MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

CBCGS(DEC- 2017)   Q.P.Code: 27636

1.(a) If a round bar of 37.5 mm diameter and 2.4 m length is stretched by 2.5 mm, find its bulk modulus and lateral contraction. Take Young's modulus= 110 GN/m^2 and shear modulus = 42 GN/m^2 for the material of the bar.   (5)

D= 37.5 mm = 37.5 x 10^{-3} m, l = 2.4 m, δl= 2.5mm = 2.5 x 10^{-3} m

E= 110 GN/m^2 = 110 x 10^9 N/m^2

G= 42 GN/m^2 = 42 x 10^9 N/m^2

Using relation, E= 2G $(1+\frac{1}{m})$

$$110 \times 10^9 = 2 \times 42 \times 10^9 \left(1+\frac{1}{m}\right)$$

$$\left(1+\frac{1}{m}\right) = \frac{110 \times 10^9}{42 \times 10^9}$$

$$\therefore \frac{1}{m} = 0.30952$$

Bulk Modulus: E= 3K $(1-\frac{2}{m})$

$$K = \frac{E}{3(1-\frac{2}{m})}$$

$$\therefore \mathbf{K = 0.96 \times 10^5\ N/mm^2}$$

Longitudinal Strain: e = $\frac{\delta l}{l}$ = **1.0416 x 10^{-3}**

Lateral contraction: $\frac{1}{m} = \frac{Lateral\ strain}{Longitudnal\ strain}$

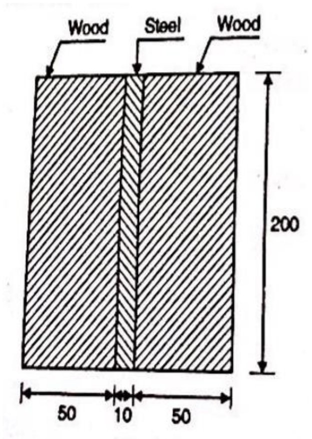
$$\therefore 0.3095 = \frac{Lateral\ strain}{1.0413 \times 10^{-3}}$$

$\therefore$ Lateral strain = 3.223 x 10^{-4}

Now, Lateral strain = $\frac{Lateral\ contraction}{original\ lateral\ dimension}$

$\therefore$ lateral contraction = 1.208 x 10^{-5} m

1.(b)A flitched beam consists of steel and timber as shown in figure. Determine the moment of resistance of the beam. Take σ_s = 100N/mm^2 and σ_w = 5N/mm^2. **(5)**



σ_s = 100N/mm^2 , σ_w = 5N/mm^2

Moment of inertia $I_{wood} = 2(\frac{BD^3}{12})$

$$= 2(\frac{50 \ x \ 200^3}{12}) = \mathbf{66.67 \ x \ 10^6 \ mm^4}$$

Moment of inertia $I_{steel} = 2(\frac{BD^3}{12})$

$$= 2(\frac{10 \ x \ 200^3}{12}) = \mathbf{6.67 \ x \ 10^6 \ mm^4}$$

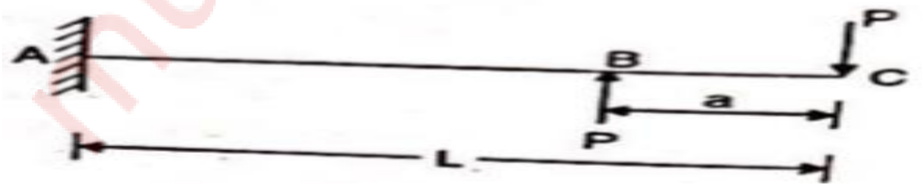
Moment of resistance M= $M_{wood} + M_{steel}$

$$= (\frac{\sigma}{y} \ x \ I) + (\frac{\sigma}{y} \ x \ I)$$

$$= \frac{5 \text{ x } 66.67 \text{ x } 10^6}{100} + \frac{1000 \text{ x } 6.67 \text{ x } 10^6}{100}$$

= **70 kN-m**

1.(c) Draw the S.F and B.M diagrams for the beam loaded shown in the figure. **(5)**



$\sum M_A = 0$

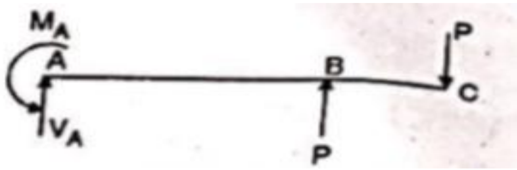
$\therefore M_A$+ PL-P(L-a) =0

$\therefore M_A$ = Pa

$\sum F_y = 0$

$\therefore V_A - P - P = 0$

$\therefore V_A = 0$



Shear force calculations:

$SF_{CR} = 0$

$SF_{BR} = P$

$SF_{BL} = 0$

$SF_A = 0$

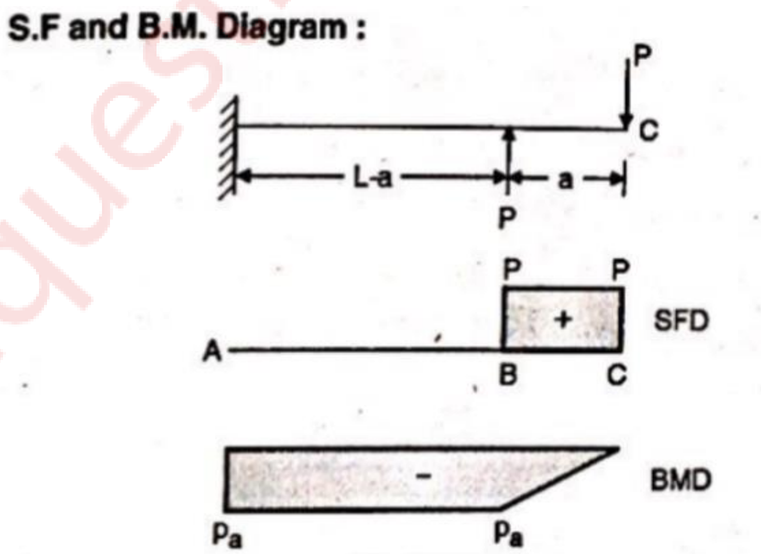
Bending moment calculations:

$BM_C = 0$

$BM_B = -P \times a = -Pa$

$BM_{AR} = -PL + P(L-a) = -Pa$

$BM_{AL} = -Pa + Pa = 0$



1.(d) Calculate the bursting pressure for a cold drawn seamless steel tubing of 60 mm inside diameter and 2 mm wall thickness. Ultimate strength of steel is 380 N/mm^2. (5)

d = 60mm,   t= 2mm

σ_h= 380 N/mm^2

$\sigma_h = \frac{pd}{2t}$

$\therefore$ 380 = $\frac{p\,X\,60}{2X2}$

$\therefore$ p = 25.33 Mpa

1.(e) Find the maximum power that can be transmitted through a 50 mm diamter shaft at 150 rpm, if the maximum permissible shear stress in the shaft is 80 N/mm^2. (5)

Polar moment of inertia = $\frac{\pi\, d^4}{32}$ = 613.59 x 10^3

Using torsional formula,

$$\frac{T}{J} = \frac{\tau}{R}$$

$\therefore$ T = 1.963 x 10^3 N-m

Now, power transmitted, P = $\frac{2\pi NT}{60}$

$\therefore$ P= 30.84 x 10^3 watt

$\therefore$ P= 30.84 kW

2.(a) A beam weighing 450 N is held horizontal by three vertical wires, one attached to the middle of the beam and the others to the ends of the beam. The outer wires are of brass with 1.25mm diameter and the central wire is of steel with 0.625 mm diameter. Estimate the stresses induced in the wires, assuming that the beam is rigid and the wires are of same length and unstretched before attaching to the beam. Take Young's moduli of brass as 8.6 x 10^4 N/mm^2 and of steel as 2.1 x 10^5 N/mm^2. (10)

E_{steel}= 2.1 x 10^5 N/mm^2

E_{brass} = 8.6 x 10^4 N/mm^2

$\delta l_s = \frac{l_s \times \sigma_s}{E_s}$

Similarly, $\delta l_b = \frac{l_b \times \sigma_b}{E_b}$

Since increase in the length of steel and brass wire is equal, therefore equating equations,

$$\frac{l_s \ x \ \sigma_s}{2.1 \ x \ 10^5} = \frac{l_b \ x \ \sigma_b}{8.6 \ x \ 10^4}$$

$\therefore \sigma_s = 2.442 \ \sigma_b$

Load supported by the three wires= 450N

$\therefore 450 = \sigma_s A_s + \sigma_b A_b$

$\therefore 450 = 1.98 \ \sigma_b$

$\therefore \boldsymbol{\sigma_b} = \mathbf{227.27 \ MPa}$

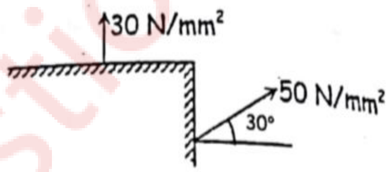
$\therefore \boldsymbol{\sigma_s} = \mathbf{555 \ MPa}$

2.(b) At a point in a material under stress, the intensity of the resultant stress on certain plane is 50 N/mm^2 inclined at 30° to the normal of that plane. The stress on a plane at right angles to this has a tensile component of intensity 30 N/mm^2. Find: (10)

(i) The resultant stress on the second plane

(ii) The principle plane and stresses

(iii) Plane of maximum shear and intensity



From the given data, we first complete the figure indicating the stresses acting at a point.

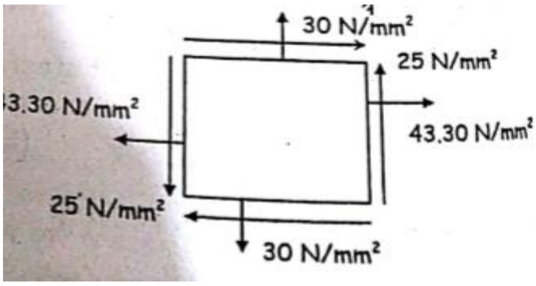
From figure,

$\sigma_x = 43.30$ N/mm^2

$\sigma_y = 30$ N/mm2

$\tau_{xy} = 25$ N/mm^2

(i)Resultant stresses on second plane



(ii)To find principal stresses P1 and P2 and their directions θ_1 and θ_2

Using $P_1, P_2 = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}{}^2}$

$\therefore$ Puting the values, we get,

$P_1 = 62.52$ N/mm^2

$P_2 = 10.78$ N/mm^2

Using $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$\therefore \theta = 37.55$

$\theta_2 = \theta_2 + 90 = 127.55°$

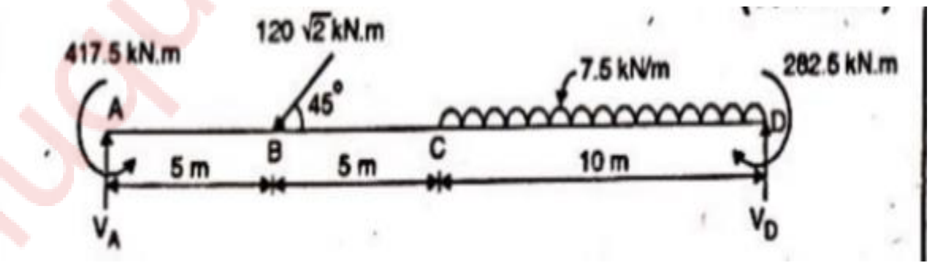
(iii)To find maximum shearing stress and their directions

Using, $\boldsymbol{\tau_{max}} = \pm \frac{\boldsymbol{P_1 - P_2}}{\boldsymbol{2}} = \pm \boldsymbol{25.87}$**N/mm^2**

$\theta_3 = \theta_1 + 45 = 82.55°$

$\theta_4 = \theta_3 + 90 = 172.55°$

3.(a) For the beam shown draw S.F. and B.M. diagram and mark important points. (10)



Step 1: Support Reactions

$\sum M = 0$

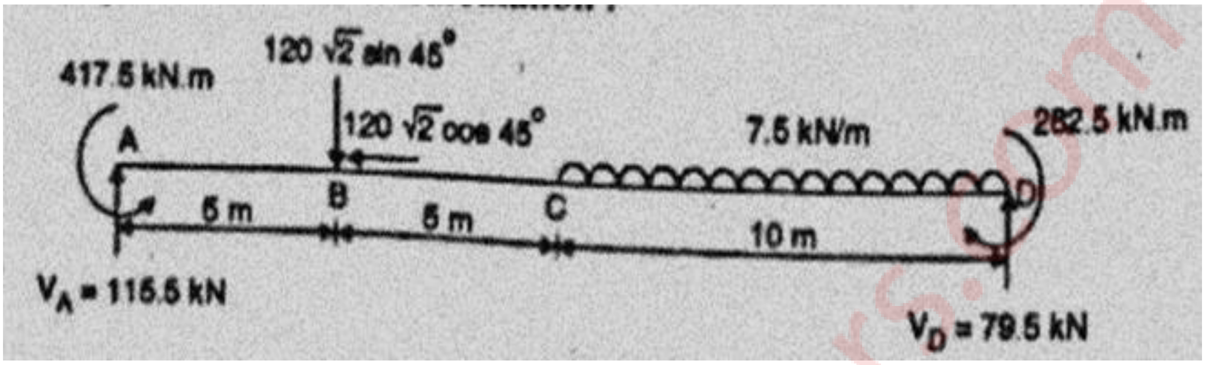
$-417.5 + 282.5 + (120\sqrt{2}\sin 45°) \times 5 + (7.5 \times 10)(5+10) - V_D \times 20 = 0$

$\therefore$ - V_D = 79.5 kN

$\sum F = 0$

$\therefore$ V_A = 115.5 kN

Step 2: Shear force calculations



SF_{AL} = 0

SF_{AR} = 115.5kN

SF_{BL} = 115.5- (120√2sin 45°) = -4.5kN

SF_C = -4.5kN

SF_{DL} = -4.5 -7.5 x 10 = -79.5 kN

SF_{DR} = -79.5 +79.5 = 0

Step 3: Bending moment calculation

BM_{AL} = 0

BM_{AR} = -417.5 kNm

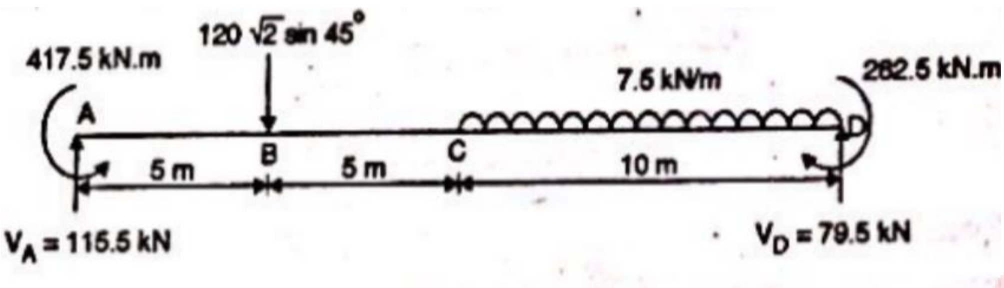
BM_B = -417.5 + 115.5 x5 = 160 kNm

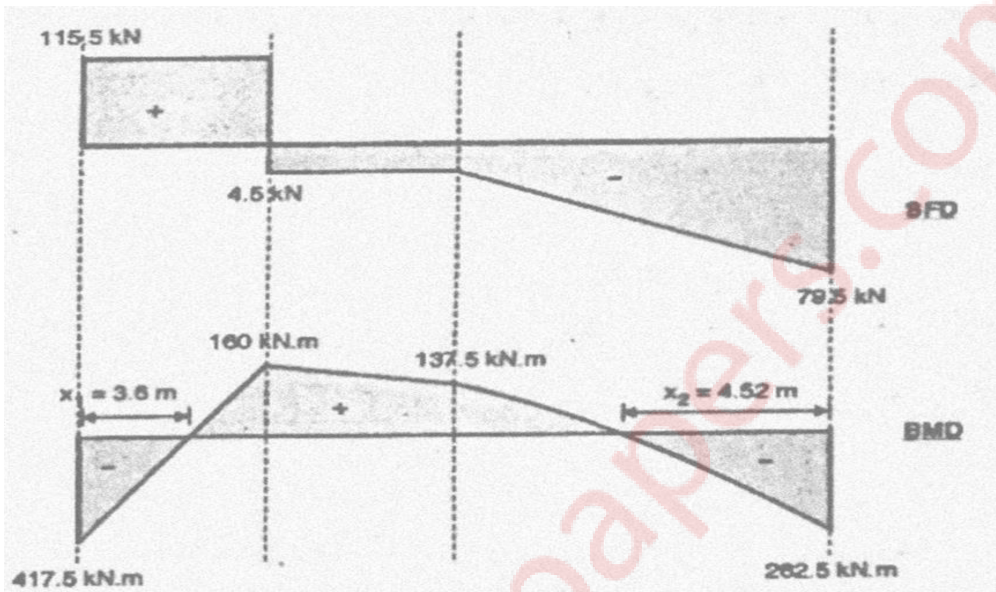
BM_C = -417.5 + 115.5 x 10 - (120√2sin 45°) x 5 = 137.5 kNm

BM_{DL} = -417.5 + 115.5 x 20 - (120√2sin 45°) x 15 -7.5 x $\frac{10^2}{2}$ = -285.5 kN

BM_{DR} = -285.5 +285.5 = 0

Step 4: Shear force and Bending moment diagram





Step 5: Point of Contraflexure

Consider BMD between portion AB by similarity of triangles,

$$\frac{x_1}{417.5} = \frac{5-x_1}{160}$$

$\therefore$ x_1 = 3.6 m from point A

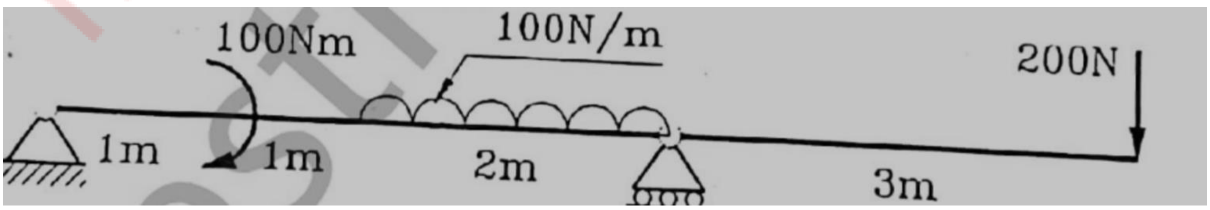
Now consider a section at distance x_2 from pt. D

$$BM_{xx} = 0$$

$$-282.5 + 79.5\, x_2 - 7.5\,\frac{x_2^2}{2} = 0$$

$\therefore$ x_2 = 4.52 m from point D

3.(b) Determine the slope and deflection at the free end of the beam loaded as shown. E=200GPa, I= 14 x 10^6 m^4 (10)



Let R_A and R_D are the support reaction at A and D.

$\sum M_A = 0$

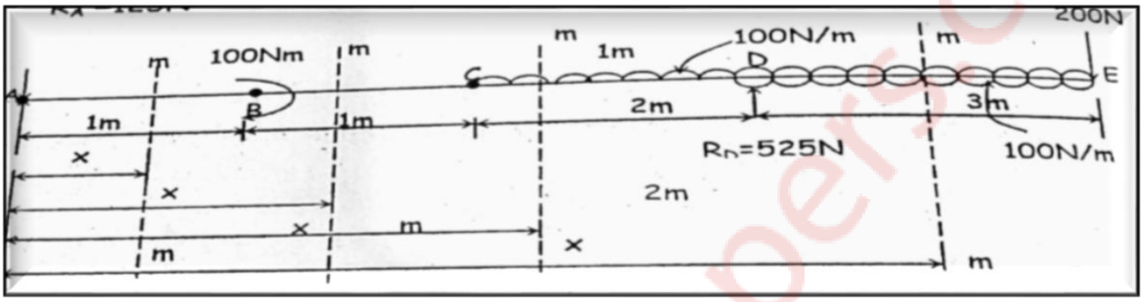
$-R_D \times 4 + 100 + (100 \times 2)(1+2) + 200 \times 7 = 0$

$\therefore R_D = 525$ kN

$\sum F_y = 0$

$R_A + R_D - 100 \times 2 - 200 = 0$

$R_A = 125$ kN



$$EI\frac{d^2y}{dx^2} = -125x\Big|_0^1 + 100(x-1)^0\Big|_0^2 - \frac{100(x-2)^2}{2}\Big|_0^4 + 525(x-4) - \frac{100(x-4)}{2}\Big|_0^7$$

Slope Equation

$$EI\frac{dy}{dx} = C_1 - \frac{125x^2}{2}\Big|_0^1 + 100(x-1)\Big|_0^2 - \frac{100(x-2)^3}{6}\Big|_0^4 + \frac{525(x-4)^2}{2} - \frac{100(x-4)^3}{6}\Big|_0^7$$

Deflection equation

$$EI.y = C_1x + C_2 - \frac{125x^3}{6}\Big|_0^1 + \frac{100(x-1)^2}{2}\Big|_0^2 - \frac{100(x-2)^4}{24}\Big|_0^4 + \frac{525(x-4)^3}{6} - \frac{100(x-4)^4}{24}\Big|_0^7$$

At x=0, y=0 $\therefore C_2 = 0$

At x=4, y=0 $\therefore C_1 = -237.5$

i)To find deflection at point E, substituting x=7 in deflection

$$EI.y = (-237.5)(7) + 0 - \frac{125(7)^3}{6} + \frac{100(7-1)^2}{2} - \frac{100(7-2)^4}{24} + \frac{525(7-4)^3}{6} - \frac{100(7-4)^4}{24}$$

$\therefore y = -2.709 \times 10^{-15}$

(ii) To find slope at end A

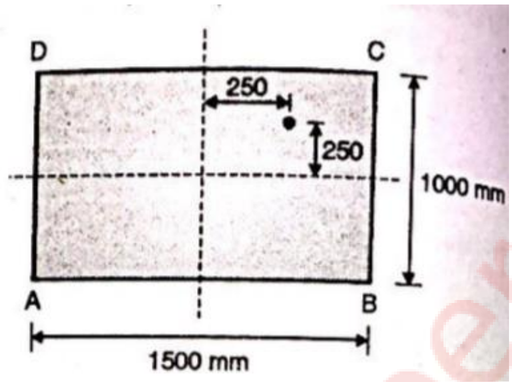
Substituting x= 7 in slope equation.

$$EI\frac{d^2y}{dx^2} = (-237.5) - \frac{127(7^2)}{2} + 100(7-1) - + \frac{525(7-4)^2}{2} - \frac{100(7-4)^2}{6}$$

$EI\frac{dy}{dx} = -2870.83$

$\therefore \frac{dy}{dx}$ **= 1.025 x 10^{-15} radians**

4.(a) A rectangular pier is subjected to a compressive load of 450 kN as shown in the figure. Find the stress intensities at the four corners of the pier. (10)



A= b x d = 1 x 1.5 = 1.5 m^2

and moment of Inertia of the pier about x-x axis

$I_{xn} = \frac{bd^3}{12} = 0.125\ m^4$

Similarly, $I_y = 0.281\ m^4$

We also know that moment due to eccentricity or load along x-x axis.

$M_x = P.\ e_x = 450 \times 0.25 = 112.5$

$M_y = P.\ e_y = 450 \times 0.25 = 112.5$

From geometry of the loading, we find that distance between y-y axis and the corners D and C

$x= \frac{1.5}{2} - 0.75 = 0.75m$

Similarly, distance between x-x axis and the corner D and A

$y= \frac{1}{2} = 0.5\ m$

From above we can calculate stresses at each corner as follows,

$\sigma = \sigma_o \pm \sigma_{bxx} \pm \sigma_{yy}$

$\sigma_A = \sigma_o - \sigma_{bxx} - \sigma_{yy} = 300 - 450 - 300 =$ **-450 kN/m^2**

$\sigma_B = \sigma_o - \sigma_{bxx} + \sigma_{yy} = 300 - 450 + 300 =$ **150 kN/m^2**

$\sigma_C = \sigma_o + \sigma_{bxx} + \sigma_{yy} = 300 + 450 + 300 =$ **1050 kN/m²**

$\sigma_D = \sigma_o + \sigma_{bxx} - \sigma_{yy} = 300 + 450 - 300 =$ **450 kN/m²**

4.(b) Internal diameter of a hollow shaft is 0.6 of its external diameter. It has to transmit 300 kW power at 80rpm. If the shear stress is not to exceed 60 N/mm², find the internal and external diameters of the shaft, assuming that the maximum torque is 1.4 times the mean torque. (10)

Hollow shaft with internal diameter (d) and outer diameter(D)

0.6D=d τ_s = 60 N/mm²

$T_{max} = 1.4\ T_{mean}$ P= 300 kW

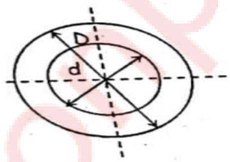
Now, P= $\frac{2\pi NT}{60}$

P = 300 x 10^3 watts N= 80 rpm

Putting these values, we get

T_{mean} = 35809.86 N-m

∴ T_{max} = 1.4 x 35809.86 = 50133.80 x 10^3 N-mm



For hollow shaft torque transmitted is given by

$$\therefore T = \frac{\pi}{16} \cdot \tau_s \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \cdot (60) \left(\frac{D^4 - (0.6D)^4}{D}\right)$$

∴ D = 169.72 mm

∴d= 101.83 mm

5.(a) A 200 kg weight is dropped on to a collar at the lower end of a vertical bar of 3 m long and 28 mm diameter. Calculate the height of drop, if the maximum instantaneous stress is not exceeding 120 N/mm².What is the corresponding instantaneous elongation? Take E= 2x 10^5 N/mm². (10)

Ans.

Given: Weight=200 kg = 200 x 9.81 = 91==1962 N

Dig of rod(d) = 28 mm

Length of rod (l)= 3m = 3000 mm

E=2 x 10^5 N/mm2

σinstantaneous = 120 N/mm2

Instantaneous stress for rod is given by,

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2EAh}{PL}}\right)$$

$$120 = \frac{1962}{\frac{\pi}{4}(28)^2}(1 + \sqrt{1 + \frac{(2)(2 \times 10^5)\left(\frac{\pi}{4} \times 28^2\right)h}{1962 \times 3000}})$$

1

h=32 mm

Now, E = $\frac{stress}{strain}$

$$(2 \times 10^5) = \frac{120}{e}$$

$$e = \frac{120}{2 \times 10^5}$$

$$e = 6 \times 10^{-4}$$

Let, instantaneous elongation = (δt)

$e = \frac{\delta t}{3000}$ $\quad\quad$ $\delta t = 1.8\ mm$

Instantaneous elongation $(\boldsymbol{\delta t}) = \mathbf{1.8}\ \boldsymbol{mm}$

5.(b) A simply supported beam, with a span of 1.3 m and a rectangular cross section of 1.3 m and a rectangular cross section of 150 mm wide and 250 mm deep, carries a concentrated load of W at center. If the allowable stresses are 7 N/mm2 for bending and 1 N/mm2 for shear, what is the value of the safe load W? (10)

Shear force and bending moment diagrams of given beam is as shown in the figure.

The maximum shear force = $\frac{W}{2}N$

The maximum BM = $\frac{WL}{4} Nm$

$\sigma_b = 7\ N/mm^2$

$\tau = 1N/mm^2$

Bending Stress criteria:

Using bending equation,

$$\frac{\sigma_b}{y_{max}} = \frac{M_{max}}{F}$$

$$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12}$$

I = 195.31 x 10^6 mm^4

$y_{max} = \frac{250}{2} = 125mm$

$$\frac{7}{125} = \frac{\frac{W(1.3 \times 10^3)}{4}}{195.31 \times 10^6}$$

W= 33653.84 N

W=33.65 kN

Shear Stress Criteria:

Maximum shear force in beam = W/2

Cross-section area of beam= 150 x 250 = 375000 mm^2

Average shear stress = $\frac{shear\ force}{shear\ area}$

$$\tau_{avg} = \frac{W/2}{37500} = (1.33 \times 10^{-5})W$$

In a rectangular beam section, the maximum shear stress should not exceed 1 MPa

Therefore, (2 x 10-5) W =1

W = 50000 N

W = 50 kN

Maximum safe load is there for minimum of above two results

W= 33.65kN

6.(a) A hollow cast iron column of 200 mm external diameter 150 mm internal diameter and a 8 m long has both ends fixed. It is subjected to axial compressive load. Taking factor of safety as 6, σ_c = 560 N/mm^2, $\alpha = \frac{1}{1600}$, determine the safe Rankine load. (10)

D= 200 mm d= 150mm

L= 8m = 800mm

Column is fixed at both ends

$L_e = \frac{L}{2}$ = 4000 mm

σ_c = 560 N/mm^2 FOS=6

M.I. = $\frac{\pi}{64}(D^4 - d^4) = 53.68 \times 10^6$ mm^4

Area = $\frac{\pi}{4}(D^2 - d^2) = 13.74 \times 10^3$ mm^2

K= $\sqrt{\frac{I}{A}} = \sqrt{\frac{53.68 \times 10^6}{13.74 \times 10^3}}$ = 62.50 mm

∴ Rankine crippling load = $\frac{\sigma_c . A}{1+\alpha(\frac{L_e}{k})^2}$ = 360.21 kN

∴ Safe Rankine Load = 360.21 kN

6.(b) A weight pf 200 kN is supported by three adjacent short pillars in a row, ecah 500 mm^2 in second. The central pillar is made of steel and the outer ones are of copper. The pillars are adjusted such that 15^{0}C each carries equal load. The temperature is then raised to 115^{0}C. Estimate the stresses in each pillar at 15^{0}C and 115^{0}C. (10)

Take $E_s = 2 \times 10^5$ N/mm^2 $E_C = 0.8 \times 10^5$ N/mm^2

$\alpha_s = 1.2 \times 10^5$ / ^{0}C $\alpha_c = 1.85 \times 10^5$ / ^{0}C

P = 200kN = 200 x 10^3 N

Now, $\delta l_s = \delta l_c$

∴ $\frac{\sigma_s . l}{2 \times 10^5} = \frac{\sigma_c . l}{0.8 \times 10^5}$

∴ $\sigma_s = 2.5\sigma_c$

Also,

$200 \times 10^3 = \sigma_s A_s + \sigma_c A_c$

Putting the values of σ_s and A_s, A_c

We get,

σ_c = 114.28 N/mm^2

$\therefore \sigma_s$= 285.7 N/mm^2

Stresses before the rise in temp = 285.7 N/mm^2

When temperature is increased,

Force in steel = Force in copper

$P_s = P_c$

$\therefore \sigma_s A_s = \sigma_c A_c$

Now, $A_s = A_c$

$\therefore \sigma_s = \sigma_c$

Now,

$\frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c} = t\,(\alpha_c - \alpha_s)$

Putting the respective values, we get

$\sigma_c(1.75 \times 10^{-5}) = 6.5 \times 10^{-4}$

$\therefore \sigma_c$= 37.14 = σ_s
