PHYSICS SOLUTION

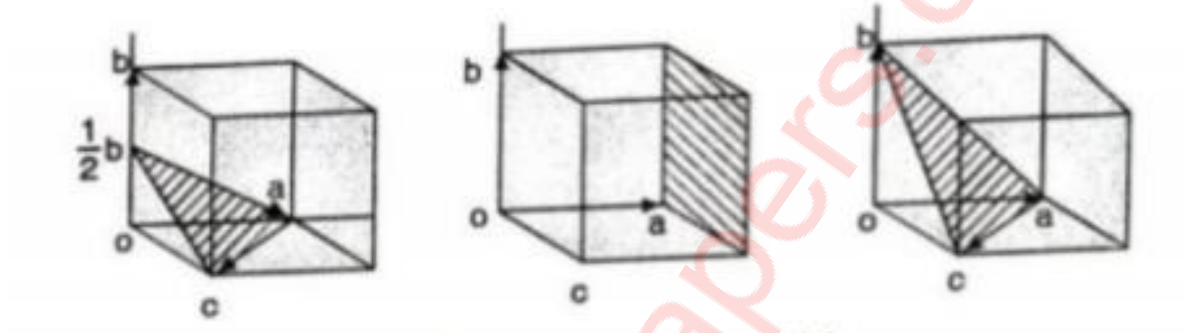
SEM-1 (REV-2019'C' Scheme DEC-2022)

Q1. Attempt any five **[15 M]**

a. Draw the following planes in a cubic unit cell (121), (100), (011).

Solution:

a. (1 2 1): Taking reciprocal: (1,½,1)
b. (1 0 0): Taking reciprocal: (1,∞,∞)
c. (0 1 1): Taking reciprocal: (∞,1,1)



(a) (b) (c)

b. The diameter of 5th dark ring in Newton's ring experiment was found to be 0.42 cm. Determine the diameter of 10th dark ring in the same set up.

Solution:

Given: Diameter of 5th dark ring = 0.42 cm

To find: Diameter of 10th dark ring =?

Formula: $D_n^2 = 4nR\lambda$

Calculations:

$$\therefore \frac{D_5^2}{D_{10}^2} = \frac{4(5)R\lambda}{4(10)R\lambda}$$

$\therefore\ 2(D_5^2) = D_{10}^2$

$\therefore D_{10} = \sqrt{2}\,(D_5\ ) = \sqrt{2}\,(0.42)$

$\therefore D_{10} = 0.594\text{ cm}$

∴ Diameter of the 10th dark ring = 0.594 cm

c. An electron is bound in a one-dimensional potential well of width 2 A° but of infinite height. Find its energy values in the ground state and in first excited state.

Solution:

Using equation for energy of the electron in one-dimensional potential well which is given by:

$$E = \frac{n^2 h^2}{8ma^2}$$

For ground state n=1 and E_0 ,

$$E_0 = \frac{1^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

∴ $E_0 = 1.5 \times 10^{-18}$ J

∴ E_0= 9.43 eV

Similarly, for first excited state, n=2 and $E=E_1$,

$$E_1 = \frac{2^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

∴ $E_1 = 6 \times 10^{-18}$ J

∴ E_1= 37.5 eV

For Second excited state, n=3 and $E=E_2$,

$$E_2 = \frac{3^2(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

∴ $E_2 = 1.35 \times 10^{-17}$ J

∴ E_2= 84.375 eV

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d. Define superconductivity and explain the terms critical temperature and critical magnetic field.

Solution:

a. Superconductivity: When normal metals are cooled their resistivity decreases with temperature. In some materials, at a lower temperature resistivity suddenly drops to zero, they are called superconductors.
Superconductivity is thus a phenomenon of sudden disappearance of electrical resistance (zero resistance) and expulsion of magnetic flux occurring in certain materials when they are cooled below a characteristic low temperature.

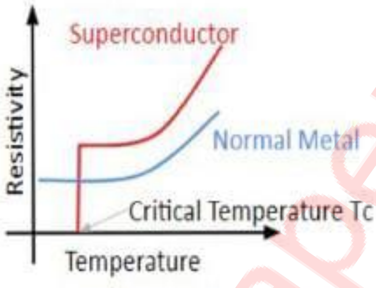

Figure 1: Resistivity Vs temperature

b. Critical Temperature: When a superconducting material is cooled below a certain temperature, its resistance suddenly drops to zero and it goes into the superconducting state from the normal state. The temperature at which a material transforms into a superconducting state is called Critical temperature 'T_c' for that material.
Different materials have different critical temperatures. The transition is reversible. When the temperature of the material is increased above the critical temperature, it passes into the normal state. For elementary solids (in extremely pure form), critical temperature (T_c) is found to be very low (e.g. Tungsten = 0.015°K, Zinc= 0.85°K), whereas, for alloys or compounds, its relatively high (e.g. NbTi= 10°K)

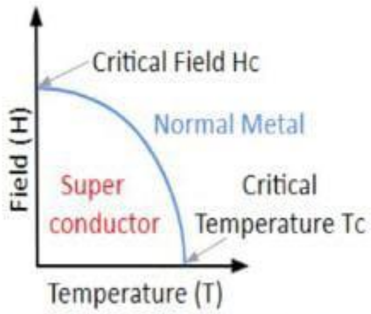

Figure 2: Tc and Hc for Superconductor

c. Critical Field: A material in its superconducting state, behaves like a diamagnetic material when placed in a weak magnetic field. But if the field strength is increased, the material may lose superconductivity even below critical temperature (T_c) .
The critical field (H_c) for a superconducting material is the minimum field value at which normal resistivity is regained by the material and it loses its superconducting state.
$H_c = H_0[1-T/T_c)^2]$

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e. Find the resistivity of intrinsic germanium at 300 K. Given density of carriers is 2.5 x 10^{19} /m^3, mobility of electrons is 0.39 m^2/volt-sec and mobility of holes is 0.19 m^2/volt-sec.

Solution:

Given:

Mobility of electrons= μ_e = 0.39 m^2/volt-sec

Mobility of holes= μ_h = 0.19 m^2/volt-sec

Density of carriers= $n_i = 2.5 \times 10^{19}$ /m^3

To find: Resistivity of intrinsic Germanium=ρ=?

Formula: 1. $\sigma_{int}= n_i(\mu_e+ \mu_h).e$

$\rho= \frac{1}{\sigma}$

Calculations: $\sigma_{int}= n_i(\mu_e+ \mu_h).e$

$\therefore \sigma_{int}= 2.5*10^{19}*1.6*10^{-19}*(0.39+0.19) = 2.32\ (ohm.m)^{-1}$

$\therefore \rho = \frac{1}{\sigma} = \frac{1}{2.32}$

$\therefore \rho = 0.431$ ohm.m

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f. What are matter Waves? State three properties of matter waves.

Solution:

Matter waves also known as de-Broglie wave describe the relationship between momentum and wavelength.

Properties of matter waves:

1. Matter waves are neither mechanical nor electromagnetic waves, they are hypothetical waves.
2. Matter waves travel faster than light because these waves depends on the velocity of the particles generating them.
3. Matter waves with different de Broglie wavelengths travel with different velocities.
4. Matter waves depends on the momentum, kinetic energy, accelerating potential for the matter particle as shown in the following formulae:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

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g. Explain the formation of colours in thin film.

Solution:

When a thin film is exposed to white light from an extended source e.g., the sun, it shows beautiful colours in the reflected system.

1. Light is reflected from the top and bottom surfaces of the thin film and the reflected rays interfere.
2. The path difference between the interfering rays depends on the thickness of the film(t) and the angle of refraction(r) and hence on the inclination of the incident ray, along with its wavelength.

3. The white light consists of continuous range of wavelengths. At a particular point on the film and for a particular position of the eye (t and r remaining constant) those wavelengths of the incident light satisfying the above condition for the constructive interference i.e. maxima will be seen in reflected light.
4. The colours will vary with t and r (i.e. position of eye w.r.t. the film). Therefore if the same point of the film is observed with an eye in different positions or different points of the film are observed with the eye in the same position, a different set of colours is observed every time.

Q2 a) State Hall Effect. Obtain an expression for Hall voltage. Calculate the mobility of charge carriers in a doped Si, whose conductivity is 100 per ohm meter and Hall coefficient is 3.6 x 10* m°/C. [8mks]

Solution:

a) State hall effect. Obtain an expression for Hall Voltage.

If a current carrying conductor or semiconductor is placed in a transverse magnetic field, a potential difference is developed across the specimen in a direction perpendicular to both the current and magnetic field. The phenomenon is called HALL EFFECT.

As shown consider a rectangular plate of a p-type semiconductor of width 'w' and thickness 'd' placed along x-axis. When a potential difference is applied along its length 'a' current 'I' starts flowing through it in x direction.

As the holes are the majority carriers in this case the current is given by:

$I = n_h A e v_d$ --(1)

Where, n_h = density of holes,

A= w*d = cross sectional area of the specimen,

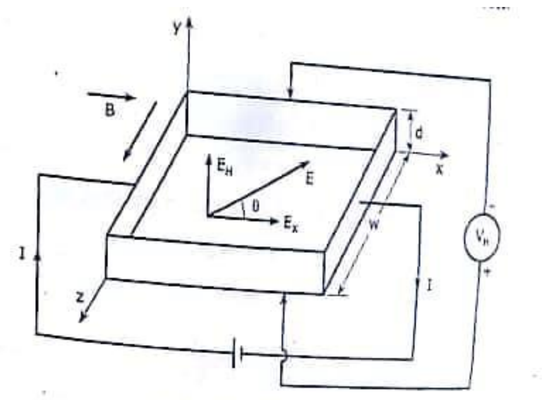


Figure 3.25 : Hall effect set up

v_d = drift velocity of the holes.

The current density is,

$J = \frac{I}{A} = n_h e v_d$ --(2)

The magnetic field is applied transversely to the crystal surface in z direction. Hence the holes experience a magnetic force

$F_m = e v_d B$ --(3)

In a downward direction. As a result of this the holes are accumulated on the bottom surface of the specimen.

Due to this a corresponding equivalent negative charge is left on the top surface.

The separation of charge set up a transverse electric field across the specimen given by,

$E_H = \frac{v_H}{d}$ --(4)

Where v_h is called the HALL VOLTAGE and the E_h HALL FIELD.

In equilibrium condition the force due to the magnetic field B and the forcedue to the electric field E_h acting on the charges are balanced. So the equation (3),

$eE_H = e v_d B$

$E_H = v_d B$ --(5)

Using equation (4) in the equation (5),

$V_H = v_d Bd$ --(6)

From equation (1) and (2), the drift velocity of holes is found as,

$v_d = \frac{1}{en_hA} = \frac{1}{en_h}$ --(7)

Hence the hall voltage can be written as,

$V_H = \frac{IBd}{en_hA} = \frac{J_xBd}{en_h}$

b) Calculate the mobility of charge carriers in a doped Si, whose conductivity is 100 per ohm meter and Hall coefficient is 3.6×10^{-4} m^3/C.

<u>Given:</u>

Conductivity= σ = 100 per ohm meter

Hall coefficient = R_H = 3.6×10^{-4} m^3/C.

<u>To find</u>: Mobility of charge carriers = ?

<u>Formula:</u> $\mu = \sigma R_H$

<u>Calculations:</u>

From the formula,

$\mu = 100 \times 3.6 \times 10^{-4}$

$= 0.036$

$\mu = 3.6 \times 10^{-2}$ $m^2V^{-1}s^{-1}$ is the mobility of the charge carriers.

Q2. b) Obtain an expression for Optical Path Difference in a thin film of uniform thickness observed in reflected light. Hence obtain conditions for maxima and minima. [7mks]

Solution:

- Consider a ray AB of monochromatic light of wavelength A from an extended source incident at B, on the upper surface of a parallel sided thin film of thickness t and refractive index u as shown in figure,
- Let the angle of incidence be i
- At B, the beam is partly reflected along BR_1 and partly refracted at an angle r along BC.
- At C, it is again partly reflected along CD and partly refracted along CT_1. Similar partial reflections and refractions occur at points D, E, etc.
- Thus we get a set of parallel reflected rays BR_1 ,DR_2, etc. and a set of parallel transmitted rays CT_1,ET_2, etc.
- For a thin film, the waves travelling along BR_1 and DR_2 in the reflected system will overlap.
- These waves originate from the same incident wave AB and are hence coherent.
- Hence they will interfere constructively or destructively according to if the path difference between them is an integral multiple of λ or an odd multiple of λ/2.

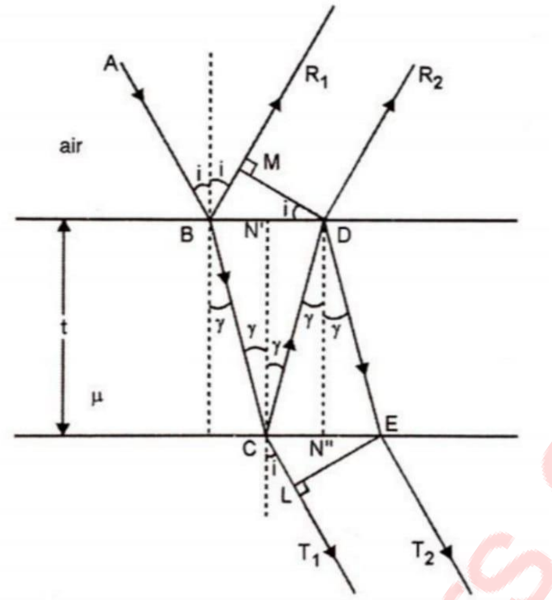


Fig. 4.1.1 : Interference in thin films

- To find the path difference between BR_1 and DR_2, draw DM perpendicular to BR_1. The paths travelled by the beams beyond DM are equal. Hence the optical path difference (optical path difference is obtained by multiplying geometrical path difference by its refractive index) between them is

Δ = Path BCD in film - Path BM in air

= μ (BC + CD) – BM

From figure, we have, BC = CD = $\frac{t}{\cos r}$

μ (BC+CD) = $\frac{2\,\mu\, t}{\cos r}$

and BM = BD.sin i = 2B N' sin i

= 2 B N' sin i

$$\therefore \text{BM} = 2\text{t}\,\frac{\sin r}{\cos r}\,.\,\sin \text{i}$$

$$= \frac{2\,\mu\, t}{\cos r}\,.\,\sin^2 \text{r} \qquad \left(\because \frac{\sin i}{\sin r} = \mu\right)$$

$\therefore$ The optical path difference between the rays is

$\Delta = \frac{2\,\mu\, t}{\cos r} - \frac{2\,\mu\, t}{\cos r}\,.\sin^2 r$

$= \frac{2\,\mu\, t}{\cos r}\,(1 - \sin^2 r)$

Or $\Delta = 2\,\mu\, t \cos r$

- The film is optically denser than the surrounding air medium. Hence the ray BR_1, originating by reflection at the denser medium suffers a phase change of n or a path change of $\lambda/2$ due to reflection at B. (No such change of phase occurs for ray DR_2, as it is a result of reflection at C)
- Hence the effective path difference between BR_1 and DR_2 is

  $2\,\mu\, t \cos r + \frac{\lambda}{2}$
- Condition for maxima and minima in reflected light

  (i) The two rays will interfere constructively if the path difference between them is an integral multiple of λ i.e.

  $2\mu\, t \cos r + \frac{\lambda}{2} = n\lambda$

  $2\mu\, t \cos r + \frac{\lambda}{2} = (2n - 1)\frac{\lambda}{2}$, where n=1,2,3,4,….(for maxima)

  Or $2\mu\, t \cos r + \frac{\lambda}{2} = (2n - 1)\frac{\lambda}{2}$ when n= 0,1,2,3…..

  When this condition is satisfied the film will appear bright in the reflected system.

(ii) The two rays will interfere destructively if the path difference between them is an odd multiple of $\frac{\lambda}{2}$ i.e.

$$2\mu\, t \cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

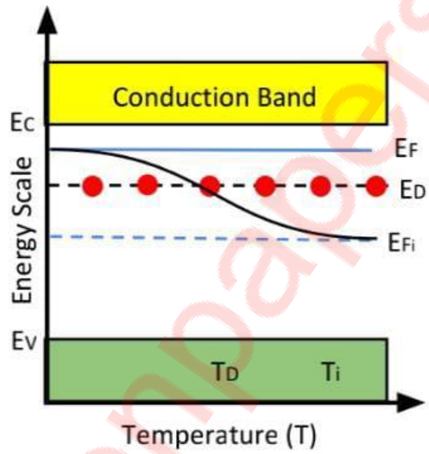
Or $2\mu\, t \cos r + \frac{\lambda}{2} = n\lambda$ (for minima) …. where n=0,1,2,3,….

Q3a) Explain with neat diagram the effect of doping and temperature on the fermi level in N type extrinsic semiconductor. What is the probability of an electron being thermally excited to the conduction band in Si at 20° C. The band gap energy is 1.12 eV. [8mrks]

Solution:

a) Explain with neat diagram the effect of doping and temperature on the fermi level in N type extrinsic semiconductor.

Effect of temperature on N-Type semiconductors:



- At low temperature: only few donor atoms get ionised and hence the Fermi level lies midway between the bottom of conduction band and donor level.
- At moderate temperature: all donor atoms get ionised, so the Fermi level moves towards the centre of forbidden gap.
- At high temperature: the concentration of electrons transferred from valence to conduction band is higher than that from the donor atoms. Hence Fermi level shifts to the middle of the forbidden gap.

Effect of doping on N-Type semiconductors:

In an N-type semiconductor as the doping concentration increases more electrons get added to the donor levels. Due to increase in the concentration of electrons Fermi level in N-type (E_{Fn}) rises indicating this increase.

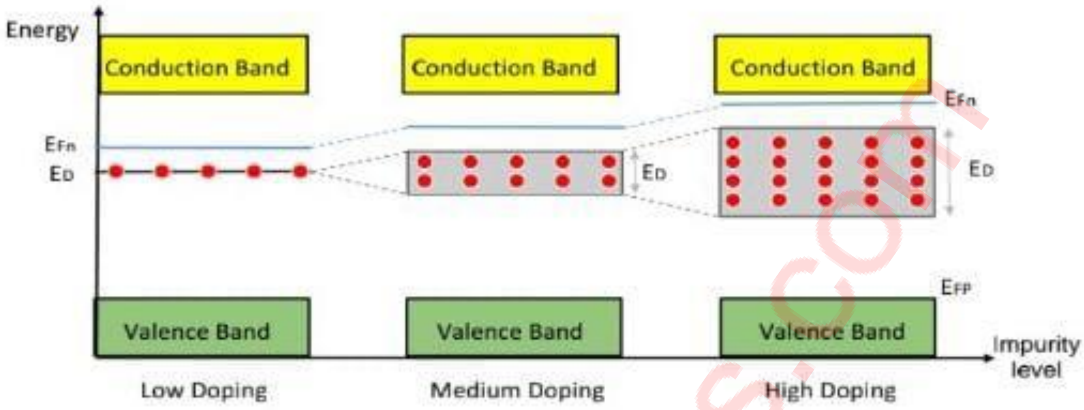


Figure 8a: Effect of Impurity on fermi level of N-type

b) What is the probability of an electron being thermally excited to the conduction band in Si at 20°C. The band gap energy is 1.12 eV.

Solution:

Given:

T= 20°C = 20 + 273.15= 293.15 K

E_g= 1.12eV

Also, K(Boltzmann constant)=1.38 x 10^{-23} J/K

$$K(\text{in eV}) = \frac{1.38 \text{ x } 10^{-23}}{1.6 \text{ x } 10^{-19}} = 86.25 \text{ x } 10^{-6} \text{ eV}$$

To find: $f(E_c)$ = ?

Formula: $f(E_c) = \frac{1}{1+\exp[(E_c + E_V)/KT]}$

Calculations:

For intrinsic semiconductor,

$$E_c - E_v = E_g/2 = \frac{1.12}{2} = 0.56 \text{ eV}$$

$$\frac{E_c - E_v}{KT} = \frac{0.56}{86.25 \text{ x } 10^{-6} \text{x } 293.15} = 22.15$$

$$f(E_c) = \frac{1}{1 + e^{22.15}} = 2.4\text{x}10^{-10}$$

$\therefore f(E_c) = 2.4\text{x}10^{-10}$

Q3. b) Show that the energy of an electron in a one-dimensional deep potential well of infinite height varies as the square of the natural numbers. [7mrks]

Solution:

- Suppose a particle of mass m is free to move in x-direction only in the region from x=0 to x=a
- Outside this region the potential energy V is taken to be infinite, and within this region it is zero. A particle does not lose energy when it collides with walls, hence its energy remains constant.
- Outside the box V= ∞ and particle cannot have infinite energy, therefore it cannot exist outside the box.

  ∴ Schrodinger's Equation is written as,

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - \infty)\Psi = 0$$

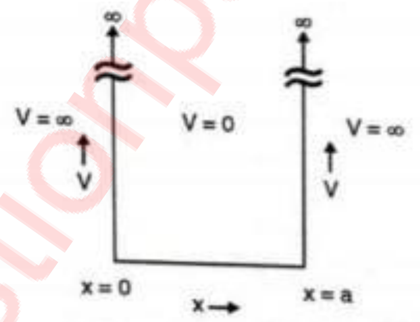


Fig. 1.13.1 : One dimensional potential well of infinite height

The above equation may be simplified as,

$$\frac{d^2\Psi}{dx^2} + K^2\Psi = 0$$

Where,

$$K^2 = \frac{8\pi^2 mE}{h^2} \ or\ K^2 = \frac{2mE}{h^2}$$

Solution of the simplified equation is written as,

$$\Psi = \mathrm{A\ cosKx + B\ sinKx}$$

When x=0 at $\Psi = 0$, we get,

0 = A cos 0+ B sin 0 ……(since cos 0 =1)

∴ A = 0

When x = a, $\Psi = 0$,

∴0 = A cos Ka+ B sin Ka

But A = 0

∴ B sin Ka = 0

Here B need not be zero,

∴ sin Ka = 0 only when

$$Ka = \frac{\sqrt{2mE}}{h} a = n\pi \ \ (where\ n = 0,1,2,3 \ldots.)$$

n= quantum number

$$\Psi_n = B \sin\left(\frac{n\pi}{a}\right) x$$

Which represents the permitted solutions. In the above equations, n=0 is not acceptable because for n=0, $\Psi = 0$, means the electron is not present inside the box which is not true.

As

$K^2 = \frac{8\pi^2 mE}{h^2}$ and $K = \frac{n\pi}{a}$

∴ $\frac{(n\pi)^2}{a^2} = \frac{8\pi^2 mE}{h^2}$

∴ $E_n = \frac{n^2 h^2}{8ma^2}$

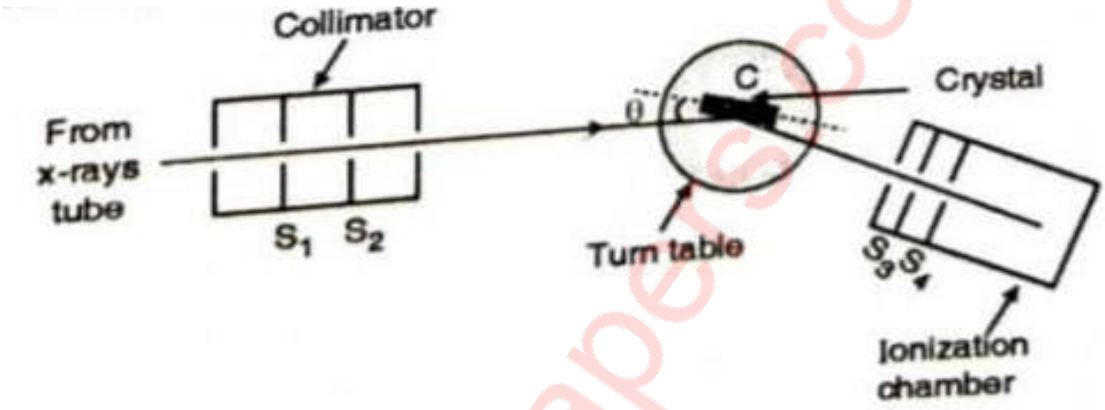
∴ $E_n \propto n^2$

This shows that the energy of the particle can have only certain values which are Eigen values.

Q4.a) Explain Bragg's spectrometer for the investigation of crystal structure with the help of a neat diagram. [5 mrks]

Solution :-

- Based upon Bragg's law an instrument called as Bragg's spectrometer was designed. This is a modified form of ordinary spectrometer to suit the use of x-rays.
- Schematic diagram of a bragg's spectrometer.



- A monochromatic x-ray beam obtained from x-ray tube is made to pass through slits S_1 and S_2 which are made up of lead. The fine beam is then made to fall on the crystal C fixed on a crystal mount exactly at the centre of circular turn table.
- The x-rays reflected are collected by ionization chamber. Since ionization chamber is sturdy, the turn table is rotated till we get a sharp increase in the intensity.
- The sudden increase in the intensity of x-ray suggests that Bragg's law is satisfied at the given angle θ of the incident beam.
- The peak in ionisation current which represents the intensity occurs more than once as θ is varied because Bragg's law states
  $n\lambda = 2d\sin\theta$ i.e. for * n = 1 ,2,3,..... we have θ_1, θ_2, θ_3,…..
- If the intensity (or ionization current) is plotted against glancing angle then we get the graph as shown in figure.
- Using graph shown above we find the angles θ_1, $\theta_{2,\ldots}$ where the peak occurs.

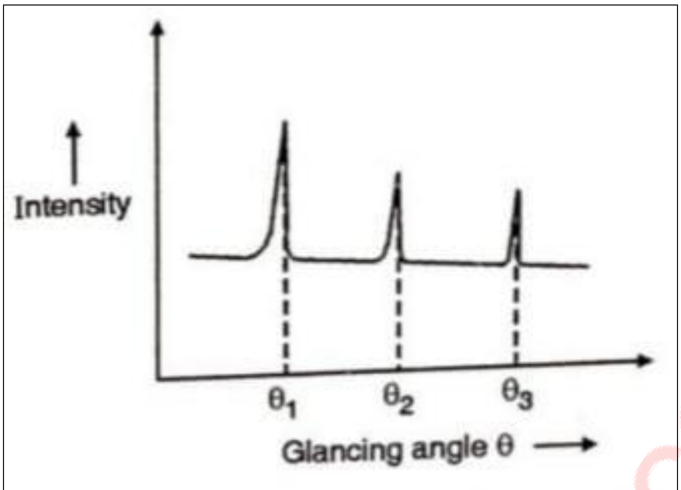


Fig. 2.5.2 : Variation of ionisation current

- Determination of crystal structure (for cubic crystals)

  Here the crystal face used for reflecting the x-rays can be so cut that it remains parallel to one set of planes, then to another and so on when placed at the centre of the turn table on Bragg's spectrometer with x-rays of known λ incident upon it. For a given plane used as reflecting surface, find the corresponding d using

  $n\lambda = 2d \sin \theta$ (take n=1)

  Similarly, find value of d for other planes as well.

  For cubic structure we select three planes viz. (100), (110), (111).

  As λ is same throughout the experiment, we get,

  $\lambda = 2d_{100} \sin \theta_1$

  $\lambda = 2d_{110} \sin \theta_2$

  $\lambda = 2d_{111} \sin \theta_3$

  $\therefore d_{100} : d_{110} : d_{111} = \frac{1}{\sin \theta_1} : \frac{1}{\sin \theta_2} : \frac{1}{\sin \theta_3}$

  Where θ_1, θ_2 and θ_3 are obtained from the graph. Intensity → θ i.e. where the peak occurs. The reason for selection of planes (100), (110) and (111) is that these are the planes rich enough in terms of atoms.

  Studies have found out the ratios of d_{100}, d_{110} and d_{111} for SC, BCC and FCC are as follows, SC $1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{3}}$

BCC $1 : \frac{2}{\sqrt{2}} : \frac{1}{\sqrt{3}}$

FCC $1 : \frac{1}{\sqrt{2}} : \frac{2}{\sqrt{3}}$

Experimentally obtained values of θ_1, θ_2 and θ_3 will provide us d_{100}, d_{110} and $d_{111.}$ By comparing their ratio with above equations, one can determine crystal structure.

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b) Derive one dimensional Schrödinger's time dependent equation for matter waves. [5 mrks]

Solution:-

- Based on de Broglie's idea of matter waves, Schrödinger developed a mathematical theory which plays the same role as Newton's laws in classical mechanics.
- Using de Broglie's hypothesis for a particle of mass m, moving with a velocity v, associated with it is wave of wavelength.

$$\lambda = \frac{h}{p}$$

- The wave equation for a de-broglie wave can be written as

$$\psi = A\, e^{-i^{\omega} t} \ldots(1)$$

Where, A = amplitude

Ω = Angular frequency

- For a one dimensional case, the classical wave equation has the following form

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2}\frac{d^2 y}{dt^2} \ldots(2)$$

- Where, y is the displacement and v is the velocity of the wave. The solution is,

$$y\,(x, t) = A\, e^{-i(kx - \omega t)} \ldots(3)$$

Where ω = 2πv

- By analogy we can write the wave equation for de-Broglie wave for the motion of a free particle as

$$\frac{d^2\Psi}{dx^2} = \frac{1}{v^2}\frac{d^2\Psi}{dt^2} \text{ ...(4)}$$

Where, ω = vk

v = phase velocity

- The solution of the above equation is,

$$\Psi\,(x,t) = A\,e^{-\frac{i(Et-px)}{h}} \text{...(5)}$$

- There we have replaced u and k of Equation (1.10.3) with E and p using Einstein and de-broglie relations.
- Differentiating w.r.t t,

$$\frac{\partial\Psi}{\partial \mathrm{t}} = \frac{\partial}{\partial \mathrm{t}}\left[A\,e^{-\frac{i(Et-px)}{h}}\right]$$

$$\frac{\partial\Psi}{\partial \mathrm{t}} = \frac{\partial}{\partial \mathrm{t}}\left[A\,e^{-\frac{i(Et-px)}{h}}.\frac{-iE}{h}\right]$$

$$\frac{\partial\Psi}{\partial \mathrm{t}} = \frac{-\mathrm{i}}{\mathrm{h}}E\,\Psi \text{ ...(6)}$$

- Similarly taking double differentiation of equation (5) w.r.t x,

$$\frac{\partial^2\Psi}{\partial \mathrm{x}^2} = \frac{-\mathrm{p}^2}{\mathrm{h}^2}\Psi \text{ ...(7)}$$

- In classical mevhanics we have energy of a free particle described as

$$E = \frac{\mathrm{p}^2}{2\mathrm{m}}$$

- Let there be a field where particle is present. Depending on its position in the field, the particle will possess certain potential energy V.

∴ Total energy of a particle is given as,

$$E = \frac{\mathrm{p}^2}{2\mathrm{m}} + V$$

Or,

$$\frac{\mathrm{p}^2}{2\mathrm{m}} = E - V \ldots(8)$$

$\therefore \frac{\mathrm{p}^2}{2\mathrm{m}}\Psi = E\Psi - V\Psi \ \ldots(9)$

But from equation (6),

$$E\Psi \ = \frac{-h}{\mathrm{i}} + \frac{\partial\Psi}{\partial \mathrm{t}}$$

And from equation (7),

$$\mathrm{p}^2\Psi \ = \ \mathrm{h}^2\frac{\partial^2\Psi}{\partial \mathrm{x}^2}$$

$\therefore$ Equation (9) becomes,

$$\frac{-\mathrm{h}^2}{2\mathrm{m}}\frac{\partial^2\Psi}{\partial \mathrm{x}^2} = \frac{-h}{\mathrm{i}}\frac{\partial\Psi}{\partial \mathrm{t}} - v\Psi\lambda$$

$$\therefore \frac{-\mathrm{h}^2}{2\mathrm{m}}\frac{\partial^2\Psi}{\partial \mathrm{x}^2} + v\Psi = \ ih\ \frac{\partial\Psi}{\partial \mathrm{t}} \ldots(10)$$

Equation (10) is the one-dimensional time-dependent schrodinger equation.

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c) White light is incident on a soap film at an angle $\sin^{-1}(4/5)$ and the reflected light is observed with a spectroscope. It is found that two consecutive dark bands correspond to wavelength 6100 A° and 6000A°. If the refractive index of the film is 4/3, calculate its thickness. [5 mrks]

<u>Solution:-</u>

We have the condition for dark band in reflected system,

$2\mu t \cos r = n\lambda$

If n and (n+1) are the orders of consecutive dark bamds for wavelength λ_1 and λ_2 respectively, then,

$2\mu t \cos r = n\lambda_1$ and

$2\mu t \cos r = (n+1)\lambda_2$ …(1)

$\therefore 2\mu t \cos r = n\lambda_1 = (n+1)\lambda_1$

$n\lambda_1 = (n+1)\lambda_1$

$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$

Put the value of n in equation (1), we have,

$\therefore 2\mu t \cos r = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

$t = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2 \, 2\mu \cos r} = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)2\mu \sqrt{1-(\frac{\sin i}{\mu})^2}}$ …(2)

Given :

$\mu = 4/3$,

$\sin i = 4/5$

as $\mu = \frac{\sin i}{\sin r}$ and $\cos r = \sqrt{1 - \sin^2 r}$

$\therefore \cos r = \sqrt{1 - (\frac{4/5}{4/3})^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Given :

$\lambda_1 = 6100$ A° $= 6.1 * 10^{-5}$ cm,

$\lambda_2 = 6000$ A° $= 6.0 * 10^{-5}$ cm,

$\mu = 4/3$

Put all these values in equation (2),

$t = \frac{6.1 * 10^{-5} * 6.0 * 10^{-5}}{(6.1-6) * 10^{-5} 2 * \frac{4}{3} * \frac{4}{5}}$

$\therefore$ **t = 0.0017 cm**

Q5. a) Find the de Broglie wavelength of (i) an electron accelerated through a potential difference of 182 Volts and (ii) 1 Kg object moving with a speed of 1 m/s. Comparing the results, explain why is the wave nature of matter not apparent in daily observations? [5 mrks]

Solution:

(i) m= 6.68*10^{-27}(mass of an α-article)

V= 182 volts

Charge q = 2e

$\therefore \lambda = \frac{h}{\sqrt{2mqV}} = \frac{6.63*10^{-34}}{\sqrt{2\times6.68\times10^{-27}\times2\times1.6\times10^{-19}\times200}}$

$\therefore \lambda = 7.17 \times 10^{-13}$ m

(ii) m = 1 Kg

v= 1 m/s

$\therefore \lambda = \frac{h}{mv} = \frac{6.63*10^{-34}}{1\times1}$

$\therefore \lambda = 6.62 \times 10^{-34}$ m

This wavelength is too small to have any practical significance which is due to extremely small value of Plank's constant (h) and the significant momentum of macroscopic objects.

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b). Derive an expression for interplanar spacing in a cubic unit cell? [5 mrks]

Solution:

- It is clear that parallel planes have same Miller indices.
- At the same time spacing between such parallel planes is an important parameter.
- It is denoted by d_{hkl} i.e. the interplanar spacing between planes with same Miller indices (hkl).

- In the Figure , we have plane ABC with Miller indices (hkl). Other plane with same Miller indices is assumed to pass through point 'O' (which is not shown in the diagram). The perpendicular spacing between these two planes is ON = d.

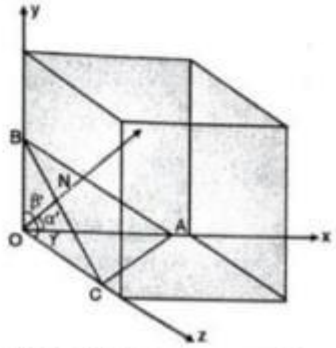

Fig. 2.2.1 : Interplanar spacing

- Let ON make an angle α' with x-axis, β' with y-axis and Υ' with z-axis.

$$\therefore \cos\alpha' = \frac{ON}{OA} = \frac{d}{OA}$$

$$\cos\beta' = \frac{ON}{OB} = \frac{d}{O}$$

$$\cos\Upsilon' = \frac{ON}{OC} = \frac{d}{OC}$$

But $\mathrm{OA} = \frac{a}{h}$

$\mathrm{OB} = \frac{a}{k}$

$\mathrm{OC} = \frac{a}{l}$

$$\therefore \cos\alpha' = \frac{d}{a/h} = \frac{dh}{a}$$

$$\cos\beta' = \frac{d}{a/k} = \frac{dk}{a}$$

$$\cos\Upsilon' = \frac{d}{a/l} = \frac{dl}{a}$$

Using relation of space geometry,

$\cos^2\alpha' + \cos^2\beta' + \cos^2\Upsilon' = 1$

$$\left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{a}\right)^2 + \left(\frac{dl}{a}\right)^2 = 1$$

$$\frac{d^2}{h^2}(h^2+k^2+l^2) = 1$$

$$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

This is the expression for interplanar spacing in terms of lattice constant a and miller indices (hkl)

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c) Explain the principle and working of Supercapacitors? [5 mrks]

Solution:

A supercapacitor is a capacitor which has very high capacitance (of the order of Farads) as compared to a normal capacitor (of the order of 10^{-6} Farads).It is capable of charging and storing energy at a higher density than normal capacitor. It is also capable of discharging to use stored energy to do work faster than the normal battery.

Construction:

A normal capacitor has two plates which are called as its electrodes. Capacitance is directly proportional to the area of plates 'A' and inversely proportional to the distance between the plates 'd'. A porous substance used to coat the metallic plates of a supercapacitor due to this the plates of a supercapacitor have a much larger effective surface area 'A'. The larger surface area of electrodes which soaked in electrolyte eventually increases the storage capacity for charge. To top this up the space between them 'd' gets effectively reduced to accommodate the unique insulating separator. Opposite charges get deposited on either side of the separator, thus creating a double layer of charge as shown in Figure 1. Hence, such capacitors are also called as double layer supercapacitor which is the most commonly used supercapacitor. In this way a Supercapacitor achieves a much higher value of capacitance than any regular capacitor.

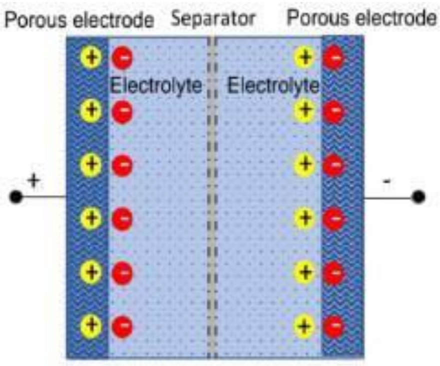
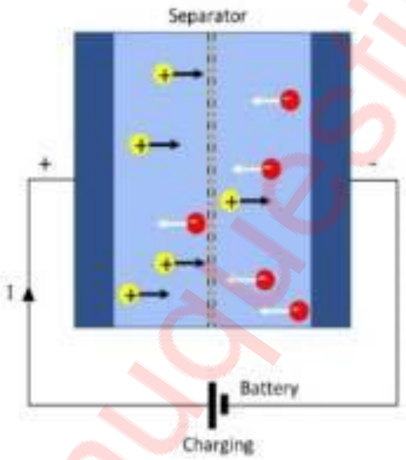
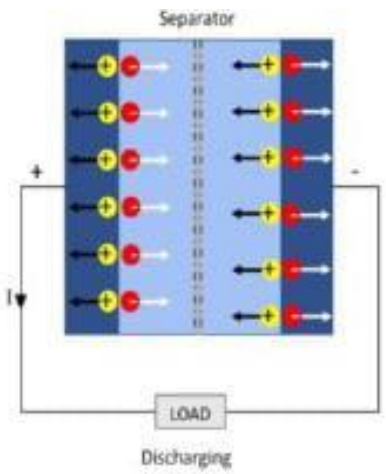


Figure 10a: Supercapacitor construction

Working:

A supercapacitor can be charged and discharged unlimited number of times. When the supercapacitor is not charged, charges in the electrolyte are distributed randomly. In order to charge a supercapacitor, it is connected to a voltage source. While charging positive charges are attracted to the negative terminal and negative charges attracted to the positive terminal as shown in Figure 2. When all the charges are deposited on the electrodes the supercapacitor is said to be fully charged as shown in Figure 1. Once charged the supercapacitor can be connected to a load for discharging as shown in Figure 3.



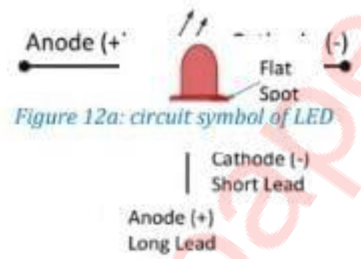


Q6.a) Explain principle, construction and working of Light Emitting Diode? [5 mrks]

Solution:

LED or Light Emitting Diode is a two terminal device which emits light when supplied with electric potential. The circuit symbol of LED is shown in Figure 1 is just like the ordinary diode with additional two arrows pointing outward indicating emission of light as its main function. The real LED component looks like Figure 2 where the longer leg indicated the anode and the flat spot on the top capsule marks the cathode side.

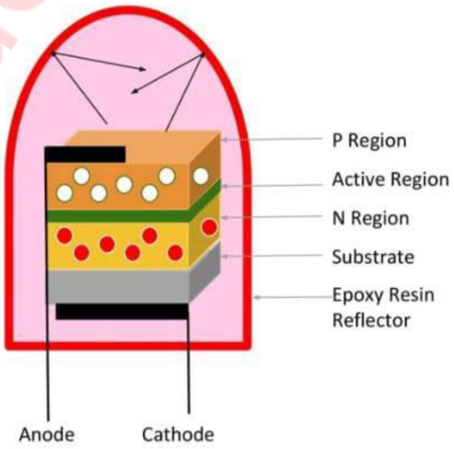
An LED is a PN junction diode which functions only in forward bias mode.



Principle of LED:

When LED is forward biased the electrons in the n-region cross the junction and recombine with the holes in the p- region releasing energy in the form of light.

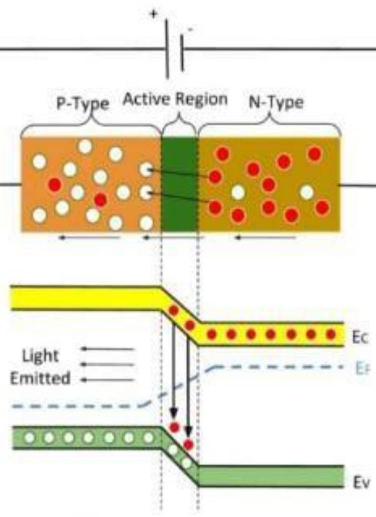
Construction:



1. The most common method of constructing an LED is to stack two semiconducting layers on the substrate.
2. Indirect band gap semiconductors like silicon and germanium are used to make ordinary diodes whereas direct band gap materials like gallium arsenide (GaAs), gallium phosphide (GaP), gallium arsenide phosphide (GaAsP) etc are used in LEDS.
3. The active region is the depletion region created between the p-type and n-type semiconductors as shown in Figure 2.
4. The positive terminal (anode) is connected on top of the p region, while the negative terminal (cathode) is connected below the n region via metal contacts.
5. Light emerges from the active region when recombination takes place. Since recombination takes place when the electrons move into the holes, the p-layer containing holes is kept above the n-layer containing electrons.
6. Light thus emitted by an LED is spread in all directions. In order to prevent spreading, the structure is placed inside a small reflective hemispherical cup made from a transparent plastic of epoxy resin.
7. The unique shape of the cup helps focus all light in one direction (the top) through reflection because of its unique shape. This makes the device more efficient.

Working of a LED:

1. When a forward bias is applied to the LED, the energy levels of the p and n region become aligned thus allowing the electrons and holes to cross the energy gap as shown in 3.
2. This happens because the electrons are repulsed by the battery, which causes them to move from the conduction band into the valence band to recombine with the holes.
3. During this process, as the conduction band has a higher energy than valence band, the electrons move from a higher energy level, to a lower energy level to attain stability.



4. This causes them to emit some energy in the form of light. This is caused by the process of recombination.
5. This is how the energy emitted from LEDs is in the form of light.

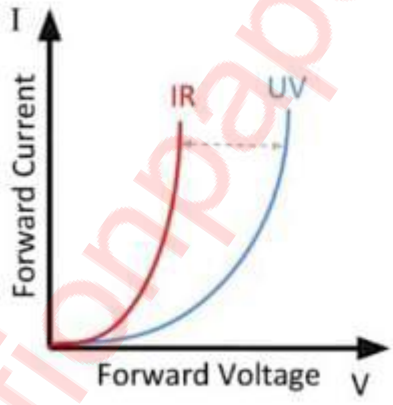


Figure 12e: IV Characteristic of LED

This represents IV characteristics of an LED through a spectrum of light. As the wavelength of light decreases, the values of forward voltage increase for the same amount of current.

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b). State Meissner's effect. Show that superconductors exhibit perfect diamagnetism. [5 mrks]

Solution:

- A superconducting material kept in a magnetic field expels the magnetic flux out of its body when cooled below the critical temperature and exhibits perfect diamagnetism. This effect is called 'Meissner effect .
- Refer Fig.(a), where a specimen is subjected to a magnetic field. The specimen is in normal state. We find that magnetic field penetrates the specimen.
- Refer Fig.(b). now the specimen is cooled below its T_c the superconductor expels field lines from its body. This is Meissner effect.
- Refer Fig.(c), when the field is switched off magnetic field will not be trapped by the superconductor cooled below T_c

(a) Magnetic flux lines (b) Expulsion (c) Magnetic field not trapped

Fig. 5.4.1 : Meissner effect

- As specimen expels the magnetic flux, it is exhibition of perfect diamagnetism, susceptibility is found out to be 1. Let's see it mathematically.
- For normal state, magnetic induction inside the specimen is given by,

  $B = \mu_0(H+M)$

  Where, μ_0 = absolute permeability

  H = external field applied

  M = magnetization produced within specimen

  At $T < T_C$, B=0 I.e. superconducting state

  $\therefore \mu_0(H+M) = 0$

  Susceptibility, $X = M/H = -1$

- It is diamagnetism which brings strong repulsion to external magnets. This has given us levitation effect and MAGLEV trains.

c). We wish to coat a flat slab of glass with refractive index 1.5 with a thinnest possible film of transparent material so that light of wavelength 600 nm incident normally is not reflected. We have two materials to choose from M1(μ= 1.21) and M2 (μ =1.6). Which one would be appropriate? What will be the minimum thickness of coating? [5 mrks]

Solution:

Given:

Wavelength (λ) = 600 nm

Refractive index of glass = μ =1.5

Refractive index of Material 1 = μ_1 = 1.21

Refractive index of Material 1 = μ_2 =1.6

To find: Minimum thickness of coating

Formula: $t_{min} = \frac{\lambda}{4\mu}$

Calculation:

For material 1,

$t_{min} = \frac{\lambda}{4\mu_1} = \frac{600 * 10^{-9}}{4(1.21)} = 1.24 \times 10^{-7}$ = 124 nm

For material 2,

$t_{min} = \frac{\lambda}{4\mu_2} = \frac{600 * 10^{-9}}{4(1.6)} = 9.38 \times 10^{-8}$ = 93 nm

The appropriate would be material 1 with minimum thickness of 124 nm.