Q.P. 70863

STRENGTH OF MATERIALS

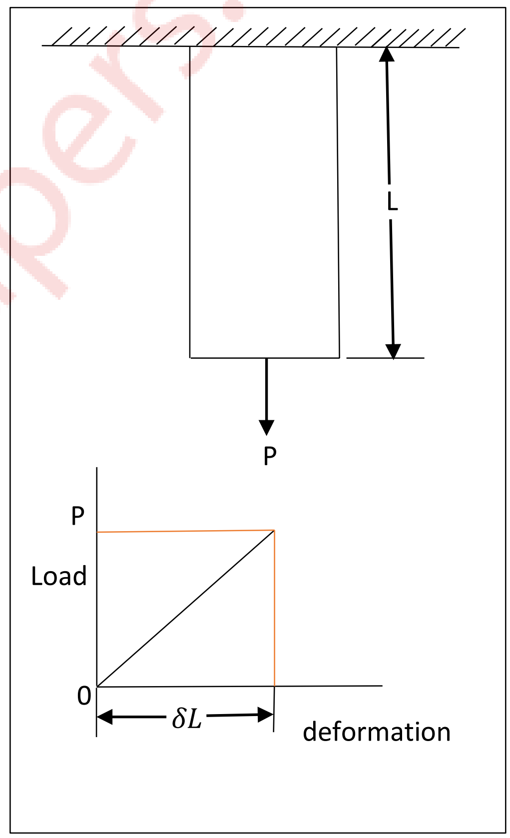
(DEC 2019 SEM3 MECHANICAL)

Q1)

a) Derive an expression for the strain energy due to suddenly applied load. (5)

Ans: Consider a member of length L, cross-sectional area A
And subjected to suddenly applied load P. let the instantaneous
Deformation of member be δL_s and the corresponding
Instantaneous stress developed be f_s. The load verses deformation
Curve would be a straight line.

Now work done by the load
= area under load-deformation curve.
$= P \times \delta L_s$.



We know strain energy U=work done.

$$U = P \times \delta L_s .$$

$$U = \frac{f^2 AL}{2E}.$$

$$P \times \delta L_s = \frac{{f_s}^2 AL}{2E}.$$

$$f_s = \frac{2P}{A}. \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

But stress due to gradually applied load = f= $\frac{P}{A}$.

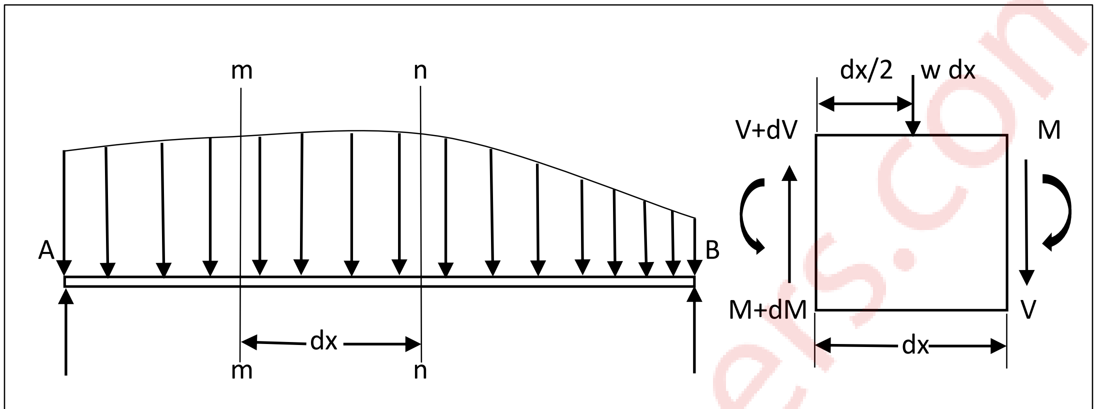
$$f_s = 2f. \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Also $\delta L_s = 2\,\delta L$.

Equations 1 and 2 indicate that sudden loads produce twice the stress thereby resulting in twice the strains as compared to the same loads when gradually applied.

b) Derive the relation between load, shear force and bending moment **(5)**

Ans:



(a) (b)

Fig. (a) shows a beam carrying a general loading. Consider an element of the beam formed by taking two cutting sections m-m and n-n distance dx apart. The general loading over the element can be assumed to be a u.d.l. of intensity w per unit run since dx is very small.

On the right face of the element the shear resistance force V and moment resistance M is developed. On the left face the shear resistance force has an increment of dV and becomes V + DV, while the moment resistance M increases by dM and becomes M + dM. Since the beam is in equilibrium, an element of the beam must also be in equilibrium.

Applying Conditions of Equilibrium to the element

Using Fy = 0 ↑ + ve

(v + dV) - V - w dx = 0

.: dV = w dx or $\frac{dV}{dx}$=W 1

Equation 1 implies that the rate of change of shear force at any section represents the rate of loading at that section.

Now using ΣM = 0 ↺ +ve

Taking moments about section m-m

$(M + dM) - M - V\,dx - (w\,dx) \times \frac{dX}{2} = 0.$

dM - V dx - w $\frac{(dX)^2}{2}$ =0.

The square of a small quantity can be neglected, we therefore have

dM = Vdx or $\frac{\mathrm{dM}}{\mathrm{dx}}$=V 2

Equation 2 implies that the rate of change of bending moment at any section represents the shear force at that section.

c) Write the assumptions made in theory of pure torsion and derive torsional formula. (5)

Ans: Assumptions made in pure torsion theory:

- The material of the shaft is homogenous and isentropic throughout its length.
- Circular sections remains circular and planar.
- The twisting moment T acts in a plane perpendicular to the shaft axis.
- At any section, the radial lines remain straight.
- Hooke’s law is valid.

Derivation of Torsion Equation

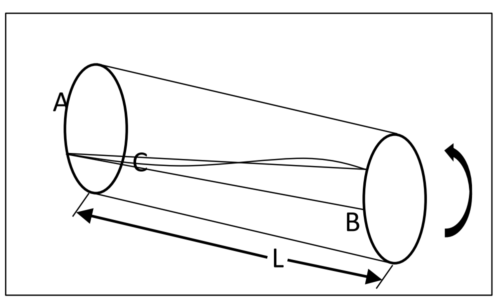
Consider a solid cylindrical shaft of radius R and length L be subjected to an equal and opposite twisting torques of magnitude T at the ends of the shaft such that the axis of the torques coincide with the shaft axis o-o. The shaft is therefore in pure torsion.

Consider a straight fibre AB on the surface of the shaft and parallel to the shaft axis o-o. Now on application of the twisting torque T, the initially straight fibre AB gets twisted into a helix AC. The corresponding angle BAC =ϕ represents the shear strain of the fibre. Let τ be the corresponding shear stress developed in the fibre.

Shear strain of the fibre = ɵ = $\frac{BC}{L}$.

Now let ɵ be the angular movement of the radial line OB to new position OC. From geometry Arc BC = R x ɵ.

.:Shear strain of the fibre = ɵ = $\frac{Rx\,\theta}{L}$.



We know , Modulus of Rigidity G=$\frac{shear\ stress}{shear\ strain}$.

G= $\frac{\tau}{\phi}$.

τ = G. ϕ

τ = G x $\frac{R\ x\ \theta}{L}$.

$\frac{\tau}{R}$=$\frac{G.\theta}{L}$ ………….(1)

consider a cutting section m-m taken to cut the shaft into two parts. Consider the equilibrium of the left part of the shaft.

Now consider an elemental area dA at a distance r from o subjected to shear stress q. Let τ be the shear stress in a fibre on the shaft surface i.e.at distance R from o.

we know that the shear stress distribution along the radial line varies linearly with the radial distance from the axis of the shaft.

Shear stress on the elemental area= q= $\frac{r}{R}$x τ .

Hence, shear resistance force developed by elemental area = qdA = $\frac{r}{R}$x τ dA.

Now. Moment of resistance developed by the elemental area =$\frac{r}{R}$x τ dA x r.

Total moment of resistance offered by the cross section of the shaft

$$=\frac{\tau}{R}\int r^2 = \frac{\tau}{R} \times J.$$

Here, $\int r^2\, dA = J$ is the polar moment of inertia of the shaft section about the axis of the shaft.

Now

Torque transmitted by shaft = Moment of resistance t developed in the Shaft

$$T = \frac{\tau}{R} \times J$$

$$\frac{T}{R} = \frac{I}{J} \text{ ……….(2)}$$

Comparing eqn 1 and 2, we get

$$\frac{\tau}{R} - \frac{T}{J} = \frac{G\theta}{L}$$

Above equation is referred to as the equation for pure torsion.

d) Draw shear stress distribution diagram for symmetry I section, T section and rectangular section. **(5)**

Ans: 1. Rectangular Section.

Consider a rectangular section of dimensions' b x d.

The N.A passes through the centroid at d/2 from the base.

Consider a layer m-m at distance y from the N.A.

area above layer m-m, $A = b \times \left(\frac{d}{2} - y\right)$

Distance of the centroid of shaded area from N.A,

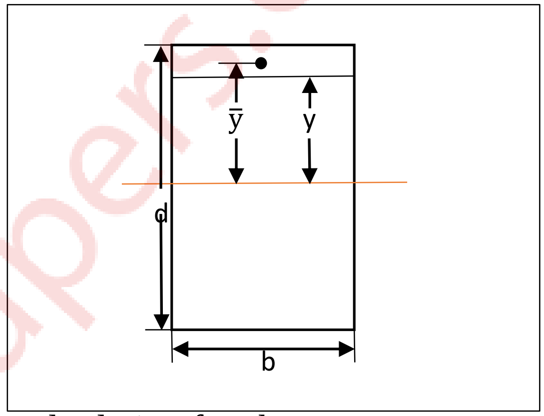
$\bar{y} = y + \frac{1}{2}\left(\frac{d}{2} - y\right)$

$= \frac{y}{2} + \frac{d}{4} = \frac{1}{2}\left(\frac{d}{2} + y\right)$

Using shear stress equation

$\tau = \frac{v \times (A x \bar{y})}{I \cdot b}$.

$\tau_{m-m} = \frac{v\left[b\left(\frac{d}{2} x y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)\right]}{\frac{b\, d^3}{12} \times b}$.



$\tau_{m-m} = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right)$ …….(1) General relation for shear stress

The above equation indicates parabolic nature of shear stress variation

At top and bottom edge $y = \frac{d}{2}$, $\tau = 0$.

Also τ would be maximum when y=0

$\bar{\tau}_{max} = \frac{3v}{2bd}$ ... (2) Relation for maximum shear stress

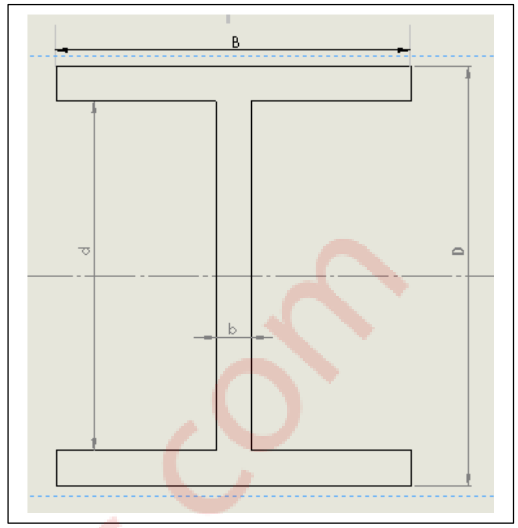
For a rectangular section (bxd) subjected to vertical shear V.

Average shearing stress = $\tau_{average} = \frac{shear\ force}{shear\ area}$. $\tau_{average} = \frac{V}{b\ X\ d}$ …..(3)

From equations (2) and (3)

$\tau_{max} = 1.5\ \tau_{average}$

This shows that for a rectangular section the maximum shear stress which occurs at the NA is 1.5 times the mean or average shear stress resisted by the section.



2. symmetry I section

Consider a symmetrical I section of flange width b, Web width b, web depth d and overall depth D.

The N>A passes through the centroid at D/2 From the base.

Shear stress analysis in the flange.
Consider a layer m-m at a distance y from N.A
Width of layer m-m=B
area above layer m-m, A=B$\left(\frac{D}{2}-y\right)$.
Distance of centroid of shaded area from N.A

$$C=\frac{D}{2}-\frac{\left(\frac{D}{2}-y\right)}{2}=\frac{\left(\frac{D}{2}+y\right)}{2}.$$

Using $\tau=\frac{V\ X\ (A\ X\ V)}{I.b}$.

$$\tau_{m-m(flange)}=\frac{vxB}{IxB}\left(\frac{D}{2}-y\right)\frac{\left(\frac{D}{2}+y\right)}{2}.$$

$$\tau_{m-m(flange)}=\frac{v}{2I}\left(\frac{D^2}{4}-y^2\right).$$ ….general relation for shear stress in flange.

Shear stress analysis in the web.
Consider a layer n–n at a distance y from the N.A
Width of layer n-n= b.
Shaded area above layer n-n, A=A_1+A_2.
Where $AA_1=B\left(\frac{D}{2}-\frac{d}{2}\right)$ and $A_2=b\left(\frac{d}{2}-y\right)$
Distance of centroid of A_1 from N.A

$$\bar{Y}_1=\frac{D}{2}-\frac{1}{2}\left(\frac{D}{2}-\frac{d}{2}\right)=\frac{D}{4}+\frac{d}{4}=\frac{\frac{D}{2}+\frac{d}{2}}{2}.$$

Distance of centroid A_2 from N.A $\bar{y}_2=y+\frac{\frac{d}{2}-y}{2}=\frac{y+\frac{d}{2}}{2}$.

Using $\tau=\frac{V\ x\ (A\ x\ \bar{y})}{I.b}$.

Using $\tau_{n-n}=V\times\frac{A_1\times\bar{y}_1+A_2\times\bar{y}_2}{I.b}$.

$$= \frac{v}{Ib}\left[B\left(\frac{D}{2}-\frac{d}{2}\right)\frac{\left(\frac{D}{2}+\frac{d}{2}\right)}{2} + b\left(\frac{d}{z}-y\right)\frac{\left(y+\frac{d}{2}\right)}{2}\right].$$

$$= \frac{V}{8Ib}\left[\,B(D^2-d^2)+b(d^2-4y^2)\right].$$ ………. general relation for shear stress

The maximum shear stress is at y=0 i.e. at N.A.

$$\tau_{max} = \frac{V}{8Ib}\left[\,B(D^2-d^2)+b^2\right]$$ …… relation for max shear stress.

Shear stress in flange at the flange and web junction i.e. at $y = \frac{d}{2}$.

$$\tau_{J-J(flange)} = \frac{V}{2I}\left(\frac{D^2}{2}-\frac{d^2}{2}\right) = \frac{V}{8I}(D^2-d^2).$$

Shear stress in web at the flange and web junction i.e. at $y = \frac{d}{2}$.

$$\tau_{J-J(web)} = \frac{V}{8Ib}\left[B(D^2-d^2)+b\left(d^2-\frac{4d^2}{4}\right)\right.$$

$$= \tau_{J-J(web)} = \frac{V}{8Ib}[B(D^2-d^2)].$$ ……… (2)

From equations (1) and (2) we understand that the stress in web drastically increases by $\frac{B}{b}$ times the corresponding stress in the flange in the junction Layer J-J of flange and web.

e) write the assumption for simple bending and derive and derive the flexural formula. (5)

Ans: assumptions made in theory of simple bending:

- The material of beam is homogenous and isotropic.
- The beam is initially straight and all the longitudinal fibres bend in circular arcs with a common centre of curvature.
- Members have symmetric cross sections and are subjected to bending in the plane of symmetry.
- The beam is subjected to pure bending and the effect of shear is neglected.
- Plane(transverse) sections through a beam taken normal to the axis of the beam remain plane after the beam is subjected to bending.
- The radius of curvature is large as compared to the dimensions of the beam.

Flexural formula:

Consider a beam section under pure sagging bending. Here the layers above The N.A are subjected to compressive forces. The magnitude of this Compressive force is proportional to the location of the layer from the N.A similarly, the layers below the N.A are subjected to the tensile forces in all the Layers about the N.A is known as the moment of resistance of the section. The moment of resistance is a result of bending and is a resisting moment to The applied resistance is result of bending and is a resisting moment to the Applied bending moment.

We know that, force on layer=$\frac{E}{R}.y.dA$.

Moment of this force about N.A =$\frac{E}{R}.y.dA.y.$

$=\frac{E}{R}.y^2.dA.$

Total moment of forces in different layers

= moment of resistance = $\int \frac{E}{R}.y^2.dA.$

$= \frac{E}{R}\int y^2.dA.$

If M is the external bending moment, then the moment of resistance should be equal to the bending M.

$$M = \frac{E}{R}\int y^2.dA.$$

But $\int y^2.dA.$ represents the second moment of area or the moment of inertia I Of the section about the N.A

$M = \frac{E}{R}\ X\ I$ or $\frac{M}{I} = \frac{E}{R}$ ………….(1)

But $\frac{f}{y} = \frac{E}{R}$

Comparing it with equation (1), we get.

$\frac{f}{y} = \frac{M}{I} = \frac{E}{R}$.

Since the stress f is due to bending we denote it as f_b and would refer to it as Bending stress.

$\frac{f_b}{y} = \frac{M}{I} = \frac{E}{R}$. …… equation of bending or flexure equation.

f) find the maximum power that can be transmitted through 50mm diameter shaft at 150 rpm, if the maximum permissible shear stress is 80 N/mm^2.(5)

Ans: N= 150 rpm.

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 15.70$ rad/s.

D= 50 mm.

$\tau = 80\ N/mm^2$.

Polar moment of inertia $J = \frac{\pi}{32} d^4$.

$\therefore\ J = \frac{\pi}{32} \times (50)^4$.

$J = 245.43\ mm^4$.

Using torsions equations

$\frac{\tau}{R} = \frac{T}{J}$

$\frac{80}{25} = \frac{T}{245.43}$.

$\therefore$ T= 785.376 N-mm.

But

P=T x ω

= 785.376 x 15.70

= 12330.4 W.

Power developed by the shaft = 1.23 kW

Q.2)

a) A bar of brass 20mm is enclosed in a steel tube of 40mm external diameter and 20mm internal diameter. The bar and the tubes are initially 1.2 m long and are rigidly fastened at both ends. If the temperature is raised by 60°C, find the Stresses induced in the bar and the tube.

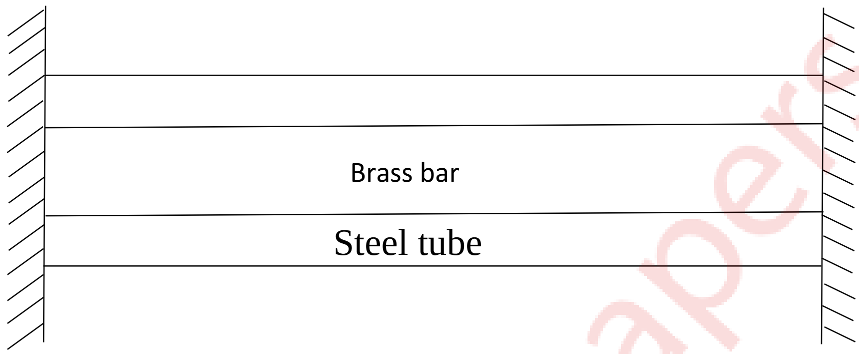
Given: $E_s = 2 \times 10^5$ N/mm^2.

$\boldsymbol{E_b = 1 \times 10^5\ N/mm^2.}$

$\boldsymbol{\alpha_s = 11.6 \times 10^{-6}/°C.}$

$\boldsymbol{\alpha_b = 18.7 \times 10^{-6}/°C.}$ **(10)**

Ans:



Brass bar

d= 20mm.

L=1.2mm=1200mm.

A=$\frac{\pi}{4} \times d^2$.

=$\frac{\pi}{4} \times (20)^2$.

= 314.15 mm^2.

$E_b = 1 \times 10^5\ N/mm^2$.

$\alpha_b = 18.7 \times 10^{-6}/°C$.

steel tube

d_0= 40mm.

d_i= 20mm.

A=$\frac{\pi}{4} \times ({d_0}^2 - d_i^2)$.

= 942.47 N/mm^2.

$E_s = 2 \times 10^5\ N/mm^2$.

$\alpha_s = 11.6 \times 10^{-6}/°C$.

T=60°C.

Load sharing relation:

Force in steel tube= force in brass bar.

$P_s = P_b$.

$f_s A_s = f_b A_b$.

$f_s \times 942.47 = f_b \times 314.15$.

$f_s = 0.333 f_b$. ……….. (1)

Strain relation:

$$\delta L_{steel} = \delta L_{brass}.$$

$[free\ expansion + induced\ expansion]_{steel} =$
$[free\ expansion - induced\ contraction]_{brass}.$

$$\left[\alpha TL + \frac{PL}{AE}\right]_{steel} = \left[\alpha TL + \frac{PL}{AE}\right]_{brass}.$$

$$\left[(11.6 \times 10^{-6}) \times 60 + \frac{f_s}{2\times 10^5}\right] = \left[(18.7 \times 10^{-6}) \times 60 - \frac{f_b}{10^5}\right].$$

$0.5f_s + f_b = 42.6.$

Substituting value of f_s from eqn 1 in eqn 2, we get.

$0.5(0.333f_b) + f_b = 42.6.$

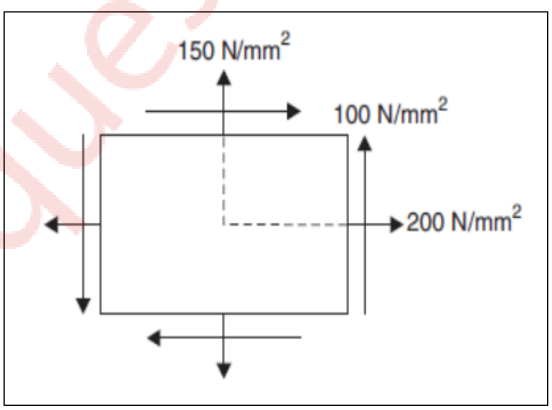
$f_b = 36.51\ N/mm^2.$

$f_s = 0.333 \times f_b$

$f_s = 0.333 \times 36.51.$

$f_s = 12.15\ N/mm^2.$

b) the state of stress at a point in a strained material is as shown in fig. Determine
(1) the direction of principal planes.
(2) the magnitude of maximum shear stress .
Indicate the direction of all the above by a sketch. (10)



Ans: given:

$\sigma_x = 200\ \text{N}/mm^2.$

$\sigma_y = 150$ N/mm^2.
$\tau_{xy} = 100$ N/mm^2.

$$\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \ .$$
$$= 175 \pm \sqrt{25^2 + 100^2} \ .$$
$$= 175 \pm 103.077.$$

$\sigma_1 = 278.077 \ N/mm^2$.
$\sigma_2 = 71.923 \ N/mm^2$.
$\tau_{max} = 103.077 \ N/mm^2$.

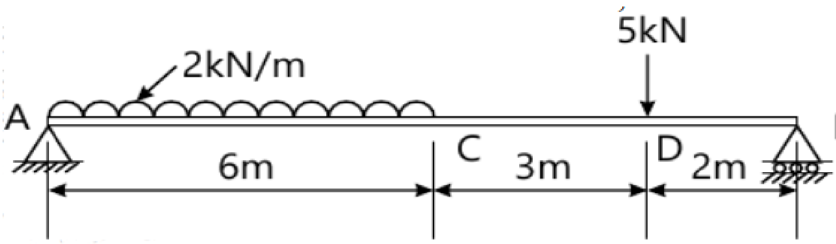
$\tan 2\theta_p = \frac{2\,\tau_{xy}}{\sigma_1 - \sigma_2}$.

$\tan 2\theta_p = \frac{2\times100}{50}$ = 4.

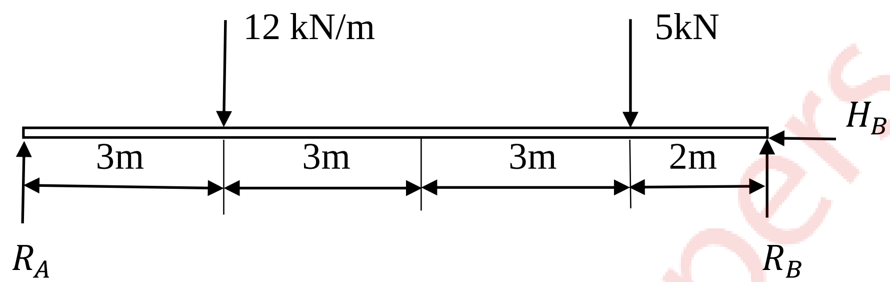
$\therefore \theta_{p\,1} = 37.98°$ and $\theta_{p2} = 90 + 37.98$= 127.98° .

Q3)

a) find slope at point A and B, deflections at points C and D for a beam as shown in fig. also find the maximum deflection. Take E= 200GPa and I=10^8 mm^4 . **(10)**



Ans:



Finding support reactions:

Taking moment about point A: +ve.

$12 \times 3 + 5 \times 9 - R_B \times 11 = 0.$

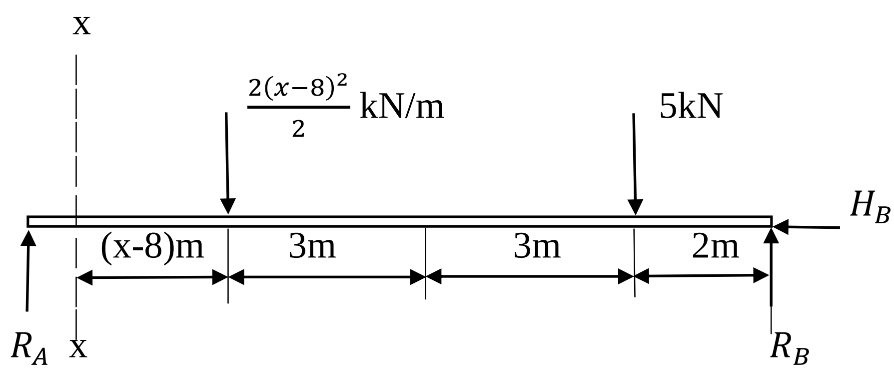
$\therefore R_B = 7.36\ kN.$

$\sum F_Y = 0.$ +ve.

$R_A + R_B - 12 - 5 = 0.$

$R_A + 7.36 - 12 - 5 = 0$

$\therefore\ R_A = 9.64\ kN.$



Taking section x-x near support A.
Distance of section x-x from point B=X
Taking moment about section x-x.

$$\therefore -R_B \times X + 5 \times (X-2) + \frac{2(X-8)}{2} \times (X-8) = M_{xx}$$

Now we have,

$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = -7.36 \times X + 5 \times (X-2) + (X-8)^2$$

Integrating both the sides w.r.t X, we get.

$$EI\frac{dy}{dx} = -\frac{7.36X^2}{2} + \frac{5(X-2)^2}{2} + \frac{(X-8)^3}{3} + c_1. \quad \text{.....(eqn for slope).}$$

Integrating both sides w.r.t X, we get.

$$EI.y = -\frac{7.36X^3}{2\times3} + \frac{5(X-2)^3}{6} + \frac{(X-8)^4}{4} + C_1X + C_2. \quad \text{.....(eqn for deflection).}$$

applying boundary conditions:

At X=0 y=0.

$$EI.y = -\frac{7.36X^3}{2 \times 3} + \frac{5(X-2)^3}{6} + \frac{(X-8)^4}{4} + C_1X + C_2.$$

$\therefore EI(0) = 0 + 0 + C_2$
$\therefore C_2 = 0.$
Now

At X=11 y=0.

$$EI.y = -\frac{7.36X^3}{2 \times 3} + \frac{5(X-2)^3}{6} + \frac{(X-8)^4}{4} + C_1X + C_2.$$

$$0 = -\frac{7.36(11)^3}{6} + \frac{5(6)^3}{6} + 0 + 8C_1 + 0$$
$C_1 = 181.58$

Now slope equation :

$$EI\frac{dy}{dx} = -\frac{7.36X^2}{2} + \frac{5(X-2)^2}{2} + \frac{(X-8)^3}{3} + 181.58.$$

Slope at points A and B.
At point A X=0
$\therefore \; EI\frac{dy}{dx} = 0 + 181.58$
$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{181.58}{EI}$

$= \frac{181.58 \times 10^9}{200 \times 10^8} = 9.079 rad$

At point B X=11

$$EI\frac{dy}{dx} = -\frac{7.36(11)^2}{2} + \frac{5(11-2)^2}{2} + \frac{(11-8)^3}{3} + 181.58.$$

$\frac{\mathrm{dy}}{\mathrm{dx}} = -2.91r\mathrm{ad}$

Deflection:

$$EI.y = -\frac{7.36X^3}{2\times 3} + \frac{5(X-2)^3}{6} + \frac{(X-8)^4}{4} + C_1X + C_2.$$

Deflection at point C

X=6

$$EI.y = -\frac{7.36(6)^3}{2\times 3} + \frac{5(6-2)^3}{6} + 181.58\times 6$$

y=43.89mm.

Deflection at point D.

X=9

$$EI.y = -\frac{7.36(9)^3}{2\times 3} + \frac{5(9-2)^3}{6} + \frac{(1)^4}{4} + 181.58\times 6$$

y= 47.74mm.

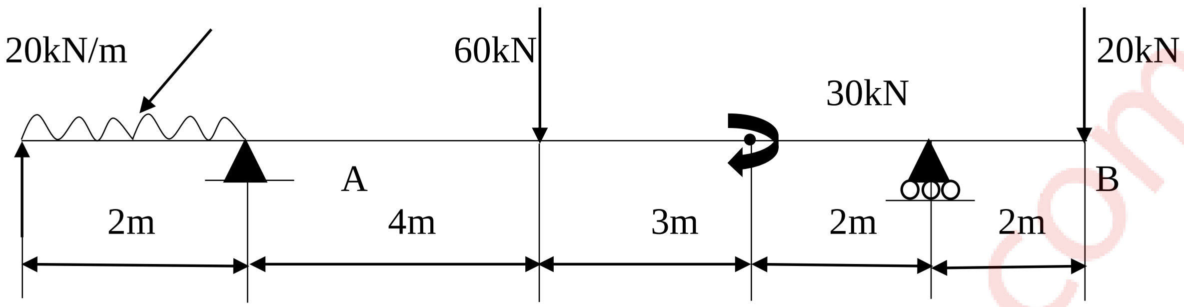
maximum deflection .

X= $\frac{11}{2}$

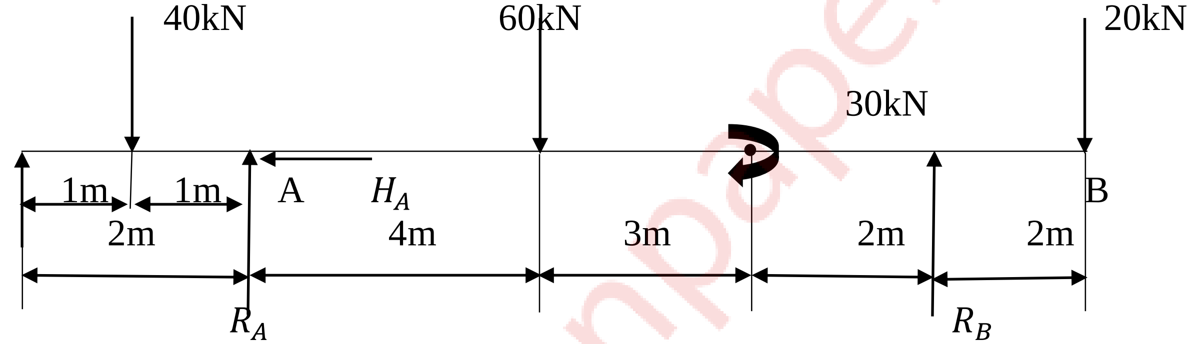
$$EI.y = -\frac{7.36(5.5)^3}{2\times 3} + \frac{5(5.5-2)^3}{6} + 181.58\times 6$$

$y_{max} = 46.056\, mm.$

b) draw SF and BM diagram for the beam shown in fig. (10)



Ans:



Support reaction calculation:

$\sum M_A = 0$ +ve.

$-20 \times 11 + R_B \times 9 - 30 - 60 \times 4 + 40 \times 1 = 0.$

$R_B = 50kN.$

$\sum F_y = 0.$ +ve.

$-40 + R_A - 60 + R_{\boldsymbol{B}} - 20 = 0.$

$R_A = 70kN.$

Shear force calculation: ↓|↑ -ve

$SF_{F(JR)} = 0.$

$SF_{F(JL)} = 20kN.$

$SF_{B(JR)} = 20kN.$

$SF_{\boldsymbol{B}(JL)} = -R_B + 20 = -50 + 20 = -30kN.$

$SF_{D(JR)} = -30kN.$

$SF_{D(JL)} = 60 - 30 = 30kN.$

$SF_{A(JR)} = 30kN.$

$SF_{A(JL)} = -70 + 30 = -40kN.$

Bending moment calculation +ve

$BM_F = 0.$

$BM_B = -20 \times 2 = -40.$

$BM_{E(JR)} = 50 \times 2 - 20 \times 4 = 20.$

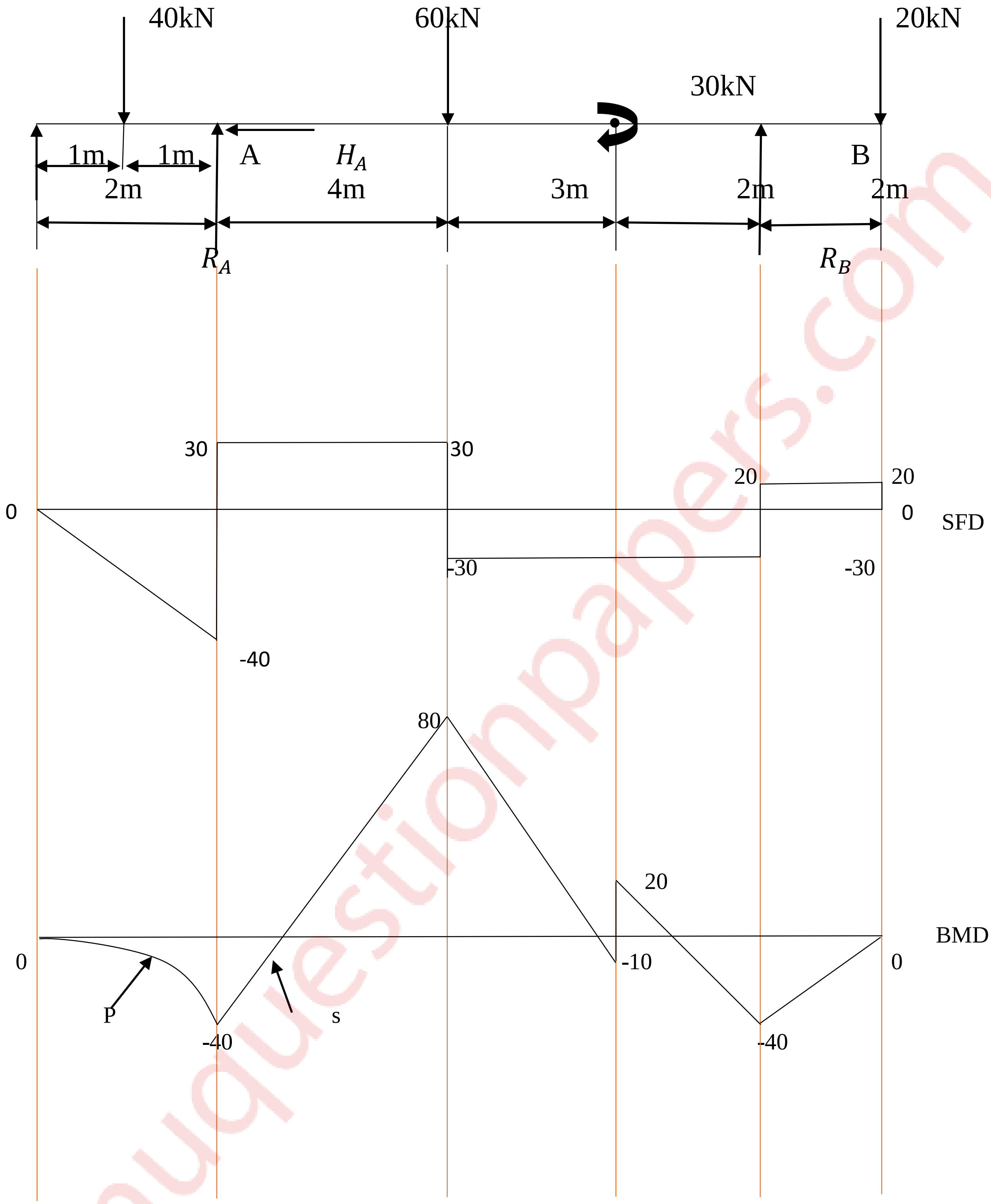
$BM_{E(JL)} = 20 - 30 = -10.$

$BM_D = -20 \times 7 + 50 \times 5 - 30$

= 80kN.

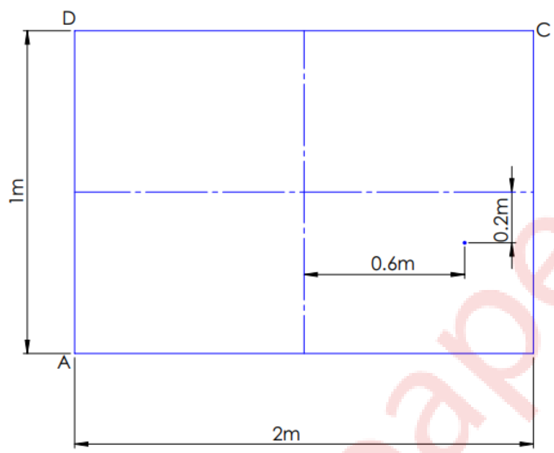
$BM_A = -20 \times 11 + 50 \times 9 - 30 - 60 \times 4.$

$= -40$.


40kN
60kN
20kN
30kN
1m
1m
A
H_A
B
2m
4m
3m
2m
2m
R_A
R_B
30
30
20
20
0
0
SFD
-30
-30
-40
80
20
BMD
0
-10
0
P
s
-40
-40

Q4)

a) A vertical column of rectangular section is subjected to a compressive load of P=800kN as shown in fig. find the stress intensities at the four corners of the column. (10)



Ans: $e_{xx} = 0.2m = 200mm.$

$e_{yy} = 0.6m = 600mm.$

Direct stress= $\frac{P}{A}$.

$= \frac{800 \times 10^3}{2000 \times 1000}$.

$= 0.4\ N/mm^2$.

Bending stress f_{bxx} due to eccentricity w.r.t x axis:

$f_{bxx} = \pm \frac{P\ e_{xx}\ y_{xx}}{I_{xx}}$.

$$I_{xx} = \frac{(bd)^3}{12} = \frac{2000\times(1000)^3}{12} = 16.6 \times 10^{10} mm^4.$$

$$f_{bxx} = \pm \frac{800\times10^3\times200\times500}{16.66\times10^{10}}.$$

$f_{bxx} = \pm 0.481\ N/mm^2$. (side AB=+ve, side CD=-ve).

Bending stress f_{byy} due to eccentricity w.r.t y axis:

$$f_{byy} = \pm \frac{P\ e_{xx}\ Y_{yy}}{I_{yy}}.$$

$$I_{yy} = \frac{bd^3}{12} = \frac{1000\times(2000)^3}{12} = 66.67 \times 10^{10}\ mm^4.$$

$$f_{byy} = \pm \frac{800\times10^3\times600\times1000}{66.67\times10^{10}}$$

$f_{byy} = \pm 0.71\ N/mm\text{^}2$. (side CB=+ve, side AD =-ve).

The resultant stresses at the corners would be addition of direct stress and bending stress along x any y axis.

$f_R = f_d + f_{bxx} + f_{byy}$.
At corner A f_{RA}= $0.4 + 0.481 - 0.71 = 0.171\ N/mm^2$ (compressive).
At corner B f_{RB}=$0.4 + 0.481 + 0.71 = 1.591\ N/mm^2$ (compressive).
At corner C $f_{RC} = 0.4 - 0.481 + 0.71 = 0.629\ N/mm^2$ (compressive).
At corner D $f_{RD} = 0.4 - 0.481 - 0.71 = -0.791\ N/mm^2$ (tensile).

b) a propeller shaft is required to transmit 50kW power at 500rpm. It is a hollow shaft, having an inside diameter 0.6 times of outside diameter and permissible shear stress for shaft material is $90 N/mm^2$. Calculate the inside and outside diameters of the shaft. **(10)**

Ans: P=50Kw = $50 \times 10^3\ W$.

N=500 rpm.

$\omega = \frac{2\pi N}{60} = 52.35\ rad/s.$

$d_i = 0.6 \times d_o.$

$[\tau] = 90\ N/mm^2$.

d_i =?

d_o =?

Polar moment of inertia $= J = \frac{\pi}{32}(d_o^4 - d_i^4).$

$$J = \frac{\pi}{32}(d_o^4 - (0.6d_o)^4).$$

$$J = 0.085\ d_O^4.$$

Using torsion equation.

$$\frac{\tau}{R} = \frac{T}{J}$$

$$\frac{\frac{90}{d_O^4}}{2} = \frac{955.10}{0.085 d_O^{4}{}^4}.$$

$d_o = 3.966\ mm.$

$d_i = 0.6 \times 3.966 = 2.37\ mm.$

Q5)

a) A cylindrical shell is 3m long and 1.2m in diameter and 12mm thick is Subjected to internal pressure of 1.8 N/mm^2 calculate change in dimensions and Volume of shell. Take $E = 210\ kN/mm^2$, 1/m=0.3. (10)

Ans: given:

L= 3m= 3000mm.

d= 1.2m=1200mm.

t= 12mm.

p= 1.8 N/mm^2.

E= $210 \times 10^3\ N/mm^2$.

1/m=0.3.

δ_d=? (change in diameter).

δ_L=? (change in length).

δ_v=? (change in volume).

Circumferential stress

$$f_c = \frac{pd}{2t} = \frac{1.8\times1200}{2\times12} = 90\ N/mm^2.$$

Longitudinal stress

$$f_l = \frac{pd}{4t} = \frac{2160}{48} = 45\ N/mm^2\ .$$

Circumferential strain

$$e_c = \frac{f_c}{E} - \frac{1}{m}\frac{f_l}{E} = 3.642 \times 10^{-4}\ .$$

Longitudinal strain

$$e_l = \frac{f_l}{E} - \frac{1}{m}\frac{f_c}{E} = 8.57 \times 10^{-5}$$

also,

$$e_c = \frac{\delta d}{d}$$

$\therefore \quad \delta d = e_c \times d$

$\delta d = 0.437mm$

Now,

$e_l = \frac{\delta L}{L}$

$\therefore \quad \delta L = e_L \times L.$

$\delta L = 0.2571\ mm.$

Volumetric strain $e_v = 2e_c + e_L$

$e_v = 2(3.642 \times 10^{-4}) + 8.57 \times 10^{-5}$

$e_v = 8.141 \times 10^{-4}$

Since,

$e_v = \frac{\delta V}{V}$

$\therefore \delta V = e_v \times V.$

$\delta V = e_v \times \frac{\pi}{4}(d)^2 L$

$\therefore \quad \delta V = 2.762 \times 10^{10}$

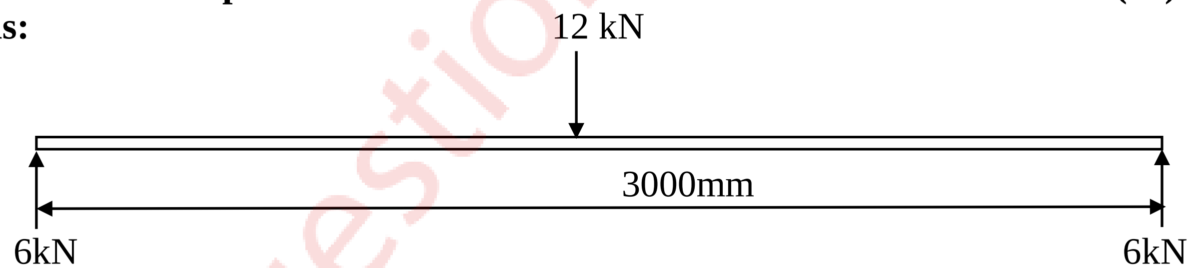
b) A simply supported beam of length 3m and a cross section of 100mm X 200mm carrying a UDL of 4kN/m, find

1) Maximum bending stress in the beam.

2) maximum shear stress in the beam.

3) the shear stress at point 1 m to the right of the left support and 25mm below the top surface of the beam. (10)

Ans:



The maximum bending moment $= \frac{wl^2}{8}$.

Using bending equation:

$\frac{\sigma_b}{y} = \frac{M}{I}$

y = 100mm.

M= $\frac{wl^2}{8} = \frac{4(3000)}{8} = 1500\ N - mm.$

$I=\frac{bd^3}{12}=\frac{100(200)^3}{120}=6666666.67mm^4.$

Substituting these values in bending to get max bending stress σ_b

$\frac{\sigma_b}{100}=\frac{1500}{3000}$

$\therefore\ \sigma_b=50\ N/mm^2.$

Max shear stress

Average shear stress $=\frac{shear\ force}{shear\ area}=\frac{wL}{100\times 200}=\frac{12000}{20000}$

$\tau_{avg}=0.6\ N/mm^2.$

$\tau_{max}=1.5\times\tau_{avg}$

$\therefore\tau_{max}=1.5\times 0.6$

$\tau_{max}=0.9\ N/mm^2$

Shear force at x=1m =1000mm.

$SF_{x=1000mm}=6000-4\times 1000=2000\ N.$=V

$\bar{y}=100mm.$

Now $\tau=\frac{V\times A\times\bar{y}}{I.b}=\frac{2000\times 100\times 200\times 100}{6666666.67\times 100}$=5.97 N/mm^2 .

$\tau\ at\ 25mm\ below\ top\ surface\ i.e.75mm\ above\ N.A$

$\therefore\ \bar{y}=100-\frac{100+75}{2}=100-87.5=12.5$

$\therefore\ \tau=\frac{2000\times 175\times 100\times 12.5}{6666666.67\times 100}=0.65\ N/mm^2.$

Q6)

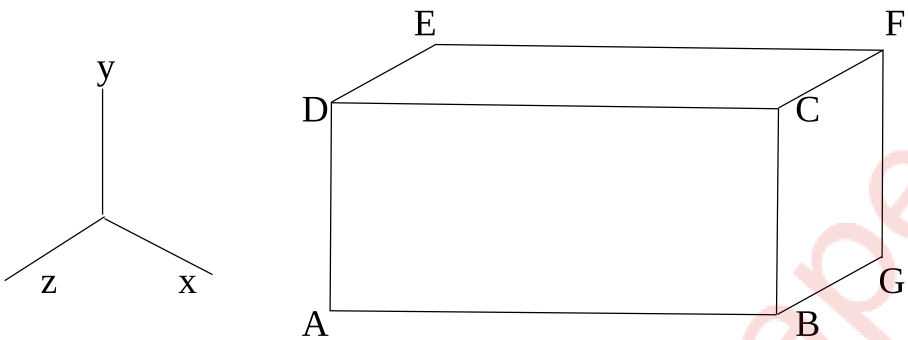
a) **A 400 mm long bar has rectangular cross section 10 X 30mm.this bar is subjected to**

1) 15 kN tensile force on 10 X 30 mm faces.

2) 80 kN compressive force on 10 X 400 mm faces.

3) 180kN tensile force on 30 X 400mm faces.

Find the change in volume if $E = 2 \times 10^5\ N/mm^2$ and 1/m=0.3. (10)



Ans: $E = 2 \times 10^5\ N/mm^2$.

1/m=0.3

AB=400mm.

BC=30mm.

BG=10mm.

Stress in x-direction

$f_x = \frac{15\times10^3}{10\times30}. = 50\ N/mm^2$ (tensile)

Stress in y-direction

$f_y = \frac{80\times10^3}{10\times400} = 20\ N/mm^2$. (compressive).

Stress in z- direction

$f_x = \frac{180\times10^3}{400\times30} = 15\ N/mm^2$.(compressive).

Strain in x direction.
$e_x = \frac{f_x}{E} - \frac{1}{m}\frac{f_y}{E} - \frac{1}{m}\frac{f_z}{E}$
$\therefore\ e_x = 3.025 \times 10^{-4}$.
Similarly
$e_y = -1.525 \times 10^{-4}$
$e_z = -1.2 \times 10^{-4}$

Now
Change in AB= L(AB) X e_x
= $400 \times 3.025 \times 10^{-4}$
= 0.121mm (increase)

Change in BC= $L(BC) \times e_y$
$= 30 \times -1.52 \times 10^{-4}$
= -4.575$\times 10^{-3}\ mm$

Change in BG = $L(BG) \times e_z$
= 10 X -1.525$\times 10^{-4}$
$= 1.2 \times 10^{-3} mm$

New lengths:
AB= 4000+0.121=400.121mm.
BC= 30-4.575$\times 10^{-3}$=29.99mm
BG= $10 - 1.2 \times 10^{-3}$=9.99mm

$change\ in\ volume = new\ volume - original\ volume.$
$\therefore \delta V = (400.121 \times 29.99 \times 9.99) - (400 \times 30 \times 10).$
$\delta V = -123.71 mm^3\ (decrease).$

b) A hollow cylinder CI column is 4m long with both end fixed. Determine the minimum diameter of the column, if it has to carry a safe load of 250kN with a FOS of 5. Take internal diameter as 0.8 times the external diameter $\boldsymbol{E = 200\ GN/m^2}$. (10)

Ans: $L_e = 0.5 \times 4 = 2m = 2000mm.$
P $= 250kN$.
FOS=5.
$d_i = 0.8 \times d_o$.

Using Euler's equation:

$$P_e = \frac{\pi^2 E I_{min}}{L_E^2}.$$

$$\therefore\ \frac{250}{5} = \frac{\pi^2 \times 200 \times 10^3 \times \frac{\pi}{64}\left(d_0^4 - d_i^4\right)}{(2000)^2}.$$

$$\therefore\ \frac{250}{5} = \frac{\pi^2 \times 200 \times 10^3 \times \frac{\pi}{64}\left(d_0^4 - (0.8 d_0^4)\right)}{(2000)^2}.$$

By solving we get.

$d_o = 43.24\ mm.$

Also

$d_i = 34.59 mm.$