Q.P Code: 29702

MUMBAI UNIVERSITY CBCGS SEM II

APPLIED PHYSICS II DEC 2019 PAPER SOLUTIONS

Q1)a) Why does an excessively thin film appear to be perfectly dark when illuminated by white light. (3M)

Ans : 1) When a thin film is exposed to white light from an extended source, it shows beautiful colours in the reflected system.

2) Light is reflected from the top and bottom surfaces of a thin film and the reflected rays interfere.

3) The path difference between the interfering rays depends on the thickness of the film and the angle of refraction r and hence on the inclination of the incident ray.

4) White light consists of a continuous range of wavelengths. At a particular point of the film and for a particular position of the eye (i.e. t and r constant) those wavelengths of incident light that satisfy the condition for the constructive interference in the reflected system will be seen in the reflected light.

5) The coloration will vary with the thickness of the film and inclination of the rays (i.e. with the position of the eye with respect to the film).Hence if the same point of the film is observed with an eye in different positions or different points of the film are observed with the eye in the same position, a different set of colours is observed each time.

Q1)b) In a plane transmission grating the angle of diffraction for the first order principal maximum is 20° for a wavelength of 6500^0A. Calculate the number of lines in 1 cm of the grating surface. (3M)

Ans : m=1 , $\lambda = 6500 \times 10^{-10}$m , $\theta = 20^\circ$,

We know that

$(a+b) \sin\theta = m\lambda$.

Hence, $(a+b) = m\lambda / \sin\theta = 1 \times 6500/\sin(20) = 7119.81 \times 10^{-8}$ cm .

Number of lines per cm = $1/(a+b) = 1/7119.81 \times 10^{-8} = 14000$

Number of lines per cm is 14000 .

Q1)c) Explain the term V-number of an optical fibre. (3M)

Ans : 1) In optical fibre, the light propagates in the same way as the electromagnetic wave propagates.

2) When confined to a duct or guide, it propagates like electromagnetic wave, but at a much higher frequency.

3) The number of modes supported by a fibre is determined by an important parameter called "cut off parameter " or "V-Number ".

4) The mathematical expression for V-number is

$$V = \pi \frac{d}{\lambda_0} \sqrt{\mu_1^2 - \mu_2^2}$$

Where λ_0 =free space wavelength

d=2a= diameter of the core (a = radius of the core)

$$\therefore V = \frac{\pi d}{\lambda}(NA)$$

$$= \frac{\pi d}{\lambda} \mu_1 \sqrt{2\Delta}$$

5) The maximum number of modes N_m supported by SI fibre is given by

$$N_m \approx \frac{1}{2} V^2$$ (provided V-number is considerably larger than unity)

Q1)d) Differentiate between Spontaneous Emission and Stimulated Emission. (3M)

Ans :

Spontaneous Emission	Stimulated Emission
1) The transition of an electron from the excited state to the ground state happens as a result of the natural tendency of the electron without the action of any external agent. The radiation produced as a result of such transitions is called as spontaneous radiation.	1) Stimulated emission of radiation is the process whereby photons are used to generate other photons that have exact phase and wavelength as that of parent photon
2) It cannot be controlled .	2)It can be controlled effectively .
3) Here, no multiplication of photons takes place .	3)Here, multiplication of photons takes place.
4) This phenomenon is found in LEDs, Fluorescent tubes.	4) This is the key process of formation of laser beam.
5) There is no population inversion of electrons in LEDs.	6) Population inversion is achieved by various 'pumping' techniques to get amplification .

Q1)e) Show that divergence of curl of a vector is zero. **(3M)**

Ans :

$$\text{Let } \bar{F} = F_x\,\bar{a}_x + F_y\,\bar{a}_y + F_z\,\bar{a}_z$$

$$\bar{F} = \nabla \times \bar{F} = \begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{bmatrix}$$

$$= \bar{a}_x \times \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \bar{a}_y\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \bar{a}_z\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$\text{Now} \quad \text{div}\,(\text{curl}\,\bar{F}) = \nabla \cdot (\nabla \times \bar{F})$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$= \frac{\partial^2 F_z}{\partial x\,\partial y} - \frac{\partial^2 F_y}{\partial x\,\partial z} - \frac{\partial^2 F_z}{\partial y\,\partial x} + \frac{\partial^2 F_x}{\partial y\,\partial z} + \frac{\partial^2 F_y}{\partial z\,\partial x} - \frac{\partial^2 F_x}{\partial z \cdot \partial y} = 0$$

Q1)f) An electron is accelerated through a potential difference of 18 Kv in a color Cathode ray tube. Calculate the kinetic energy and the speed of the electron. **(3M)**

Ans : $eV = ½(mv^2)$

Kinetic Energy = $1.6 \times 10^{-19} \times 18 \times 10^{3}$

= 28.8×10^{-16} Joules .

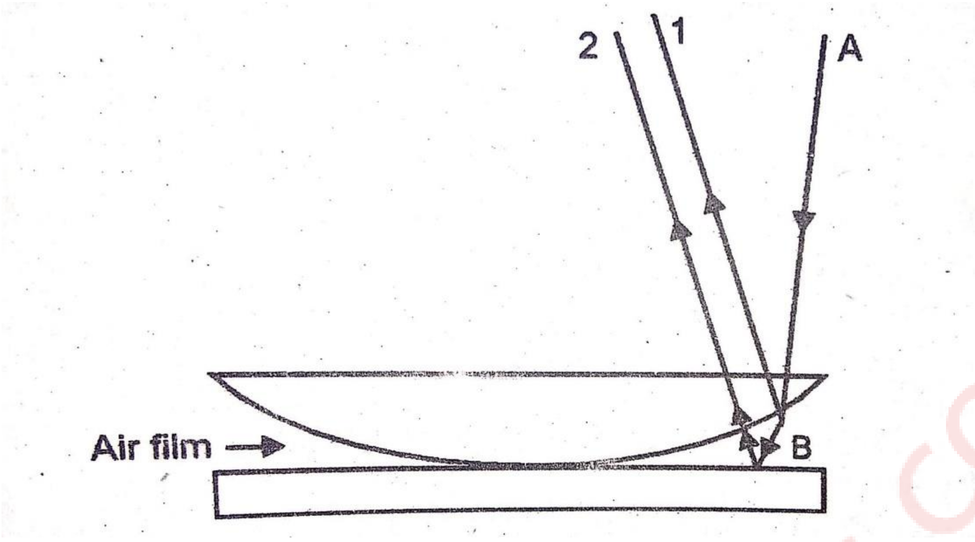
$v = (2eV/m)^{1/2} = (2 \times 1.6 \times 10^{-19} \times 18 \times 10^{3} / 9.1 \times 10^{-27})^{1/2}$

= 795.5×10^{3} m/sec .

The Kinetic Energy of the electron is 28.8×10^{-16} Joules and its speed is 795.5×10^{3} m/sec .

Q1)g) What will happen when a liquid is introduced between the plano convex lens and glass plate in Newton's rings experiment. **(3M)**

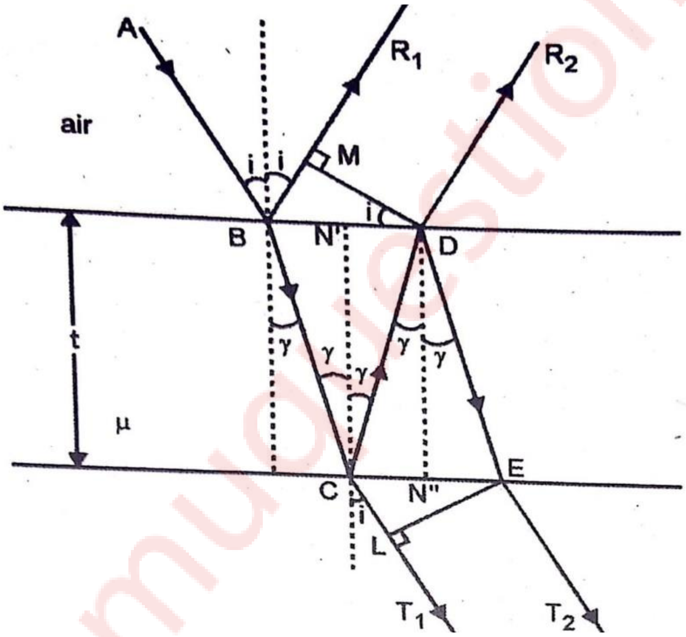
Ans : When a plano-convex lens with large radius of curvature is placed on a plane glass plate such that its curved surface faces the glass plate, a wedge air film (of gradually increasing thickness) is formed between the lens and the glass plate. The thickness of the air film is zero at the point of contact and gradually increases away from the point of contact .



Q2)a) What do you mean by thin film? Obtain the conditions for the maxima and minima of the light reflected from a thin transparent film of uniform darkness. (8M)

Ans :

A thin film is a layer of material with thickness in the sub- nanometer to micron range. As light strikes the surface of a film it is either transmitted or reflected at the upper surface. Light that is transmitted reaches the bottom surface and may once again be transmitted or reflected. Thin films can be engineered to control the amount of light reflected or transmitted at a surface for a given wavelength.



i) The two rays will interfere consecutively if the path difference between them

is an integral multiple of λ i.e.

$2\mu t \cos r + \lambda/2 = n\lambda$

Or

$2\mu t \cos r = (2n-1)\lambda/2$

Where,

n= 1, 2, 3, 4, ……….. (for maxima)

or

$2\mu t \cos r = (2n-1)\lambda/2$

Where n = 0, 1, 2, 3, 4, ………………….

When this condition is satisfied the film will appear bright in the reflected system.

ii) The two rays will interfere destructively if the path difference between them is an odd multiple of λ/2 i.e.

$$2\mu t \cos r + \lambda/2 = (2n-1)\,\lambda/2$$

Or

$$2\mu t \cos r = n\lambda \qquad \text{(for minima)}$$

iii. The film which appears bright in reflected light appears dark in transmitted light and vice versa.

iv. For the transmitted light, the intensity of maxima is about 100% and the minima is about 85%. The result in poor contrast between bright and dark where in poor contrast between where as in reflected light, minima is having its intensity zero and maxima is nearly 15% of incident energy. This result in good contrast.

Q2)b) Explain Step index and Graded index fibre. A Step Index Fibre has a core diameter of $2.9*10^{-6}$ m , the refractive indices of core and cladding are 1.52 and 1.5189 respectively .If the light of wavelength 1.3 μm is transmitted through the fibre determine the normalized frequency and number of modes supported by the fibre. (7M)

Ans :

Step Index Fibre is the simplest type of an optical fibre. It consists of a thin cylindrical structure of transparent glossy material of uniform refractive index μ_1 surrounded by cladding of another material of uniform but slightly lower refractive index μ_2 .These fibres are called as the step index fibres due to the discontinuity of the index profile at the core cladding surface.

The graded index fibre has its core refractive index gradually decreasing in a nearly parabolic manner from a maximum value at the center of the core to a content value at the core-cladding interface. The variation in refractive index is achieved by using concentric layers of different refractive indices. Such a profile causes a periodic focusing of the light propagating through the fibre.

Numerical : For an optical fibre

$$V= \frac{\pi d}{\lambda} NA$$

Where V is V-Number or normalized frequency , d is diameter of the core ,NA is the numerical aperture and λ is the wavelength of light used .

$$V = \frac{3.14\ X\ 2.9\ X10^{-6}}{1.3X10^{-6}} X\sqrt{1.52^2 + 1.5189^2}$$

V =15.05 .

For Step Index fibre, the number of modes is

$$N = V^2/2 = 113 \ .$$

The normalized frequency is 15.05 Hz and the number of modes is 113, i.e it is a multimode fibre .

Q3)a) With neat energy level diagram describe the construction and working of Nd-YAG laser (8M)

Ans : Neodymium-doped Yttrium Aluminum Garnet (Nd: YAG) laser is a solid state laser in which Nd: YAG is used as a laser medium. Nd: YAG laser is a four-level laser system, which means that the four energy levels are involved in laser action. These lasers operate in both pulsed and continuous mode. Nd: YAG laser generates laser light commonly in the near-infrared region of the spectrum at 1064 nanometers (nm).

Construction :

Nd : YAG laser consists of three important elements: an energy source, active medium, and optical resonator. The laser is an elliptically cylinder reflector with both of its axis occupied by a flash lamp and Nd : YAG rod respectively. The light leaving one focus of the ellipse will certainly pass through the other focus after reflection from reflecting surface. Hence, entire light generated by flash tube is focussed on the Nd:YAG rod. The optical resonator is formed by highly silvered reflecting surfaces. Each mirror is silvered or coated differently. One mirror is fully silvered whereas, another mirror is partially silvered.

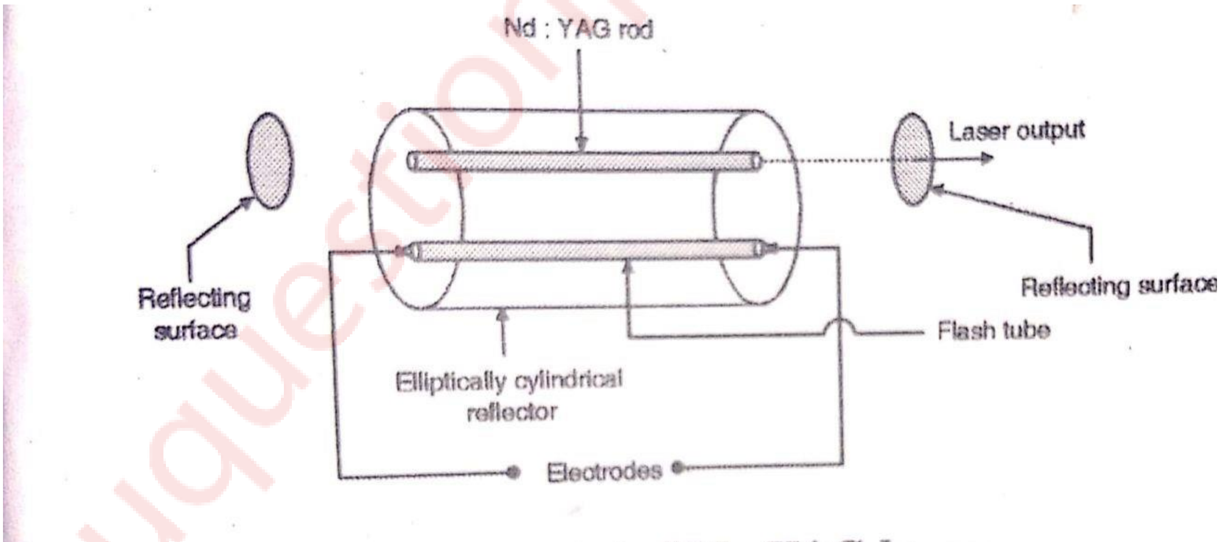


Fig. 4.12.1 : Nd : YAG laser

Working :

1) Nd: YAG laser is a four-level laser system, which means that the four energy levels are involved in laser action. The light energy sources such as flashtubes or laser diodes are used to supply energy to the active medium. In Nd:YAG laser, the lower energy state electrons in the neodymium ions are excited to the higher energy state to achieve population inversion.

2) Consider a Nd:YAG crystal active medium consisting of four energy levels E_1, E_2, E_3, and E_4 with N number of electrons. The number of electrons in the energy states E1, E_2, E_3, and E_4 will be N_1, N_2, N_3, and N_4. Let us assume that the energy levels will be $E_1 < E_2 < E_3 < E_4$. The energy level E_1 is known as ground state, E_2 is the next higher energy state or excited state, E_3 is the metastable state or excited state and E_4 is the pump state or excited state. Let us assume that initially, the population will be $N_1 > N_2 > N_3 > N_4$.

3) When flashtube or laser diode supplies light energy to the active medium (Nd:YAG crystal), the lower energy state (E_1) electrons in the neodymium ions gains enough energy and moves to the pump state or higher energy state E_4.

4) The lifetime of pump state or higher energy state E_4 is very small (230 microseconds (Âµs)) so the electrons in the energy state E_4 do not stay for long period. After a short period, the electrons will fall into the next lower energy state or metastable state E_3 by releasing non-radiation energy (releasing energy without emitting photons).

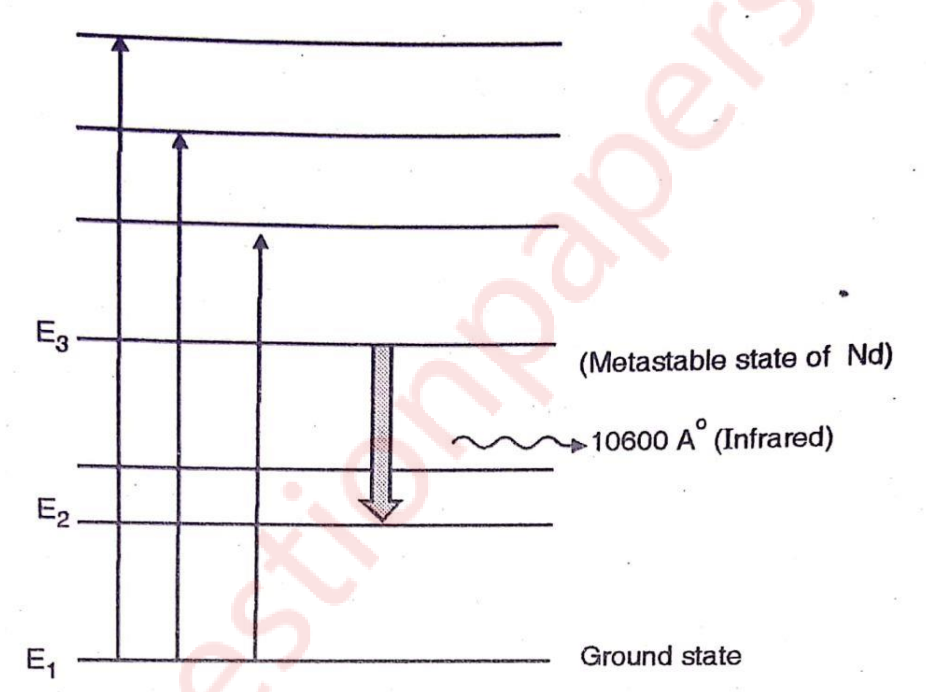


Fig. 4.12.2 : Energy levels of Nd along with YAG

5) The lifetime of metastable state E_3 is high as compared to the lifetime of pump state E_4. Therefore, the electrons reach E_3 much faster than they leave E_3. This results in an increase in the number of electrons in the metastable E_3 and hence population inversion is achieved.

6) After some period, the electrons in the metastable state E_3 will fall into the next lower energy state E_2 by releasing photons or light. The emission of photons in this manner is called spontaneous emission. Population Inversion takes place between E_3 and E_2 .A continuous laser of 10600˚A in infrared region is given out due to stimulate emission taking place between E_3 and E_2 .

Q3)b) What is grating element. The visible spectrum ranges from 4000 ̊A to 5000 ̊A .Find the angular breadth of the first order visible spectrum produced by a plane grating having 6000 lines/cm when light is incident normally on the grating. **(7M)**

Ans : A grating element is a diffraction grating. These can take many forms but the most basic is a repeated array of lines where the spacing from line to line is on the order of the wavelength. When the period of the grating is less than the wavelength, the grating acts like an effective medium and exhibits anisotropy and an average refractive index. Gratings can be used beam splitters.

The angles of the diffraction orders depend on wavelength so gratings can also be used to separate colors of light for spectroscopy, multiplexing, and more.

Numerical :

Given : 11 = 4000 Å = 4000 x 10-10 m,

12 = 7000 Å = 7000 x 10-10 m,

a + b = 6000 cm; m=1

Formula : (a + b) sin 0 = mλ

Hence, for λ, we have

$\sin\Theta_1 = 1 \times 4000 \times 10^{-10} \times 6000 \times 10^{-2} = 0.24$

$\Theta_1 = \sin^{-1}(0.24) = 13° 53'$ ·

For λ_2 we have

$\sin\Theta_1 = 1 \times 7000 \times 10^{-10} \times 6000 \times 10^{-2} = 0.42$

$\Theta_2 = \sin^{-1}(0.42) = 24° 50'$

Angular breadth of the first order visible spectrum

$= \Theta_2 - \Theta_1 = 24° 50' - 13° 53' = 10° 57'$

Ans : The angular breadth is 10° 57' .

Q4)a) Explain with neat diagram, construction and working of SEM. **(5M)**

Ans : Scanning electron microscope is an improved model of an electron microscope. SEM is used to study the three dimensional image of the specimen. When the accelerated primary electrons strikes the sample , it produces secondary electrons . these secondary electrons are collected by a positive charged electron detector which in turn gives a 3- dimensional image of the sample.

Construction :

It consists of an electron gun to produce high energy electron beam. A magnetic condensing lens is used to condense the electron beam and a scanning coil is arranged in-between magnetic condensing lens and the sample. The electron detector (Scintillator) is used to collect the secondary electrons and can be converted into electrical signal.

Working :

Stream of electrons are produced by the electron gun and these primary electrons are accelerated by the grid and anode. These accelerated primary electrons are made to be incident on the sample through condensing lenses and scanning coil. These high speed primary electrons on falling over the sample produces low energy secondary electrons. The collection of secondary electrons are very difficult and hence a high voltage is applied to the collector. These collected electrons produce scintillations on to the photo multiplier tube are converted into electrical signals. These signals are amplified by the video amplifier and is fed to the CRO.

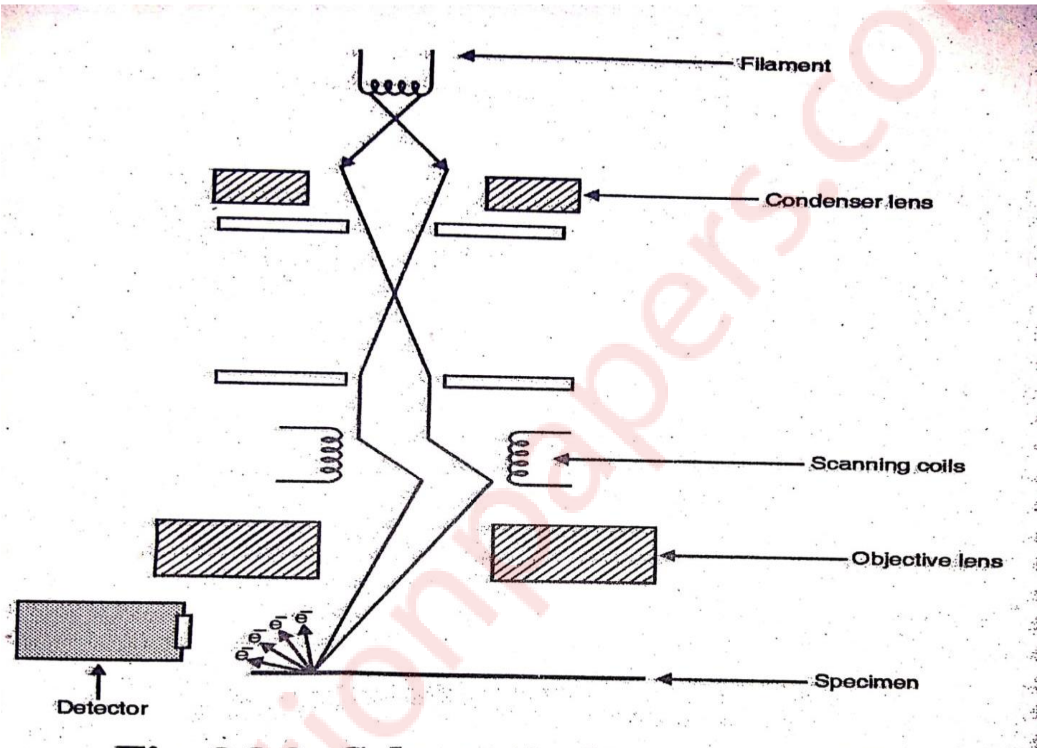


Fig. 8.3.2 : Schematic diagram of SEM

Q4)b) Explain spherical co-ordinate system. State the transformation relation between Cartesian and spherical co-ordinates. (5M)

Ans : Spherical coordinates determine the position of a point in three-dimensional space based on the distance ρ from the origin and two angles θ and φ. If one is familiar with polar coordinates, then the angle is essentially the same as the angle θ from polar coordinates.

Relationship between spherical and Cartesian coordinates :

Spherical coordinates are defined as indicated in the following figure, which illustrates the spherical coordinates of the point P. For spherical coordinates system also x, y, z axes are used for reference. Imagine a sphere of radius r with centre at origins. Any point on the sphere is at the same distance r from origin, therefore the spherical surface is defined as r= constant surface.

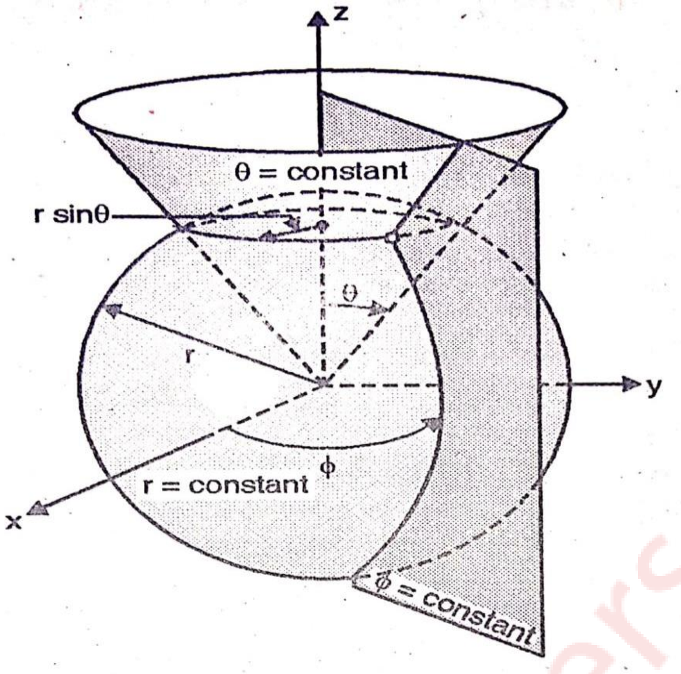


Fig. 6.1.8 : Spherical Coordinates

Now consider a line from origin making angle θ with z-axis. Rotate this line about z-axis fixing the end at the origin. This forms a cone with angle θ, this conical surface is defined as a θ = constant surface. When a sphere with centre at origin intersects with the vertical cone with vertex at origin, the intersection is a horizontal circle with radius equal to r sin θ. We want to locate a point in spherical coordinate system. Imagine a Ø = constant plane similar to in cylindrical system. A horizontal circle with centre onz-axis, Ø=constant plane. The intersection is the point.

Because r=constant, θ=constant and Ø=constant surface intersects at a point, the point is defined as (r, Ø, θ). In spherical system varriations of angle θ is

from 0 to 180 and variation of Ø is from 0 to 360 .

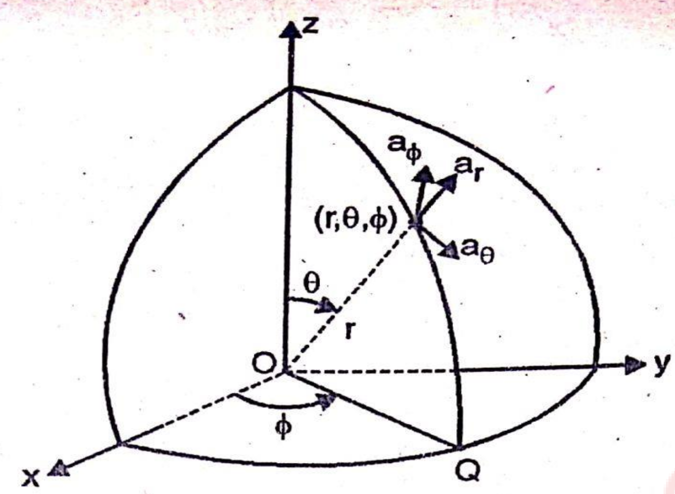


Fig. 6.1.10 : Unit vectors in spherical System

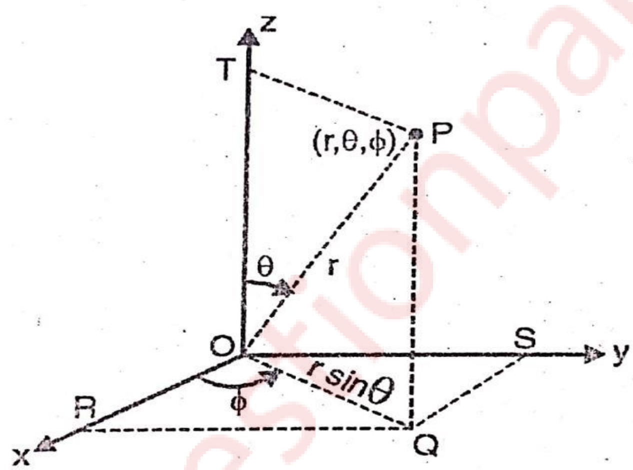


Fig. 6.1.11 : Relation between Spherical and Cartesian coordinates

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Q4)c) What is holography? Distinguish between holography and ordinary photography. (5M)

Ans : Holography is a photographic technique that records the light scattered from an object, and then presents it in a way that appears three-dimensional. The principle behind holography is that during the recording process one superimposes on the scattered wave (emanating from the object) another coherent wave (called reference beam) of the same wavelength .

Holography	Photography
1)) In holography, both intensity as well as phase of light wave is recorded, thus holography gives three dimensional picture of the object.	1) In photography, only intensity is recorded so photography produces two dimensional picture of the object .
2) If the hologram is broken into parts, each part is capable of reconstructing the entire object.	2) In photography the destruction of even very small portion of negative or photography results in a irrepareable loss of information.
3) Holography has high information capacity as compared to photography.	3) Photography has low information capacity.
4) A photograph can be recorded using normal light sources (sunlight or electric lighting) e.g. lens .	4) In holography, the light from the object is scattered directly onto the recording medium e.g. laser .

Q5)a) Show that diameter of Newton's dark ring is directly proportional to square root of natural number. (5M)

Ans : 1.When a plano-convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of the lens and upper surface of the plate.

2.The thickness of the film gradually increases from the point of contact outwards.

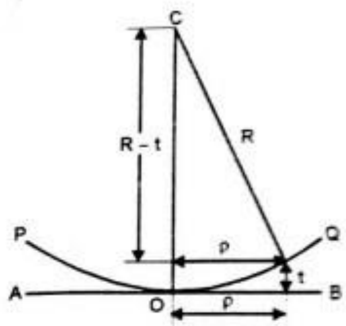
3.If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with centre dark is formed in the air film.

4.These rings were first studied by newton and hence they are known as Newton’s rings.

5.They can be seen through a low power microscope focused on the film.

6.Newton’s rings are formed as a result of interference between the waves reflected from the top and bottom surfaces of the air film formed between the lens and the plate.

Let p be the radius of a newton’s rings corresponding to the constant film thickness t.



Path difference between two interfering rays= 2µtcos(r+Ɵ)+λ/2

$R^2 = P^2 + (R-t)^2$

$p^2 = 2Rt - t$

$t \ll R$

$2t = \frac{p^2}{R}$

Path difference between the interfering rays is $\frac{P^2}{R} + \frac{\lambda}{2}$

For dark rings :

Path difference = $\frac{p^2}{R} + \frac{\lambda}{2}$

$= (2n+1)\frac{\lambda}{2}$

If D is the diameter of newton's rings

$P = \frac{D}{2}$

$\frac{D^2 n}{4R} = n\lambda$

$\boldsymbol{D_n^{\alpha}\sqrt{n}}$

Thus , it is proved that the diameter of nth dark rings are proportional to square root of natural numbers.

Q5)b) What are the different techniques to synthesize nanomaterial and explain any one of them in detail. (5M)

Ans : The different techniques to synthesize nanomaterials are :

1) Ball Milling
2) Sputtering

2) Vapour Deposition
3) Sol Gel Technique
4) Electro Deposition
5) Mechanical Crushing
6) Laser Synthesis
7) Inert gas condensation

i) Ball Milling is a process where a powder mixture placed in the ball mill is subjected to high energy collision from the balls.
ii) Planetary ball mill is frequently used system for mechanical alloying since only a very few amount of powder is required. In simple language, a ball mill consists of a hollow cylindrical shell rotating about its axis.
iii) The axis of the shell may be either horizontally or at a small angle to the horizontal. It is partially filled with balls which makes grinding media and made up of steel, stainless steel or ceramic .
iv) The inner surface of the shell is made up of an abrasion resistant material. When continuously operated, the shell rotates and lifts the ball up and drops them from near the top of the shell which causes the grinding of the particles inside .
v) It has been used in size comminutions of ore, mineral dressing, preparing talc powders and many other applications.
vi) Ball milling is a grinding method that grinds nanotubes into extremely fine powders. During the ball milling process, the collision between the tiny rigid balls in a concealed container will generate localized high pressure. Usually, ceramic, flint pebbles and stainless steel are used.

Q5)c) In a Newton's ring experiment,the diameter of nth and (n+12)th rings are 4.3 mm and 6.8 mm respectively. Radius of curvature of plano-convex lens is 1m.Find the wavelength of light. (5M)

Given : D^2_n=4.3X4.3=18.49 mm^2 , D^2_{n+2}=6.8x6.8=46.24 mm^2 ,R=1000mm

To find : λ

Solution :

Diameter of nth dark ring is given by

$$D^2_n = 4nR\lambda$$

$$\frac{D_n^2}{D_{n+2}^2} = \frac{18.49}{46.24} = \frac{4nR\lambda}{4(n+2)R\lambda} \quad .$$

$$\frac{18.49}{46.24} = \frac{n}{n+2} \quad .$$

18.49(n+2) = 46.24n .

$18.49n + 36.98 = 46.24n$.

$27.75n = 36.$

$n = 1.297 \approx 1$

Now, $18.49 = 4nR\lambda = 4x1x1x\lambda$.

$\lambda = 18.49/4 = 4.622$ m

Ans : The wavelength of light used is 4.622 m .

Q6)a) Explain the physical significance of divergence and curl of a vector field. (5M)

Ans : The divergence of a vector field $\mathbf{F}$, denoted $\operatorname{div}(\mathbf{F})$ or $\nabla \cdot \mathbf{F}$ (the notation used in this work), is defined by a limit of the surface integral

$$\nabla \cdot \mathbf{F} \equiv \lim_{V \to 0} \frac{\oint_S \mathbf{F} \cdot d\,\mathbf{a}}{V}$$

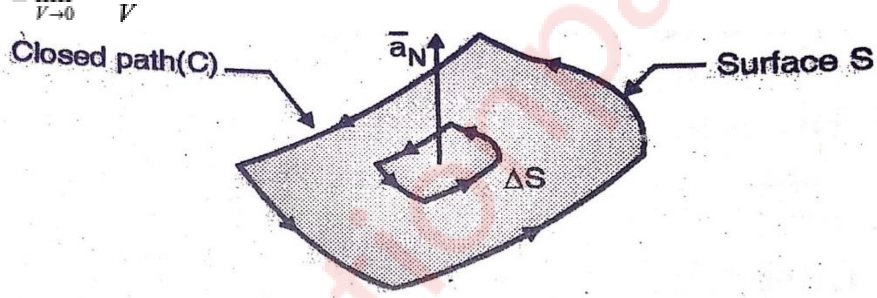


Fig. 6.3.6 : Curl of a vector

where the surface integral gives the value of F integrated over a closed infinitesimal boundary surface $S = \partial V$ surrounding a volume element V, which is taken to size zero using a limiting process. The divergence of a vector field is therefore a scalar field. If $\nabla \cdot \mathbf{F} = 0$, then the field is said to be a divergenceless field. The symbol ∇ is variously known as "nabla" or "del."

The physical significance of the divergence of a vector field is the rate at which "density" exits a given region of space. The definition of the divergence therefore follows naturally by noting that, in the absence of the creation or destruction of matter, the density within a region of space can change only by having it flow into or out of the region. By measuring the net flux of content passing through a surface surrounding the region of space, it is therefore immediately possible to say how the density of the interior has changed. This property is fundamental in physics, where it goes by the name "principle of continuity." When stated as a formal theorem, it is called the divergence theorem, also known as Gauss's theorem. In fact, the definition in equation (1) is in effect a statement of the divergence theorem.

For example, the continuity equation of fluid mechanics states that the rate at which density ρ decreases in each infinitesimal volume element of fluid is proportional to the mass flux of fluid parcels flowing away from the element, written symbolically as

$$\nabla \cdot (\rho\, \mathbf{u}) = -\frac{\partial \rho}{\partial t}, \tag{2}$$

where u is the vector field of fluid velocity. In the common case that the density of the fluid is constant, this reduces to the elegant and concise statement

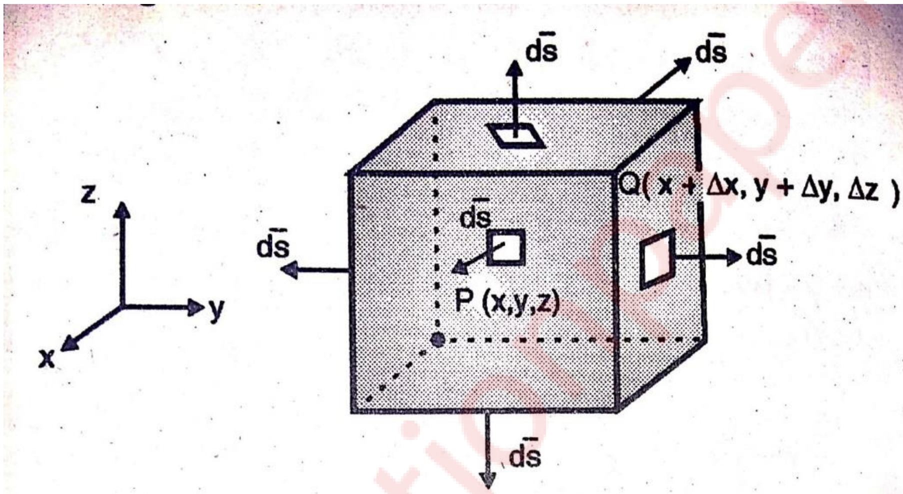
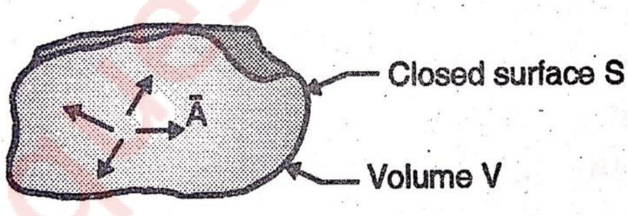


Fig. 6.3.3 : Divergence in Cartesian

(3)



$$\nabla \cdot \mathbf{u} = 0,$$

which simply says that in order for density to remain constant throughout the fluid, parcels of fluid may not "bunch up" in any place, and so the vector field of fluid parcel velocities for any physical system must be a divergenceless field.

Divergence is equally fundamental in the theory of electromagnetism, where it arises in two of the four Maxwell equations,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (5)$$

where MKS units have been used here, E denotes the electric field, ρ is now the electric charge density, ϵ_0 is a constant of proportionality known as the permittivity of free space, and B is the magnetic field. Together with the two other of the Maxwell equations, these formulas describe virtually all classical and relativistic properties of electromagnetism.

A formula for the divergence of a vector field can immediately be written down in Cartesian coordinates by constructing a hypothetical infinitesimal cubical box oriented along the coordinate axes around an infinitesimal region of space. There are six sides to this box, and the net "content" leaving the box is therefore simply the sum of differences in the values of the vector field along the three sets of parallel sides of the box. Writing $\mathbf{F} = (F_x, F_y, F_z)$, it therefore following immediately that

$$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}. \quad (6)$$

This formula also provides the motivation behind the adoption of the symbol $\nabla \cdot$ for the divergence. Interpreting ∇ as the gradient operator $\nabla = (\partial / \partial x, \partial / \partial y, \partial / \partial z)$, the "dot product" of this vector operator with the original vector field $\mathbf{F} = (F_x, F_y, F_z)$ is precisely equation (6).

While this derivative seems to in some way favor Cartesian coordinates, the general definition is completely free of the coordinates chosen. In fact, defining

$$\mathbf{F} \equiv F_1 \, \hat{\mathbf{u}}_1 + F_2 \, \hat{\mathbf{u}}_2 + F_3 \, \hat{\mathbf{u}}_3, \quad (7)$$

the divergence in arbitrary orthogonal curvilinear coordinates is simply given by

$$\nabla \cdot \mathbf{F} \equiv \frac{1}{h_1 \, h_2 \, h_3} \left[\frac{\partial}{\partial u_1} (h_2 \, h_3 \, F_1) + \frac{\partial}{\partial u_2} (h_3 \, h_1 \, F_2) + \frac{\partial}{\partial u_3} (h_1 \, h_2 \, F_3) \right]. \quad (8)$$

The divergence of a linear transformation of a unit vector represented by a matrix A is given by the elegant formula

$$\nabla \cdot \frac{\mathbf{A}\,\mathbf{x}}{|\mathbf{x}|} = \frac{\mathrm{Tr}\,(\mathbf{A})}{|\mathbf{x}|} - \frac{\mathbf{x}^{\mathrm{T}}\,(\mathbf{A}\,\mathbf{x})}{|\mathbf{x}|^3}, \tag{9}$$

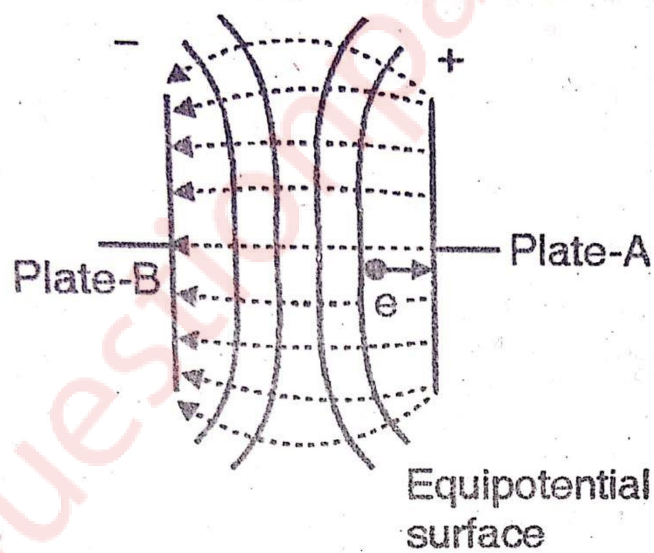
where $\mathrm{Tr}\,(\mathbf{A})$ is the matrix trace and $\mathbf{x}^{\mathrm{T}}$ denotes the transpose.

The concept of divergence can be generalized to tensor fields, where it is a contraction of what is known as the covariant derivative, written

$$\nabla \cdot A \equiv A^{\alpha}_{;\alpha}. \tag{10}$$

Q6)b) State Bethe's Law and explain electrostatic focusing of electron beam. (5M)

Ans : Diagram above represents the electrostatic focusing. A and B are two co-axial cylinders with potentials V1 and V2 such that V2>V1.R is the equipotential ring placed between A and B.



Working :

(1) Consider electron beam 1:

It will remain normal to all the equipotential surfaces and hence it is simply accelerated without any deviation of the path.

(2) Consider electron beam 2:

It will have following 2 effects:

i) On the Left Hand Side of R : The parallel component of F will move the electrons towards right while the normal component of F will move the electron downward by applying Fleming's Left Hand Rule at point C .

ii) On the Right Hand Side of R : The horizontal and vertical component of F will move the elctrons towards right and towards left respectively by applying Fleming's Left Hand Rule at point D .

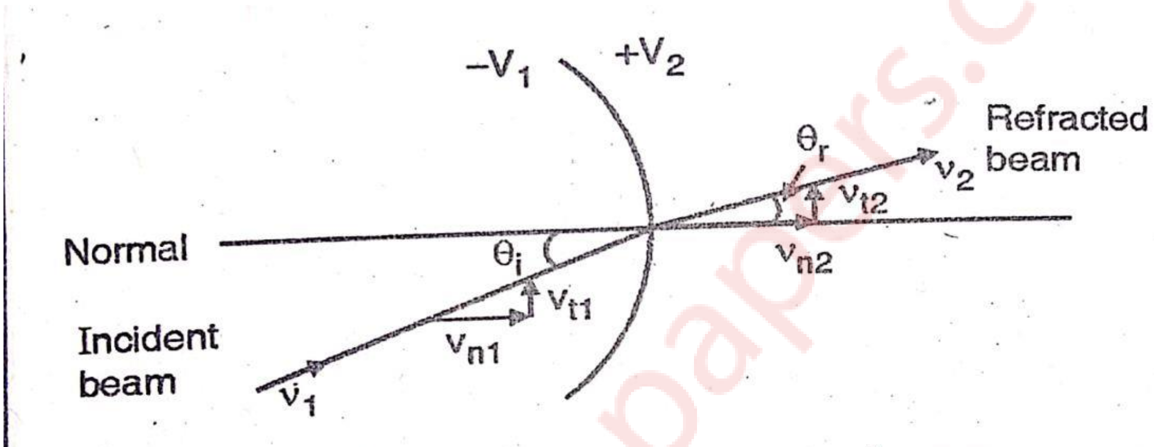


Fig. 7.7.2 : Refraction of a beam

(3) Consider electron beam 3:

It's path will be as shown with same case as case(2).

The focal length can be changed by varying V1 and V2

(4) Bethe's laws is also followed in electrostatic focusing .

(5) Electrostatic focusing is used for accelerating and focusing electron beams.

Because the electric field exists only in the y-direction, the vertical component (y-component) of electron changes while the tangential component (x-component) remains constant. If V1>V2, v_{1y} increases while if V2 > V1, v_{2y} increases. This is known as **Bethe's law** of electron refraction.

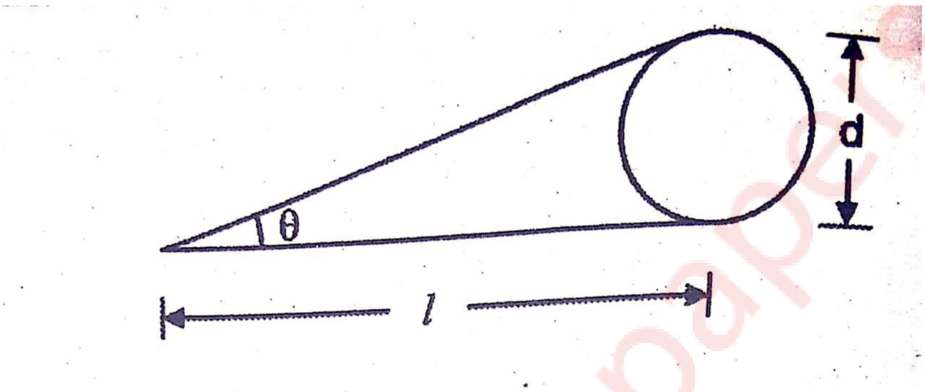
Q6)c) Two glass plates enclose a wedge-shaped air film touching at one end are separated by wire of 0.03mm diameter at distance 15 cm from the edge. Monochromatic light of wavelength λ=6000 Å from a broad source falls normally on the film. Calculate the fringe width. (5M)

Given :

l = 15 cm , r = 0.03 mm = 0.003 cm ,d = 0.006 cm , λ = 6000 Å

To find : fringe width (β)

Solution : Assuming that (i)wedge angle is very small , (ii) the medium is air , (iii) Incident of light in normal



$\tan\Theta = d/l = \Theta$. (for very small Θ)

and $\beta = \lambda/2\Theta$

$\beta = 15 \times 6000 \times 10^{-8}/2 \times 0.006$

β = 0.075 cm

Ans : The value of fringe width is 0.075 cm .