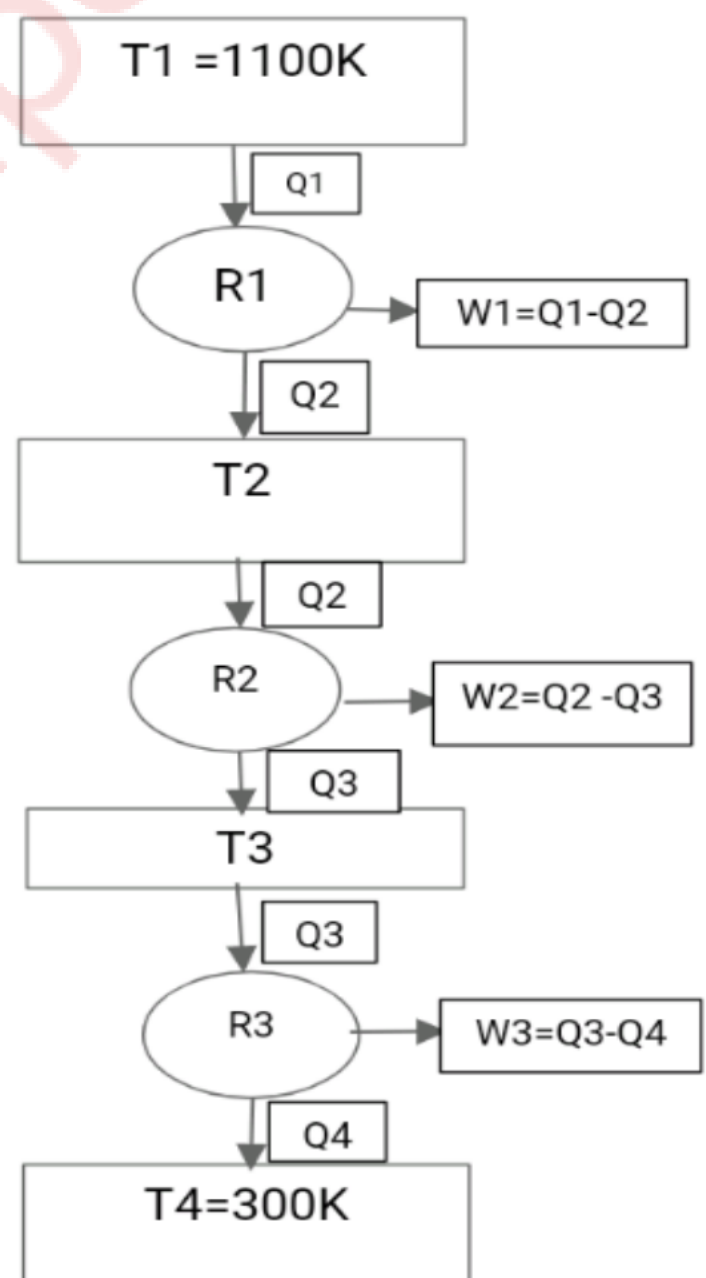
Divide (3) by (4), we get

$$W2/W3 = \frac{T2 - T3}{T3 - T4} = 2/1$$

$$\gg -T2 + 3T3 = 600 \quad \dots(6)$$

On solving equation (5) and (6) we get,

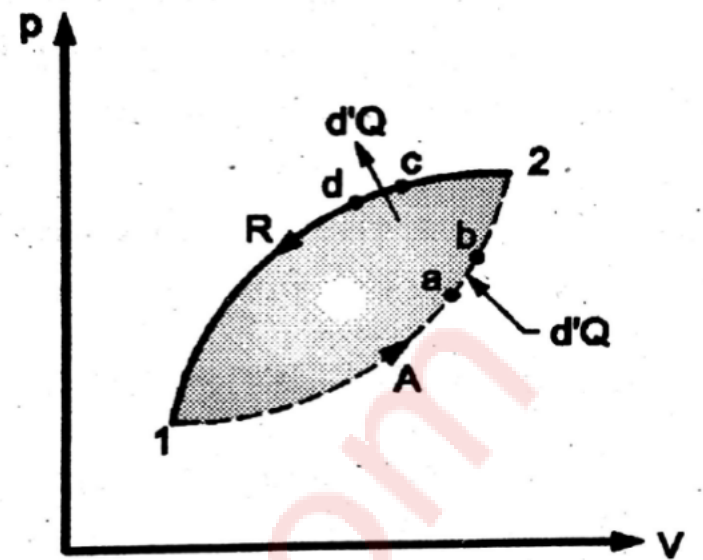
Intermediate Temperatures, $T2 = 700 \text{ K}$ and $T3 = 433.3 \text{ K}$



(b) Explain the principle of increase of entropy.

(5M)

Ans. Consider a cycle in which the system is taken from state 1 to 2 by an arbitrary process (Reversible or irreversible) along the path A and the cycle is completed by a reversible process by bringing the system from state 2 to 1 along the path R as shown in figure.



Applying Clausius inequality,

$$\oint \frac{d'Q}{T} \leq 0$$

$$\text{i.e.} \quad \int_{1A}^{2A} \frac{d'Q}{T} + \int_{2R}^{1R} \frac{d'Q}{T} \leq 0$$

Assume that during the arbitrary process A the system absorbs heat from a heat reservoir at temperature T, the entropy of surroundings will decrease by an amount given by the equation,

$$\int_{1A}^{2A} \frac{d'Q}{T} = -(\Delta S)_{\text{surr}}$$

Since the process (2R1) is reversible, it implies that the temperature of the system and surroundings at any instant is same and the heat energy will be transferred from the system to their surroundings during this process. Therefore, the entropy of the system will decrease and it is given as,

$$\int_{2R}^{1R} \frac{d'Q}{T} = -(\Delta S)_{\text{system}}$$

From the values of entropy change we get,

$$-(\Delta S)_{\text{surr}} - (\Delta S)_{\text{system}} \leq 0$$

$$(\Delta S)_{\text{surr}} + (\Delta S)_{\text{system}} \geq 0$$

$$\text{i.e. } (\Delta S)_{\text{universe}} \geq 0$$

Above equations show that the entropy change of the system and its surroundings i.e universe during any process between two equilibrium states is always equal to or greater than zero. This is known as principle of increase of entropy.

(c) Derive the first and second T-dS equations. (5M)

Ans. Entropy of a gas can be expressed as a function of any two properties out of the three measurable properties P, V, T. It leads to first and second entropy equations.

Consider the entropy S as a function of temperature and volume: $S = S(T, V)$:

$$dS = \left(\frac{\partial S}{\partial T} \right)_P dT + \left(\frac{\partial S}{\partial P} \right)_T dP$$

We apply the definition of the heat capacity to the first term and a Maxwell relation to the second, and obtain

$$dS = \frac{C_p}{T} dT - \left(\frac{\partial V}{\partial T} \right)_P dP \quad \text{or}$$
$$TdS = C_p dT - T \left(\frac{\partial V}{\partial T} \right)_P dP \quad (\text{second } TdS \text{ equation})$$

The second TdS equation follows from considering S as a function of temperature and pressure: $S = S(T, P)$:

$$dS = \left(\frac{\partial S}{\partial T} \right)_P dT + \left(\frac{\partial S}{\partial P} \right)_T dP$$

We again use the definition of heat capacity and a Maxwell relation to obtain

$$dS = \frac{C_v}{T} dT + \left(\frac{\partial p}{\partial T} \right)_v dV \quad \text{or}$$

$$TdS = C_v dT + T \left(\frac{\partial p}{\partial T} \right)_v dV \quad (\text{first } TdS \text{ equation})$$

The TdS equations are frequently useful in deriving relationships among various thermodynamic derivatives.

Q.4 (a) In a thermal power plant operating on an ideal Rankine cycle, superheated steam produced at 5 MPa and 500°C is fed to a turbine where it expands to the condenser pressure of 10 kPa. If the net power output of the plant is to be 20 MW, determine: i) heat added in boiler in kJ/k ii) the thermal efficiency. iii) the mass flow rate of steam in kg/sec. (10M)

Ans. Pressure of boiler, $P_b = 5 \text{ MPa} = 5 \text{ bar}$

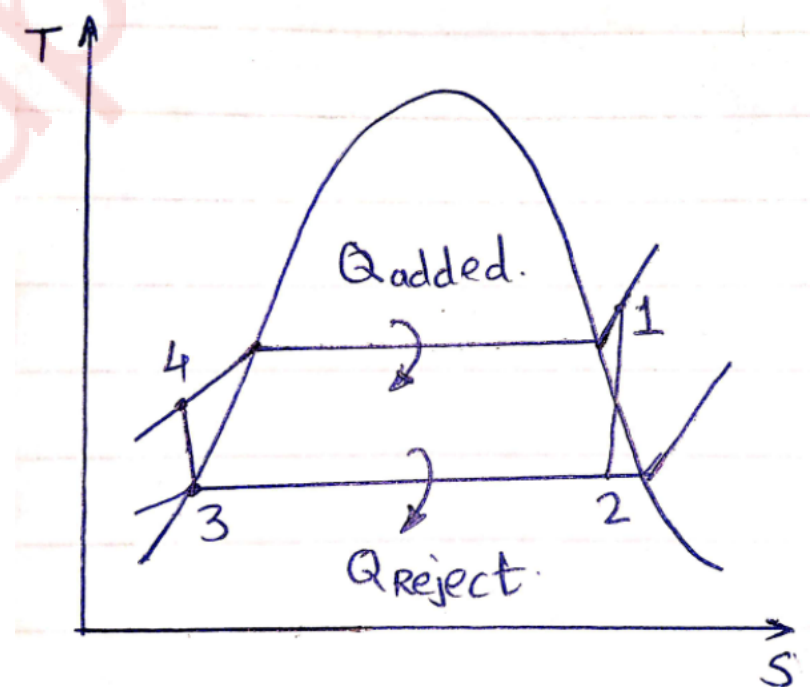
Temperature at boiler, $T = 500^\circ\text{C}$

Pressure of condenser, $P_c = 10 \text{ kPa} = 0.1 \text{ bar}$

Net power output, $W = 20 \text{ MW} = 20000 \text{ kW}$

It is given that steam is supersaturated.

For finding heat added in boiler, efficiency and mass flow rate we have to first find enthalpies at all the points (1,2,3,4) shown in figure.



1) Therefore, from steam table

At $P_b = 5 \text{ bar}$, $h_1 = 3433.7 \text{ kJ}$

$s_1 = 6.977 \text{ J/K}$

2) Since, Entropy at point 1 = Entropy at point 2

$$\gg s_1 = s_2$$

$$\gg 6.977 = s_f + (x) s_{fg}$$

$$\gg x = 0.85$$

3) Now , $h_2 = h_f + (x)h_{fg}$

$$= 209.3 + (0.85) \cdot (2382.8)$$

$$h_2 = 2234.68 \text{ KJ}$$

4) Since, $h_f = h_3 = 209.3 \text{ KJ}$

5) Work done by pump, $W_p = h_4 - h_3$

$$V_3(P_4 - P_3) = h_4 - h_3$$

$$V_f(P_b - P_c) = h_4 - h_3$$

$$0.001012(50 - 0.1) = h_4 - 209.3 \quad \gg \quad h_4 = 209.35 \text{ KJ}$$

6) $Q_{\text{added in boiler}} = h_1 - h_4 = 3224.35 \text{ KJ} \quad \dots \text{ Ans}$

$$Q_{\text{rejected from condenser}} = h_2 - h_3 = 2025.33 \text{ KJ}$$

7) Thermal efficiency

$$\eta = 1 - Q_{\text{added}} / Q_{\text{rejected}} = 0.37 = 37\% \quad \eta = 37\% \quad \dots \text{ Ans}$$

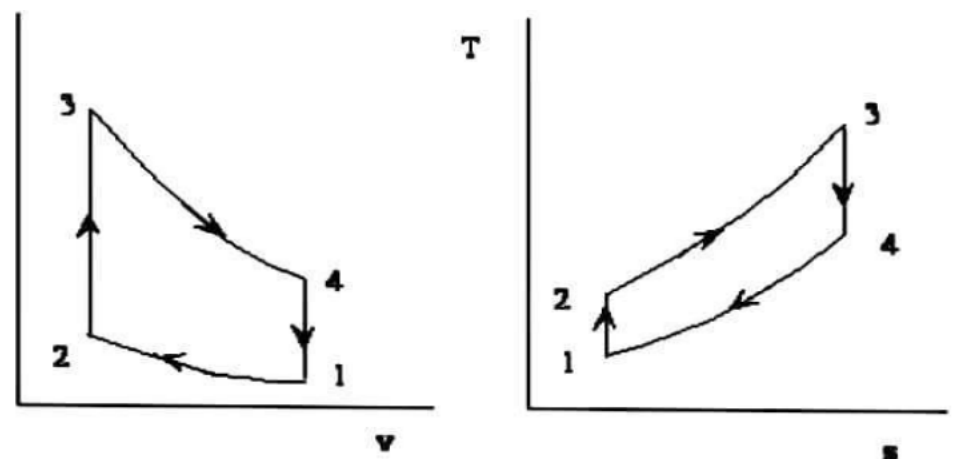
8) Steam rate $= 3600 / W_s = 3600 / (h_1 - h_2)(h_4 - h_3) = 3 \text{ kg/kWh}$

... Ans

(b) Show that the efficiency of the Otto cycle depends only on compression ratio.

(5M)

Ans. The cycle is also called a constant volume or explosion cycle. This is the equivalent air cycle for reciprocating piston engines using spark ignition. Figures 1 and 2 show the P-V and T-s diagrams respectively.



Analysis of cycle

$$\text{Net workdone, } W = \text{Heat supplied} - \text{Heat rejected} = m C_v (T_3 - T_2) - m C_v (T_4 - T_1)$$

Air standard efficiency,

$$\eta = \frac{\text{Net workdone}}{\text{Heat supplied}} = \frac{m C_v (T_3 - T_2) - m C_v (T_4 - T_1)}{m C_v (T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Compression ratio, $r_c = r = \frac{V_1}{V_2}$

Expansion ratio, $r_e = r = \frac{V_4}{V_3} = \frac{V_1}{V_2}$

As the compression and expansion processes are isentropic with the same volume ratio, we can write

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \text{and} \quad \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

Since $V_2 = V_3$ and $V_1 = V_4$ it follows that,

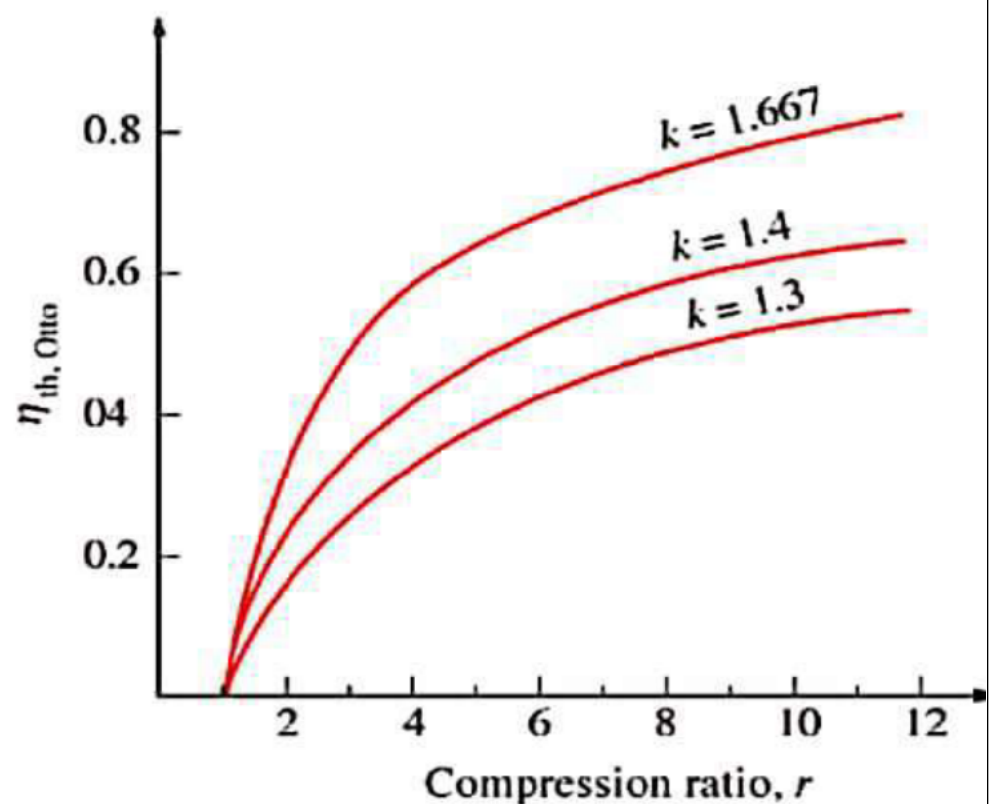
$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r)^{\gamma-1}$$

$$\eta = 1 - \left[\frac{\left(\frac{T_3}{(r)^{\gamma-1}} - \frac{T_2}{(r)^{\gamma-1}}\right)}{(T_3 - T_2)} \right]$$

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}}$$

In a true thermodynamic cycle, the term expansion ratio and compression ratio are synonymous. However, in a real engine, these two ratios need not be equal because of the valve timing and therefore the term expansion ratio is preferred sometimes.

Equation 4 shows that the thermal efficiency of the theoretical Otto cycle increases with increase in compression ratio and specific heat ratio but is independent of the heat added (independent of load) and initial conditions of pressure, volume and temperature. Figure 3 shows a plot of thermal efficiency versus compression ratio for an Otto cycle. It is seen that the increase in efficiency is significant at lower compression ratios.



c) Define volumetric efficiency of a compressor. On what factors does it depend?

(5M)

Ans. The volumetric efficiency represents the efficiency of a compressor cylinder to compress gas. It may be defined as the ratio of the volume of gas actually delivered to the piston displacement, corrected to suction temperature and pressure.

$$\eta_v = \text{Volume of air actually compressed} / \text{piston displacement}$$

❖ Factors on which it depends

- Due to increased pressure ratio and clearance ratio
- Pressure drop at inlet passage
- Leaky piston rings
- High temperature of air at suction of first stage
- Dirty air Filter
- Lack of water supply

Q.5 (a) One kg of air at 1bar and 300K is compressed adiabatically till its pressure becomes 5 times the original pressure. Subsequently it is expanded at constant pressure and finally cooled at constant volume to return to its original state. Calculate the heat and work interactions and change in internal energy for each process and for the cycle. (10M)

Ans. $m=1\text{kg}$, $C_v= 0.718\text{kJ/kg}$,

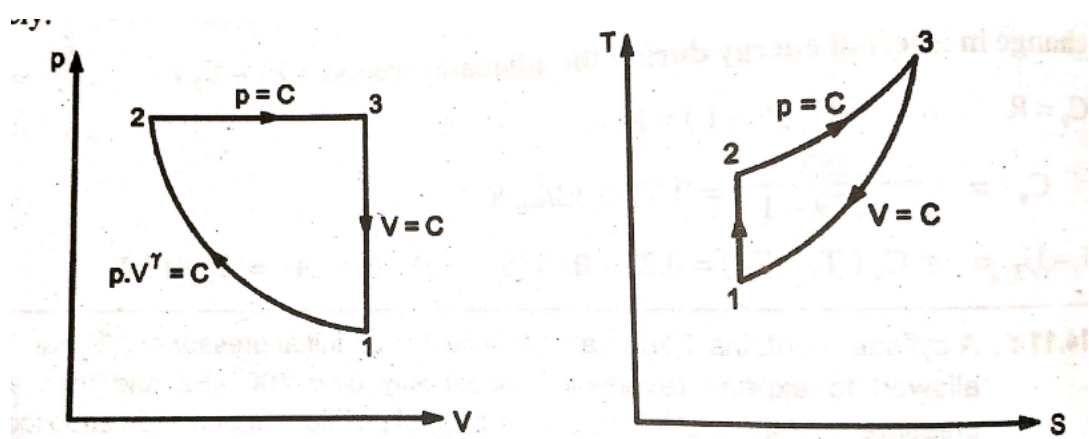
$$\gamma = 1.4 , C_p=1.0052 \text{ kJ/kg}$$

From gas equation,

$$v_1 = \frac{mRT_1}{p_1} = \frac{1 \times 287 \times 300}{1 \times 10^5}$$

$$\therefore v_1 = 0.861 \text{m}^3$$

1) Heat transfer Q_{12} and change in internal energy ($U_2 - U_1$) in adiabatic process(1 -2)



$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} = 300 (5)^{(1.4-1)/1.4} = 475.15 \text{ K}$$

Due to adiabatic process, heat transfer $Q_{12}=0$...Ans

$$\begin{aligned} \text{Change in internal energy, } (U_2 - U_1) &= mC_v(T_2 - T_1) = 1 \times 0.718(475.15 - 300) \\ &= 125.76 \text{ kJ ...Ans} \end{aligned}$$

2) Constant pressure process(2-3)

$$\begin{aligned} \therefore P_1 V_1^\gamma &= P_2 \cdot V_2^\gamma \\ V_2 &= \left(\frac{P_1}{P_2} \right)^{1/\gamma} V_1 = \left(\frac{1}{5} \right)^{1/1.4} \times 0.861 = 0.2727 \text{ m}^3 \end{aligned}$$

For constant pressure process(2-3)

$$\begin{aligned} \frac{T_3}{T_2} &= \frac{V_3}{V_2} = \frac{V_1}{V_2} \text{ i.e. } T_3 = T_2 \left(\frac{V_1}{V_2} \right) \\ T_3 &= 475.15 \times \frac{0.861}{0.2727} = 1500.2 \text{ K} \end{aligned}$$

$$\begin{aligned} \text{Heat transfer } Q_{23} &= mC_p(T_3 - T_2) = 1 \times 1.0052(1500.2 - 475.15) \\ &= 1030.38 \text{ kJ ...Ans} \end{aligned}$$

$$\begin{aligned} \text{Change in internal energy, } (U_3 - U_2) &= mC_v(T_3 - T_2) = 1 \times 0.718(1500.2 - 475.15) \\ &= 735.99 \text{ kJ ...Ans} \end{aligned}$$

3) Constant volume Process(3-1)

$$\begin{aligned} \text{Change in internal energy, } (U_1 - U_3) &= mC_v(T_1 - T_3) = 1 \times 0.718(300 - 1500.2) \\ &= -861.74 \text{ kJ ...Ans} \end{aligned}$$

$$\text{Workdone, } W_{31} = \int p \cdot dV = 0 \text{ (since } dV=0)$$

$$\text{Heat transfer, } Q_{31} = W_{31} + (U_1 - U_3) = 0 + (-861.74) = -861.74 \text{ kJ ...Ans}$$

4) Heat transfer and change in internal energy for the cycle

$$\Sigma Q = Q_{1-2} + Q_{2-3} + Q_{3-1} = 0 + 1030.38 - 861.74 = 168.64 \text{ kJ ...Ans.}$$

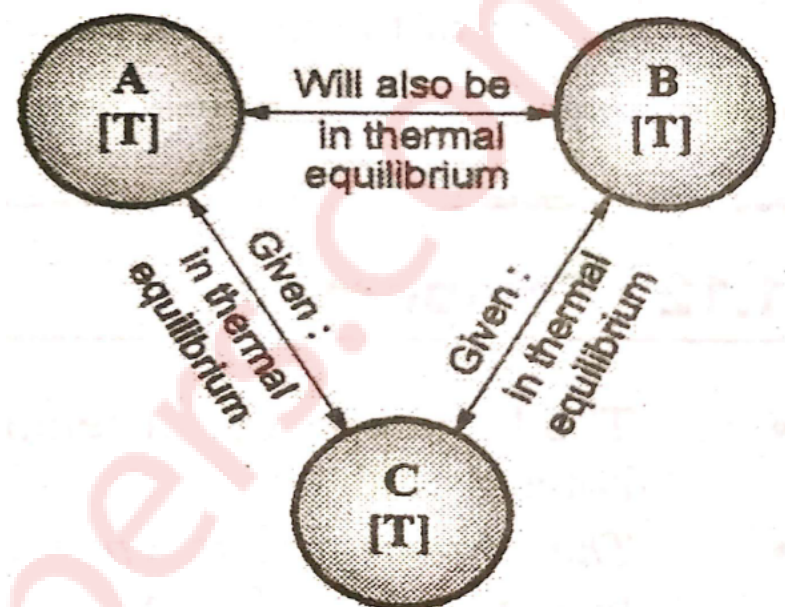
$$\Sigma \Delta U = (U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 125.76 + 735.99 - 861.74 = 0.01 \approx 0 \text{ ...Ans.}$$

(b) State the Zeroth law of thermodynamics. What is its significance? (5M)

Ans. If two bodies A and B are individually in thermal equilibrium with a third body C, then the two bodies A and B will also be in thermal equilibrium with each other. Above statement is known as zeroth law of thermodynamics.

This statement is represented schematically in given figure

Explanation : Since body A is in thermal equilibrium with body C, therefore, they are at equality of temperature T each. Similarly, the body B being in thermal equilibrium with body C, the temperature of body B will also be T.



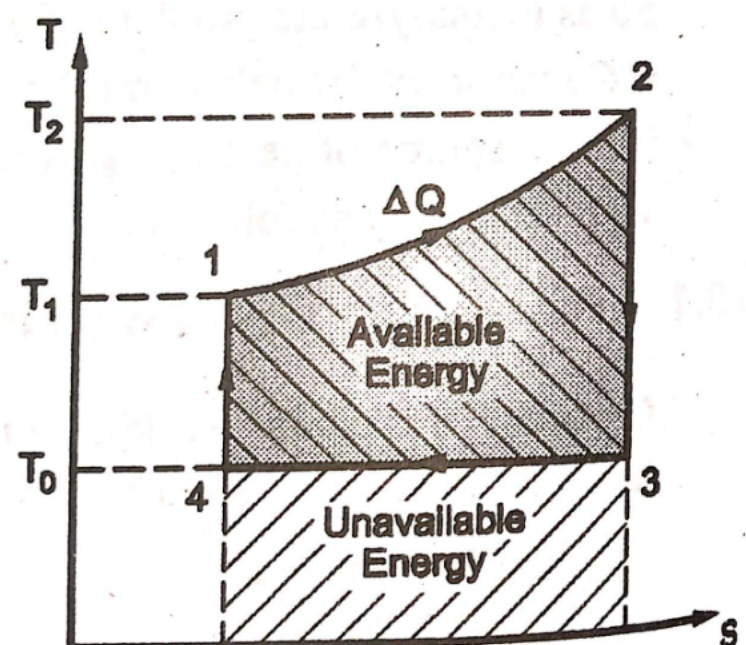
Significance :

- It has wide applications in thermometry.
- If body C is a thermometer and it's used to measure temperatures of A and B. If it shows both the readings as same, then C is in fact showing its own temperature.

(c) Deduce the expression for available energy from a finite energy source at temperature T when ambient temperature is T_0 . (5M)

Ans. In this case the heat withdrawn is not at constant temperature but during the process (1-2), the temperature of working fluid will increase while that of the heat source will decrease being of a finite size as shown in figure.

Whereas, the heat rejection still takes place at surroundings temperature, T_0 . Assuming that the heat ΔQ withdrawn is in reversible manner, i.e. the temperature of the source and working fluid remains same at any instant, the availability of the heat energy is equal to the reversible work during a reversible cycle completed with the help of two reversible adiabatic processes (2-3) and (4-1).



$$\text{Availability} = (\Delta W)_{\text{rev}}$$

$$= (\text{Heat supply to cycle}) + (\text{Heat reject by cycle})$$

$$= \Delta Q + T_0 (S_4 - S_3) = \Delta Q - T_0 (S_2 - S_1)$$

$$\therefore \text{Availability, } A = \Delta Q - T_0 \cdot \Delta S$$

Where, ΔS represents the increase in entropy of working fluid during the reversible heat addition process which also represents the entropy decreases during the heat rejection process

$$\text{Unavailable energy} = \text{Heat supplied} - \text{Availability}$$

$$= \Delta Q - (\Delta Q - T_0 \cdot \Delta S)$$

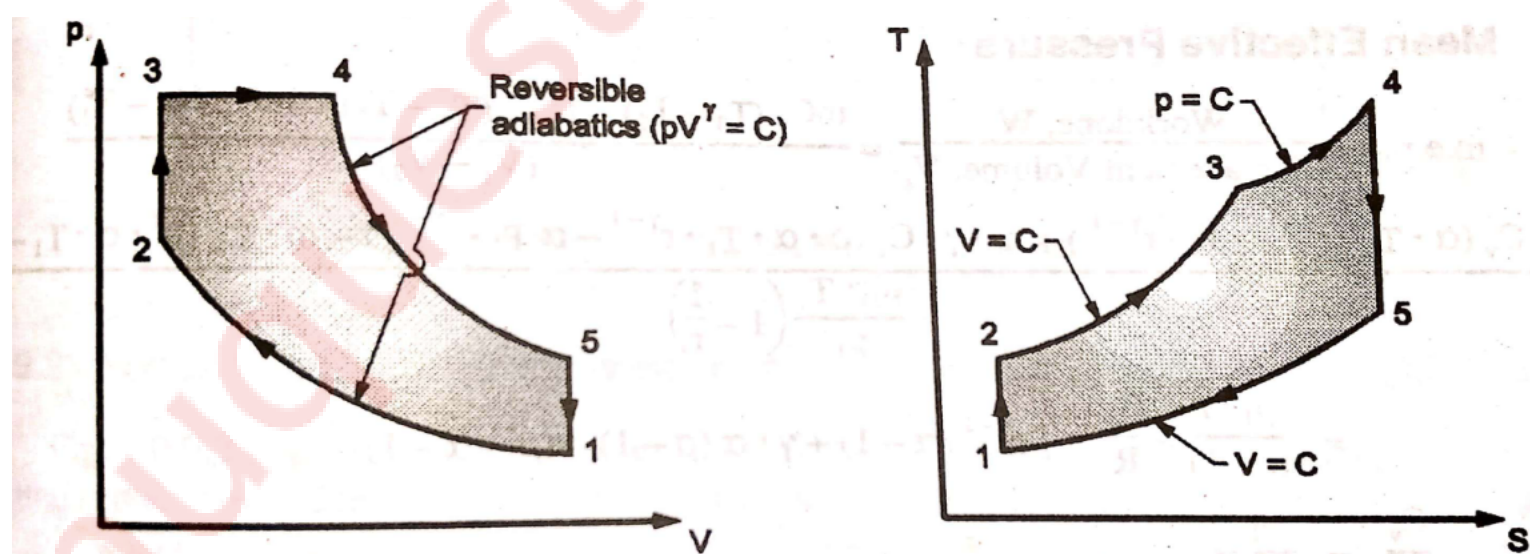
$$\text{Or, Unavailable energy} = T_0 \cdot \Delta S$$

Q.6 (a) An oil engine takes in air at 1.01 bar, 20°C and the maximum cycle pressure is 69 bar. The compression ratio is 18. Calculate the air standard thermal efficiency based on the dual combustion cycle. Assume that the heat added at constant volume is equal to the heat added at constant pressure. (10M)

Ans. $p_1 = 1.01 \text{ bar}$, $T_1 = 20^\circ = 293 \text{ K}$, $p_3 = p_4 = 69 \text{ bar}$

$$r = v_1/v_2 = 18, \quad \eta = ? \quad \gamma = C_p/C_v = 1.4$$

$$(\text{Heat added})_{\text{constat volume}} = (\text{Heat added})_{\text{constant pressure}} \dots (1)$$



1) from equation (1)

$$mC_v(T_3 - T_2) = mC_p(T_4 - T_3) \quad \therefore (T_3 - T_2) = \gamma(T_4 - T_3) \dots \text{equation (2)}$$

$$T_2 = T_1 \cdot (r)^{(\gamma - 1)} = 293(18)^{0.4} \quad \therefore T_2 = 931.05 \text{ K}$$

$$2) \quad p_1 \cdot v_1^\gamma = p_2 \cdot v_2^\gamma$$

$$\therefore p_2 = 1.01(18)^{1.4} = 57.77 \text{ bar}$$

$$3) \quad p_2/T_2 = p_3/T_3$$

$$\gg T_3 = p_3 \times T_2 / p_2$$

$$\gg T_3 = 69 \times 931.05 / 57.7 \quad \therefore T_3 = 1112.03 \text{ K}$$

4) From equation (2)

$$1112.03 - 931.05 = 1.4(T_4 - 1112.03) \quad \therefore T_4 = 1241.3 \text{ K}$$

5) At constant pressure

$$T_4/T_3 = v_4/v_3 = 1241.3/1112.03 = 1.116$$

$$v_4/v_3 = (v_5/v_3)/(v_4/v_3) = (v_1/v_2)/(v_4/v_3) = 18/1.116 = 16.13 = 18/1.116 = 16.13$$

$$6) \quad T_5 = T_4(v_4/v_5)^{(\gamma - 1)}$$

$$T_5 = 1241.3(1/16.13)^{1.4 - 1} \gg T_5 = 408.15 \text{ K}$$

7) Efficiency of cycle,

$$\eta = 1 - \text{Heat rejected/Heat added} = 1 - Q_{51}/2 \times Q_{23} \quad (\text{Since, } Q_{23} = Q_{34})$$

$$= 1 - mC_v(T_5 - T_1)/2 \times mC_v(T_3 - T_1)$$

$$= 1 - (408.15 - 293/1112.03 - 931.05)$$

$$\therefore \eta = 0.36 = 36\% \quad \dots \text{Ans}$$

(b) A single stage single acting air compressor running at 1000 rev/min delivers air at 25 bar. For this purpose the induction and free air conditions can be taken as 1.013 bar and 15°C. and the FAD as 0.25 m³/min. The clearance volume is 3% of the swept volume and the bore/stroke ratio is 1.2/1.

Calculate: i) the bore and stroke ii) the volumetric efficiency iii) the indicated power iv) the isothermal efficiency . Take the index of compression and re-expansion as 1.3. (10M)

Ans. With clearance volume

Single stage single acting air compressor

$N=1000 \text{ rpm/min}$,

$p=p_1 = 1.013 \text{ bar} = 1.013 \times 10^5 \text{ N/m}^2$

$p_2 = 25 \text{ bar} = 25 \times 10^5 \text{ N/m}^2$

$T_1 = 15^\circ = 273 + 15 = 288 \text{ K}$,

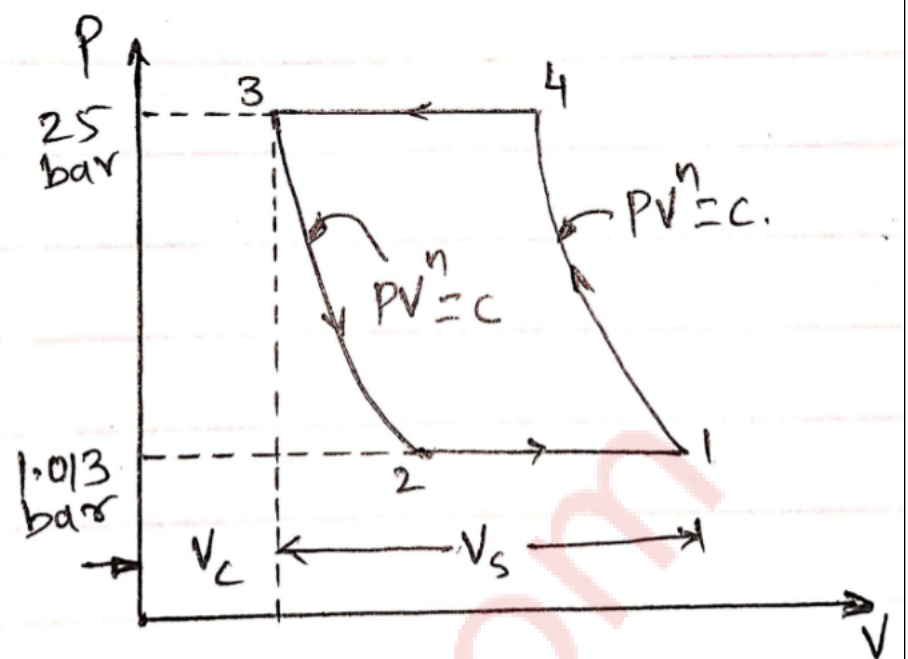
$V_c = 0.03 V_s$,

$V_c/V_s = k = 0.03$,

Bore(L)/Stroke (D) = 1/1.2 $\gg L = 1.2D$

FAD = 0.25 m³/min

$n = 1.3$, $PV^n = C$ (polytropic process)



1) Volumetric Efficiency

$$\eta_v = 1 + k - k(p_2/p_1)^{1/n}$$

$$= 1 + 0.03 - 0.03 (25/1.013)^{1/1.3}$$

$$\eta_v = 0.6766 = 67.66\% \quad \dots \text{Ans}$$

$$2) V_s = \text{FAD}/\eta_v = 0.25/0.6766 = 0.369 \text{ m}^3/\text{min}$$

$$\text{Now, } V_s = \pi/4 \cdot D^2 \cdot L$$

$$\therefore 0.369 = \pi/4 \cdot D^2 \cdot 1.2D \gg \therefore D = 1071 \text{ mm} \quad \dots \text{Ans}$$

$$\text{So, } L = 1.2D = 1.2 \times 1071 = 1285.2 \text{ mm} \quad \dots \text{Ans}$$

$$V_c/V_s = 0.03 \gg V_c = 0.03 \times 0.369 = 0.01107 \text{ m}^3/\text{min}$$

$$\text{But, } V_c = v_3 \gg \therefore v_3 = 0.01107 \text{ m}^3/\text{min}$$

$$v_1 = V_c + V_s = 0.38007 \text{ m}^3/\text{min}$$

3) 3-4 is Expansion

$$PV^n = C$$

$$p_4/p_3 = (v_3/v_4)^n$$

$$v_4/v_3 = (p_3/p_4)^{1/n} = (p_2/p_1)^{1/n}$$

$$v_4 = v_3 (p_2/p_1)^{1/n} = 0.01107(25/1.013)^{1/1.3}$$

$$\therefore v_4 = 0.13036 \text{ m}^3/\text{min}$$

4) Indicated Power (I.P)

$$W_{\text{poly}} = \frac{n}{n-1} p (v_1 - v_4) [(p_2/p_1)^{(n-1)/n} - 1]$$

$$= (1.3/1.3-1) \cdot 1.013 \times 10^5 (0.38007 - 0.13036) \cdot [(25/1.013)^{(1.3-1)/1.3} - 1]$$

$$\therefore W_{\text{poly}} = 120092.793 \text{ N}\cdot\text{min}$$

$$\text{Now, I.P} = W_{\text{poly}} \times N/60$$

$$= 120092.793 \times 100/60$$

$$\therefore \text{I.P} = 200146.546 \text{ kW} \quad \dots \text{Ans}$$

5) Isothermal efficiency (η_{iso})

$$\eta_{\text{iso}} = W_{\text{iso}}/W_{\text{poly}}$$

$$W_{\text{iso}} = p_1 (v_1 - v_2) \ln(p_2/p_1)$$

$$= 1.013 (0.38007 - 0.13036) \ln(25/1.013)$$

$$\therefore W_{\text{iso}} = 81096.75 \text{ kW}$$

$$\therefore \eta_{\text{iso}} = 81096.745/120092.793$$

$$\eta_{\text{iso}} = 0.675 = 67.5\% \quad \dots \text{Ans}$$