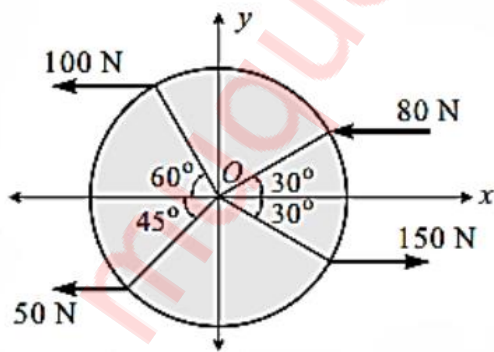


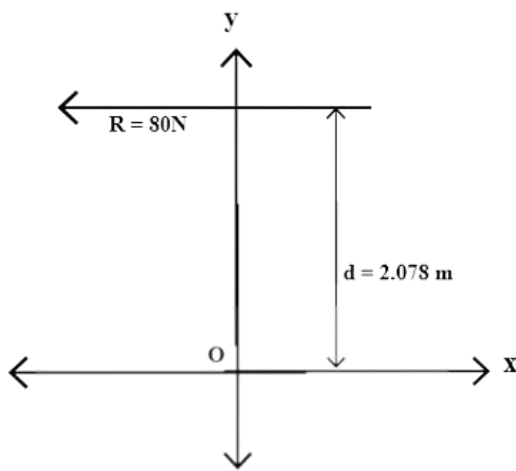
MECHANICS SOLUTION OF QUESTION
PAPER REV 2019'C SCHEME (DEC-
2022)

Q1. Attempt Any Five

- a. For the force system shown. Find the resultant and locate it with respect to O if the radius of plate is 1m. [4]



Solution:



FBD of the diagram

i) Magnitude of the Resultant R

$$R = -100 - 80 + 150 - 50$$

$$R = -80\text{N}$$

$$\therefore R = 80\text{N} (\leftarrow)$$

ii) $\sum M_o = (100 \times 0.866) + (80 \times 0.5) + (150 \times 0.5) - (50 \times 0.707)$

$$\sum M_o = 166.25 \text{ Nm } \curvearrowright$$

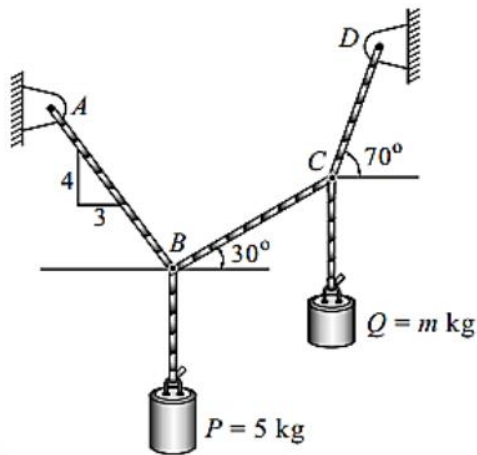
iii) Applying Varignon's theorem,

$$d = \frac{\sum M_o}{R}$$

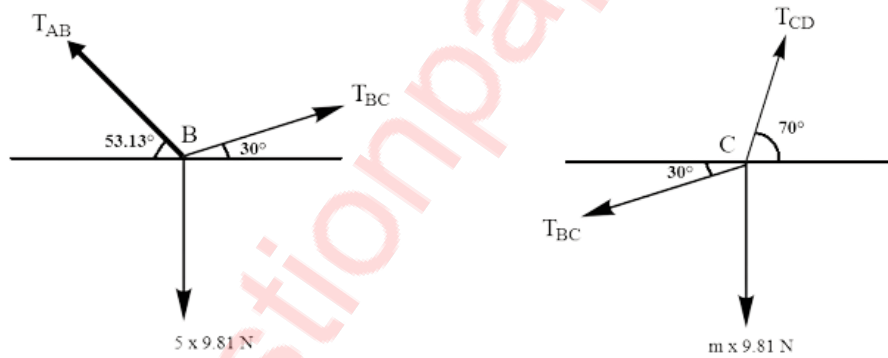
$$d = \frac{166.25}{80}$$

$$\therefore d = 2.078 \text{ m}$$

b. For the system shown in fig. Determine mass m to maintain the equilibrium. [4]



Solution:



By Lami's theorem,

$$\frac{5 \times 9.81}{\sin 96.87^\circ} = \frac{T_{AB}}{\sin 120^\circ} = \frac{T_{BC}}{\sin 143.13^\circ}$$

$$T_{AB} = 42.79 \text{ N}$$

$$T_{BC} = 29.64 \text{ N}$$

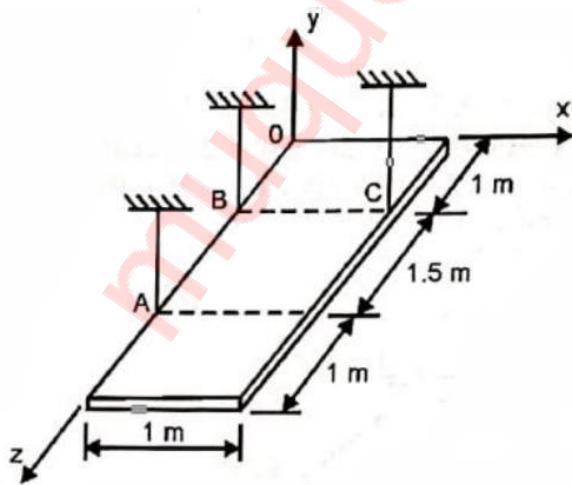
By Lami's theorem,

$$\frac{m \times 9.81}{\sin 140^\circ} = \frac{T_{BC}}{\sin 143.13^\circ}$$

$$m = 5.678 \text{ kg}$$

c. Define laws of Friction.**[4]**

1. The frictional force is always tangential to the contact surface and acts in a direction opposite to that in which the body tends to move.
2. The magnitude of frictional force is self-adjusting to the applied force till the limiting frictional force is reached and at the limiting frictional force the body will have impending motion.
3. Limiting frictional force F_{MAX} directly proportional to normal reactions (i.e $F_{MAX} = \mu_s N$).
4. For a body in motion, kinetic frictional force F_K developed is less than that of limiting frictional force F_{MAX} and the relation $F_K = \mu_K N$ is applicable.
5. Frictional force depends upon the roughness of the surface and the material in contact.
6. Frictional force is independent of the area of contact between the two surfaces.
7. Frictional force is independent of the speed of the body.
8. Coefficient of static friction μ_s is always greater than the coefficient of kinetic friction μ_K .

d. A rectangular plate weighing 500 N is suspended in the horizontal plane using three cables. Find the tension in each cable.**[4]**

Solution:

Equating moments at xx axis to zero.

$$\Sigma M_{xx} = 0$$

$$-(T_A \times 2.5) - (T_B \times 1) - (T_C \times 1)$$

$$500 \times 1.75 = 0$$

$$2.5 T_A + T_B + T_C = 875 \quad \dots\dots\dots(1)$$

Equating moments at zz axis to zero.

$$\Sigma M_E = 0$$

$$-T_C \times (1 - 500 \times 0.5) = 0$$

$$T_C = 250 \text{ N}$$

$$\Sigma F_Y = 0$$

$$T_A + T_B + T_C - 500 = 0 \quad \dots\dots\dots(2)$$

Substituting value of T_C in equation (2)

$$T_A + T_B + 250 - 500 = 0$$

$$T_A + T_B = 250 \quad \dots\dots\dots(3)$$

Substituting value of T_C in equation (1)

$$2.5 T_A + T_B + 250 = 875$$

$$2.5 T_A + T_B = 625 \quad \dots\dots\dots(4)$$

Solving equations (3) and (4), we get

$$T_A = 250 \text{ N}$$

$$T_B = 0 \text{ N}$$

e. The acceleration of the particle is given by the equation $a = -0.05v^2 \text{ m/s}^2$ where, v is the velocity in m/s and x is the displacement in m . Knowing at $v=20 \text{ m/s}$ at $x=0$ determine

(i) the position of the particle at $v= 15 \text{ m/s}$.

(ii) acceleration at $x=50 \text{ m}$.

[4]

Solution:

Using $a = \frac{v dv}{dx}$

$$\therefore \frac{v dv}{dx} = -0.05 v^2 \dots\dots\dots (1)$$

$$\therefore \frac{v dv}{-0.05 v^2} = dx$$

Integrating taking lower limits $V = 20$ m/s and $x = 0$

$$\int_{20}^v -20 \frac{1}{v} dv = \int_0^x dx$$

$$\therefore -20 [\log_e v - \log_e 20]_{20}^v = [x]_0^x$$

$$\therefore -20 [\log_e \frac{v}{20}] = x \dots\dots\dots (2)$$

Substituting,

$v = 15$ m/s in above equation we get,

$x = 5.745 \text{ m}$

Now, substituting $x = 50$ m in equation (2)

$$50 = -20 \log_e \frac{v}{20}$$

$$\log_e \frac{v}{20} = -2.5$$

$$\frac{v}{20} = e^{-2.5}$$

$$\therefore v = 1.642 \text{ m/s}$$

Substituting,

$v = 1.642$ m/s in equation (1)

$$a = -0.05 (1.642)^2$$

$$\therefore a = -0.1348 \text{ m/s}^2$$

f. Define General Plane motion and ICR. What are the properties of an ICR.

[10]

Solution:

General Plane motion is the combination of translation motion and rotational motion happening simultaneously.

Properties of ICR :

Instantaneous Centre is defined as the point about which the G.P body rotates at given instant.

This point keeps on changing as the G.P body performs its motion.

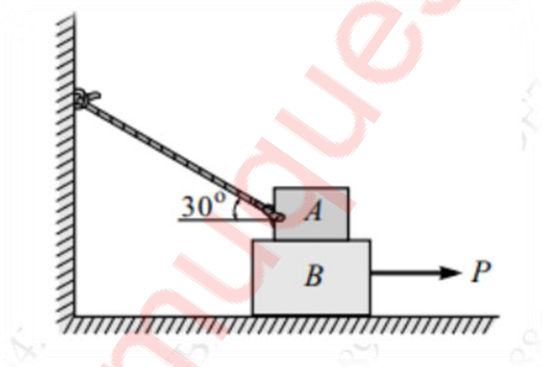
The locus of instantaneous centres during the motion is known as centrode.

Instantaneous Centre may be denoted by letter I.

Q2.

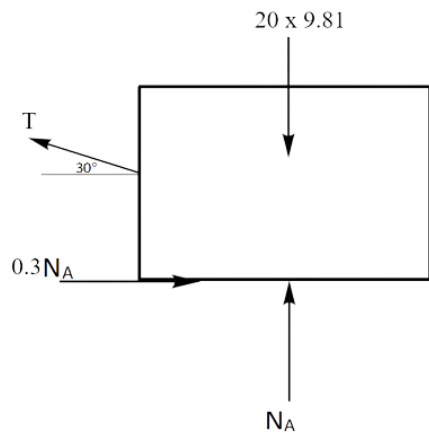
- a. Find the minimum force P required to pull the block. Take the Coefficient of friction between A and B as 0.3 and between B and floor as 0.25.

(10)



Solution:

- i) Consider the F.B.D of block A



FBD of block A

$$\sum F_y = 0$$

$$N_A + T \sin 30^\circ - 20 \times 9.81 = 0$$

$$N_A = 20 \times 9.81 - T \sin 30^\circ$$

$$\sum F_x = 0$$

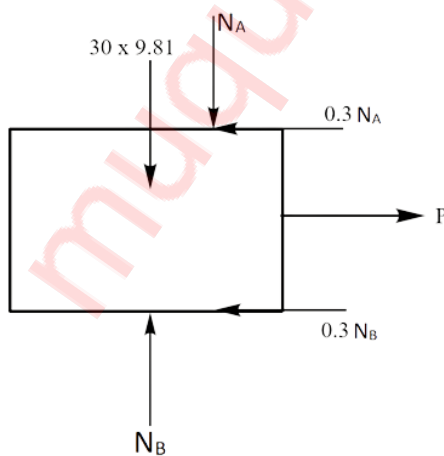
$$T \cos 30^\circ - 0.3 N_A = 0$$

$$T \cos 30^\circ - 0.3 (20 \times 9.81 - T \sin 30^\circ) = 0$$

$$T = 57.93 \text{ N}$$

$$N_A = 167.23 \text{ N}$$

ii) Consider the F.B.D of block A



FBD of block B

$$\sum F_y = 0$$

$$N_B - 30 \times 9.81 - 167.06 = 0$$

$$N_B = 461.53 \text{ N}$$

$$\sum F_x = 0$$

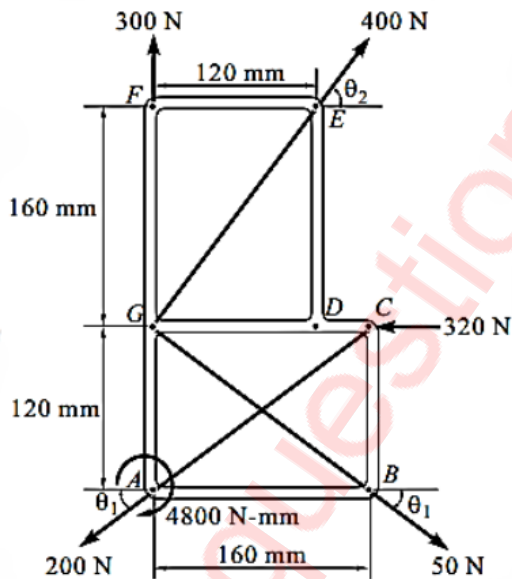
$$P - 0.3N_A - 0.25N_B = 0$$

$$P = (0.3 \times 167.23) - (0.25 \times 461.36) = 0$$

$$P = 165.55 \text{ N}$$

b. For given system find resultant and its point of application with respect to point A.

(6)



Solution:

$$(i) \tan \theta_1 = \frac{1200}{1600}$$

$$\therefore \theta_1 = 36.87^\circ$$

$$\tan \theta_2 = \frac{1600}{1200}$$

$$\therefore \theta_2 = 53.13^\circ$$

$$\sin \theta_1 = 0.6$$

$$\sin \theta_2 = 0.8$$

$$\cos \theta_1 = 0.8$$

$$\cos \theta_2 = 0.6$$

$$(ii) \quad \Sigma F_x = (-200 \times 0.8) + (50 \times 0.8) - 320 + (400 \times 0.6)$$

$$\Sigma F_x = -200 \text{ N}$$

$$\therefore \Sigma F_x = 200 \text{ N } (\leftarrow)$$

$$(iii) \quad \Sigma F_y = -(200 \times 0.6) - (50 \times 0.6) + (400 \times 0.8) + 300$$

$$\Sigma F_y = 470 \text{ N } (\uparrow)$$

$$(iv) \quad R = \sqrt{(200)^2 + (470)^2}$$

$$\therefore R = 510.78 \text{ N}$$

$$(v) \quad \theta = \tan^{-1} \left(\frac{470}{200} \right)$$

$$\theta = 66.95^\circ$$

$$(vi) \quad \Sigma M_A = -4800 - (50 \times 0.6 \times 160) + (320 \times 120) - (400 \times 0.6 \times 280) + (400 \times 0.8 \times 120)$$

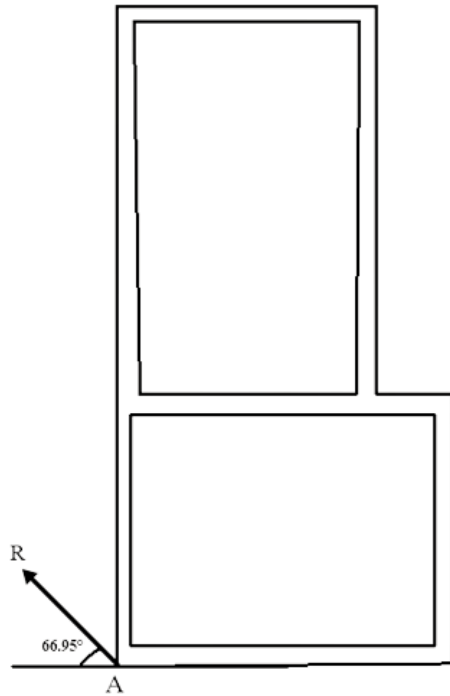
$$\therefore \Sigma M_A = 0$$

(vii) Applying Varignon's theorem,

$$\Sigma M_A = R \times d$$

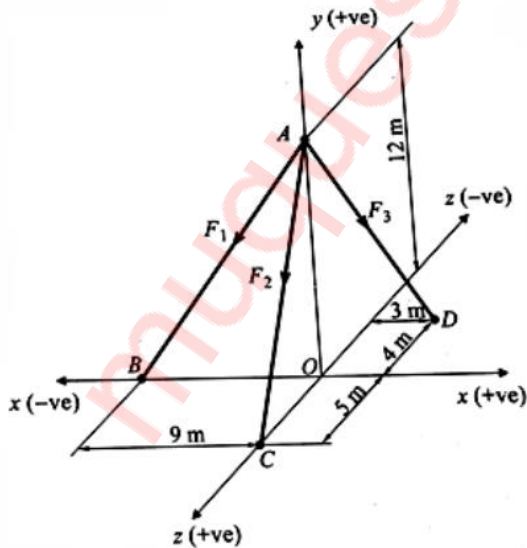
$$d = \frac{\Sigma M_A}{R} = \frac{0}{510.78}$$

$$\therefore d = 0$$



c. The resultant of the three concurrent space forces at A is $R = 788j$ N. Find magnitude of F_1 , F_2 , F_3 forces.

(4)



Solution:

From the figure the coordinates are,

A (0, 12, 0) m, B (-9, 0, 0) m, C (0, 0, 5) m and D (3, 0, -4) m.

Putting the forces in vector form.

$$\bar{F}_1 = F_1 \cdot \hat{e}_{AB}$$

$$\bar{F}_1 = F_1 \left[\frac{-9\mathbf{i} - 12\mathbf{j}}{\sqrt{9^2 + 12^2}} \right]$$

$$\bar{F}_1 = F_1(-0.61\mathbf{i} - 0.8\mathbf{j}) \text{ N}$$

$$\bar{F}_2 = F_2 \cdot \hat{e}_{AC}$$

$$\bar{F}_2 = F_2 \left[\frac{-12\mathbf{j} - 5\mathbf{k}}{\sqrt{12^2 + 5^2}} \right]$$

$$\bar{F}_2 = F_2(-0.923\mathbf{j} - 0.385\mathbf{k}) \text{ N}$$

$$\bar{F}_3 = F_3 \cdot \hat{e}_{AD}$$

$$\bar{F}_3 = F_3 \left[\frac{-3\mathbf{i} - 12\mathbf{j} - 4\mathbf{k}}{\sqrt{3^2 + 12^2 + 4^2}} \right]$$

$$\bar{F}_3 = F_3(-0.231\mathbf{i} - 0.923\mathbf{j} - 0.308\mathbf{k}) \text{ N}$$

$$\bar{R} = F_1 + F_2 + F_3$$

$$0\mathbf{i} - 788\mathbf{j} + 0\mathbf{k} = F_1(-0.6\mathbf{i} - 0.8\mathbf{j}) + F_2(-0.923\mathbf{j} + 0.385\mathbf{k}) + F_3(0.231\mathbf{i} - 0.923\mathbf{j} - 0.308\mathbf{k})$$

$$0\mathbf{i} - 788\mathbf{j} + 0\mathbf{k} = (-0.6F_1 + 0.231F_3)\mathbf{i} + (-0.8F_1 - 0.923F_2 - 0.923F_3)\mathbf{j} + (0.385F_2 - 0.308F_3)\mathbf{k}$$

Equating the coefficients

$$- 0.6 F_1 + 0.231 F_3 = 0 \quad \dots\dots\dots(1)$$

$$- 0.8F_1 - 0.923 F_2 - 0.923 F_3 = - 788 \quad \dots\dots\dots(2)$$

$$0.385 F_2 - 0.308 F_3 = 0 \quad \dots\dots\dots(3)$$

Solving equations (1), (2) and (3) we get,

$$F_1 = 154 \text{ N,}$$

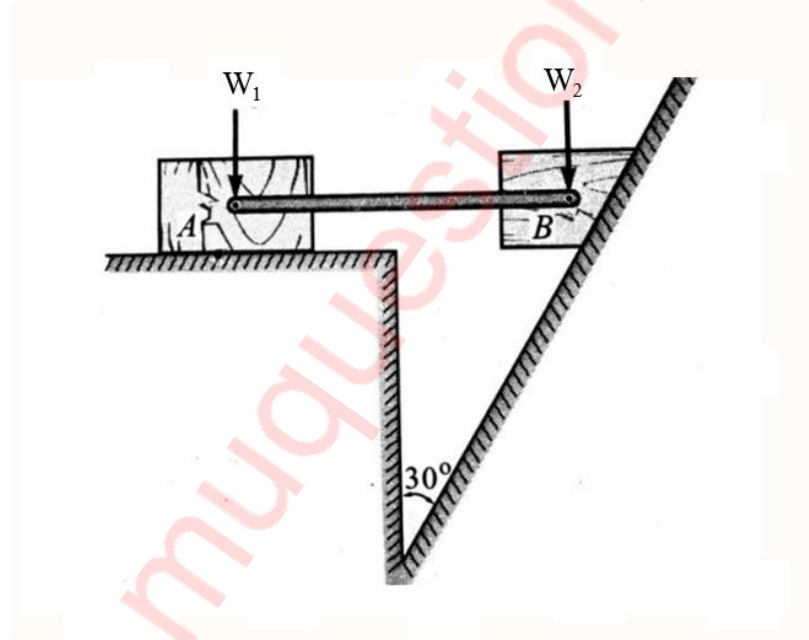
$$F_2 = 320 \text{ N,}$$

$$F_3 = 400 \text{ N}$$

Q.3

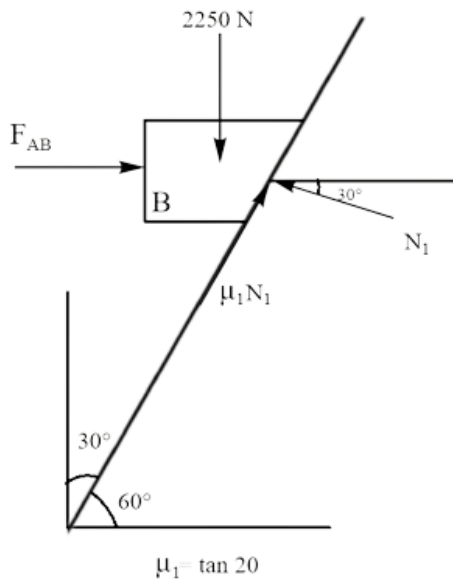
a. Two blocks W_1 and W_2 connected by a horizontal bar AB are supported on rough planes as shown in fig. Considering the coefficient of friction between block A and ground as 0.4 and angle of friction for block B is 20° . Find the smallest weight W_1 for which the equilibrium can exist, if W_2 is 2250 N .

(8)



Solution:

(i)



Consider the F.B.D. of block B as shown in figure,

$$\Sigma F_y = 0$$

$$N_1 \sin 30^\circ + \mu_1 N_1 \sin 60^\circ - 2250 = 0$$

$$N_1 (\sin 30^\circ + \tan 20^\circ \sin 60^\circ) = 2250$$

$$N_1 = 2760.03 \text{ N}$$

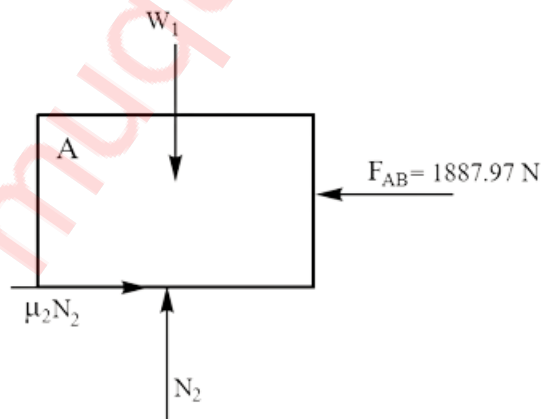
$$\Sigma F_x = 0$$

$$F_{AB} + \mu_1 N_1 \cos 60^\circ - N_1 \cos 30^\circ = 0$$

$$F_{AB} = 2760.03 \cos 30^\circ - \tan 20^\circ \times 2760.03 \cos 60^\circ$$

$$F_{AB} = 1887.97 \text{ N}$$

(ii)



Consider the F.B.D. of block A as shown in figure,

$$\Sigma F_x = 0$$

$$\mu_1 N_1 - F_{AB} = 0$$

$$0.4 N_2 = 1887.97$$

$$N_2 = 4719.93 \text{ N}$$

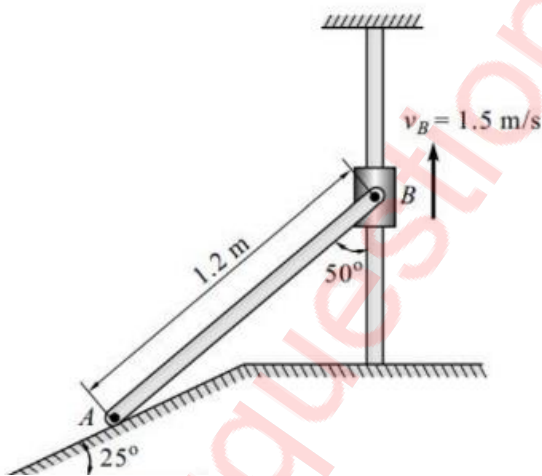
$$\Sigma F_y = 0$$

$$N_2 - W_1 = 0$$

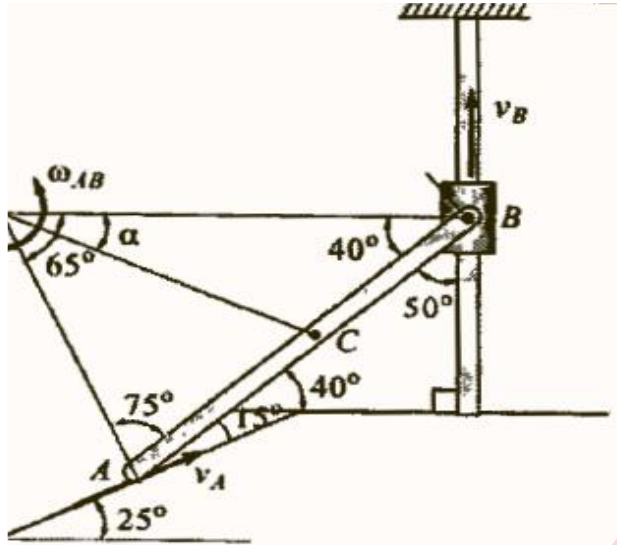
$$W_1 = 4719.93 \text{ N}$$

b. For the system shown in fig. if the collar is moving upwards with a velocity of 1.5m/s. Locate the ICR for the instant shown. Determine angular velocity of rod AB, Velocity of A and velocity at the midpoint of AB.

(8)



Solution:



FBD of the diagram

(i) In ΔIAB , using sine rule,

$$\frac{1.2}{\sin 65^\circ} = \frac{IA}{\sin 40^\circ} = \frac{IB}{\sin 75^\circ}$$

$$IA = 0.851 \text{ m}$$

$$IB = 1.28 \text{ m}$$

(ii) Rod AB (Performs general plane motion)

At the given instant point I is the ICR

$$V_B = (IB) (\omega_{AB})$$

$$\omega_{AB} = \frac{1.5}{1.28}$$

$$\omega_{AB} = 1.172 \text{ rad/s } \curvearrowright$$

$$V_A = (IA) (\omega_{AB})$$

$$V_A = (0.851) (1.172)$$

$$V_A = 1 \text{ m/s}$$

In ΔICB ,

$$(IC)^2 = (IB)^2 + (CB)^2 - 2(IB)(CB) \cos 40^\circ = (0.851)^2 + (0.6)^2 - 2(1.28)(0.6) \cos 40^\circ$$

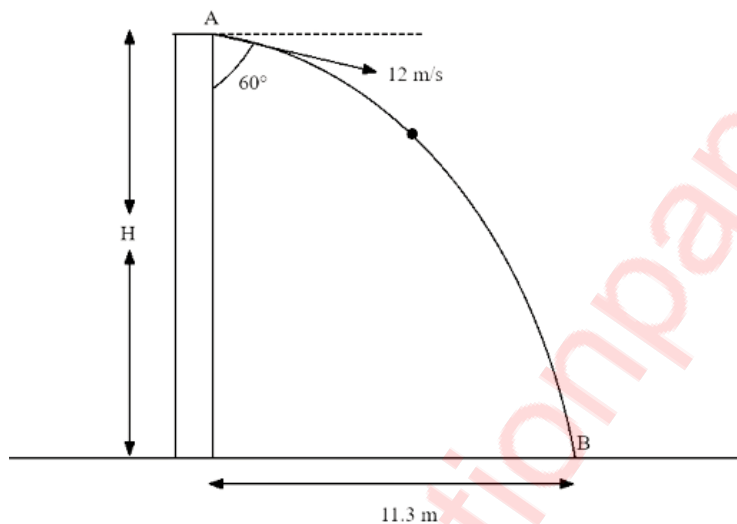
$$IC = 0.906 \text{ m}$$

$$V_C = (IC) \omega_{AB} = (0.906) \times 1.17$$

$$V_c = 1.06 \text{ m/s}$$

c. A ball thrown with a speed of 12m/s at an angle of 60° with a building strikes the ground 11.3m horizontally from the foot of the building as shown in fig. Determine the height of the building.

(4)

**Solution:**

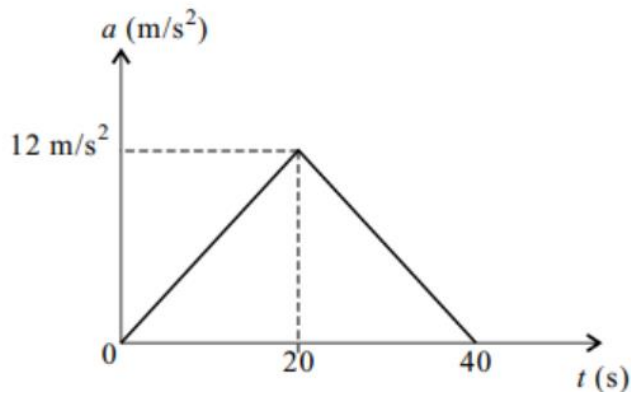
$$y = x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$$

$$-h = 11.3 \tan (-30)^\circ - \frac{9.81 \times 11.3^2}{2 \times 12^2} = [1 + \tan^2 (-30)^\circ]$$

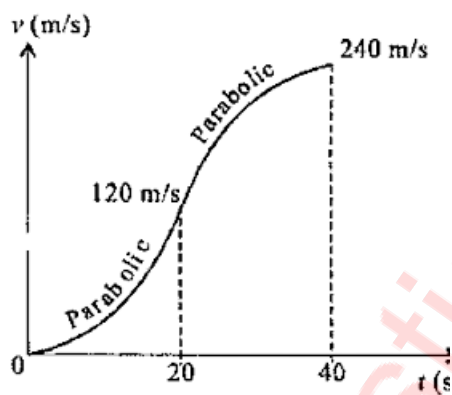
$$h = 12.32 \text{ m}$$

Q.4.

a. A car moves along a straight road such that its acceleration time motion is described by the graph shown in fig. construct v-t and s-t graphs and determine the maximum speed and maximum distance covered.

**Solution:**

(i) Velocity-Time graph



Change in velocity = Area under a-t diagram

(a) At $t = 20$ s

$$V_{20} - V_0 = \frac{1}{2} \times 20 \times 12$$

$$V_{20} = 120 \text{ m/s} \quad (\because v_0 = 0)$$

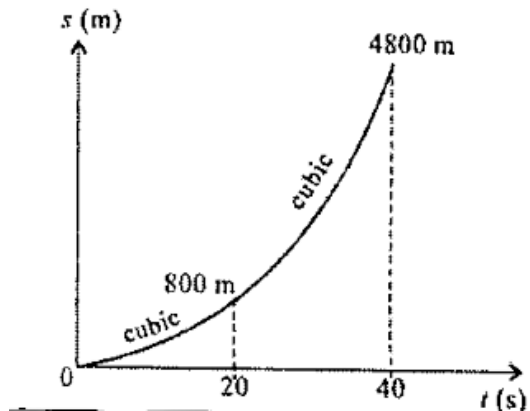
(b) At $t = 40$ s

$$V_{40} - V_{20} = \frac{1}{2} \times 20 \times 12$$

$$V_{40} = 120 + 120$$

$$V_{40} = 240 \text{ m/s}$$

(ii) Displacement-Time graph



Change in displacement = Area under v- t diagram

(a) At $t=20$ s

$$S_{20} - S_0 = \frac{1}{3} 20 \times 120$$

$$S_{20} = 800 \text{ m} \quad (\because S_0 = 0)$$

(b) At $t = 40$ s

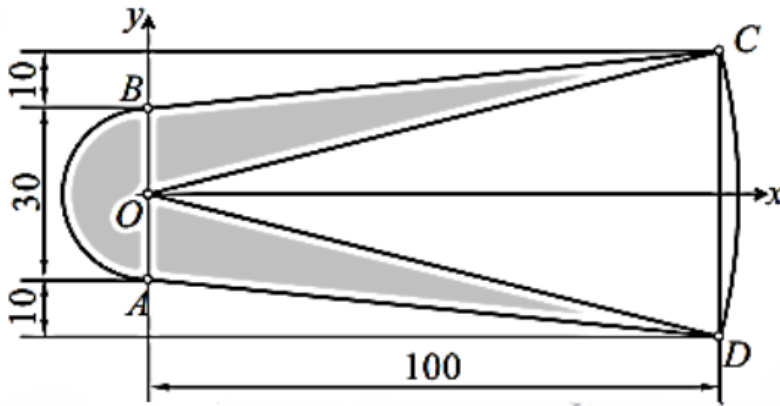
$$S_{40} - S_{20} = 20 \times 120 + \frac{2}{3} \times 20 \times 120$$

$$S_{40} = 800 + 2400 + 1600$$

$$S_{40} = 4800 \text{ m}$$

b) Determine the centroid of the shaded area.

(8)

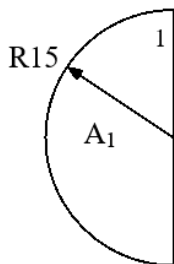


Solution:

(i) The given figure is symmetric about the x-axis.

$$\therefore \bar{y} = 0.$$

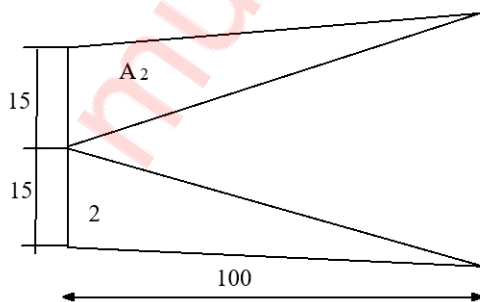
(ii) Consider semicircle:



$$A_1 = \frac{\pi \times 15^2}{2} = 353.43 \text{ cm}^2$$

$$x_1 = \frac{-4 \times 15}{3\pi} = -6.37 \text{ cm}$$

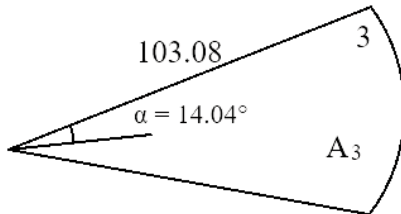
(iii) Consider two equal triangles:



$$2(A_2) = 2 \left(\frac{1}{2} \times 15 \times 100 \right) = 2(750) \text{ cm}^2$$

$$x_2 = \frac{100}{3} = 33.33 \text{ cm}$$

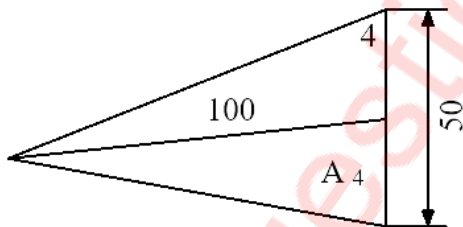
(iv) Consider a sector of circle:



$$A_3 = 103.08^2 \times 14.04 \times \frac{\pi}{180} = 2603.71 \text{ cm}^2$$

$$x_2 = \frac{2 \times 103.08 \sin 14.04}{3 \times 14.04 \times \frac{\pi}{180}} = 68.03 \text{ cm}$$

(v) Consider triangle:



$$-A_4 = \left(\frac{1}{2} \times 50 \times 100 \right) = -2500 \text{ cm}^2$$

$$x_4 = \frac{2}{3} \times 100 = 66.67 \text{ cm}$$

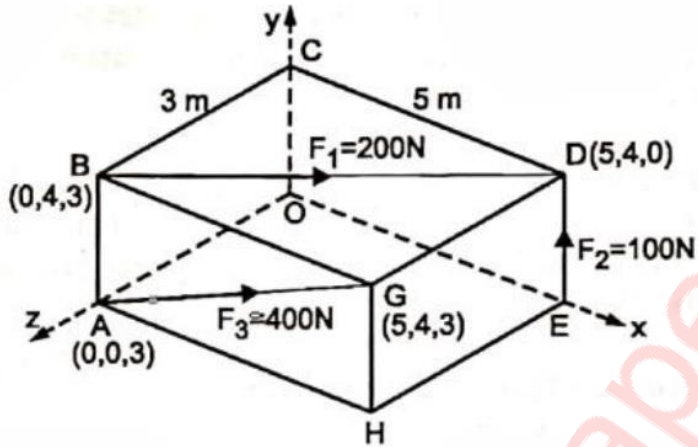
$$\bar{x} = \frac{353.43 \times (-6.37) + 2(750 \times 33.33) + 2603.71 \times 68.03 + (-2500) \times (66.67)}{353.43 + 2(750) + 2603.71 - 2500}$$

$$\bar{x} = 29.74 \text{ cm}$$

∴ coordinates of centroid w.r.t. origin O are G (29.74, 0) cm.

C. A rectangular parallelepiped carries four forces shown in fig. Reduce the force system to a resultant force applied at the origin.

(4)



Solution:

Putting the forces in vector form.

$$\vec{F}_1 = F_1 \cdot \hat{e}_{BD}$$

$$\vec{F}_1 = 200 \left[\frac{5i+0j-3k}{\sqrt{5^2+3^2}} \right]$$

$$\vec{F}_1 = 171.51 i - 102.9 k \text{ N}$$

$$\vec{F}_2 = 100 j \quad \text{..... Since the force acts along the y axis in the + ve sense.}$$

$$\vec{F}_3 = F_3 \cdot \hat{e}_{AG}$$

$$\vec{F}_3 = 400 \left[\frac{5i+4j-0k}{\sqrt{5^2+4^2}} \right]$$

$$\vec{F}_3 = 312.3 i - 249.9 j \text{ N}$$

The Resultant force

$$\vec{R} = F_1 + F_2 + F_3$$

$$\vec{R} = (171.5 i - 102.9 k) + (100 j) + (312.31 i + 249.9 j)$$

Or

$$\vec{R} = 483.81 i + 349.91 j - 102.9 k \text{ N}$$

The resultant moment

$$\bar{M}_O = \bar{M}_O^{F1} + \bar{M}_O^{F2} + \bar{M}_O^{F3}$$

$$\bar{M}_O = (-411.61i + 514.5j - 686k) + (500k) + (-749.7i + 936.9j)$$

$$\bar{M}_O = -1161.3i + 1451.4j - 186k \text{ Nm}$$

The resultant of General force system is

$$\bar{R} = 483.81i + 349.91j - 102.9k \text{ N}$$

And the resultant

$$\bar{M}_O = -1161.3i + 1451.4j - 186k \text{ Nm}$$

$$\bar{M}_O^{F1} = \bar{r}_{OB} \times \bar{F}_1$$

$$\bar{M}_O^{F1} = (4i + 3k) \times (171.5i - 102.9k)$$

$$\bar{M}_O^{F1} = -411.6i + 514.5j - 686k \text{ Nm}$$

$$\bar{M}_O^{F2} = \bar{r}_{OD} \times \bar{F}_2$$

$$\bar{M}_O^{F2} = (5i + 4j) \times (100j)$$

$$\bar{M}_O^{F2} = 500k \text{ Nm}$$

$$\bar{M}_O^{F3} = \bar{r}_{OA} \times \bar{F}_1$$

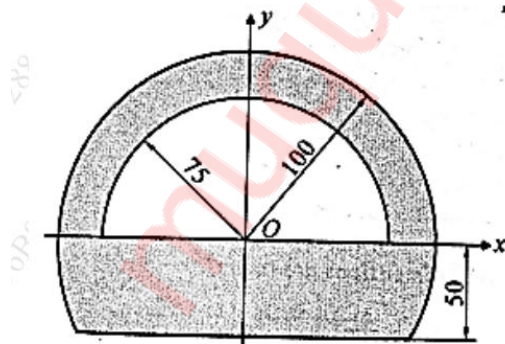
$$\bar{M}_O^{F3} = (3k) \times (312i - 249.9j)$$

$$\bar{M}_O^{F3} = -749.7i + 936.9j \text{ Nm}$$

Q.5

a. Find the centroid of the shaded area.

(8)



Solution:

i) The given figure is symmetric about the y-axis.

$$\therefore \bar{x} = 0$$

$$\text{In } \triangle COE, \frac{OE}{OC} = \cos \theta$$

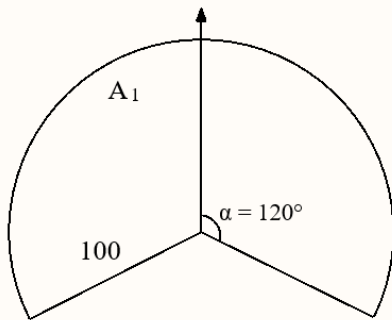
$$\cos \theta = \frac{50}{100}$$

$$\therefore \theta = 60^\circ$$

$$\therefore \angle COE = 60^\circ$$

Divide the figure into three parts as shown.

ii) consider *CAFBDO* :



$$A_1 = \left(\frac{120 \times \pi}{180} \right) \times 100^2 = 20944 \text{ mm}^2$$

$$A_1 = \left(\frac{2 \times 100 \sin 120}{3 \times \left(\frac{120 \times \pi}{180} \right)} \right) = 27.57 \text{ mm}$$

iii) Consider Triangle *COD* :



$$CE = \sqrt{100^2 - 50^2} = 86.6 \text{ mm}$$

$$\therefore CD = 173.2 \text{ mm}$$

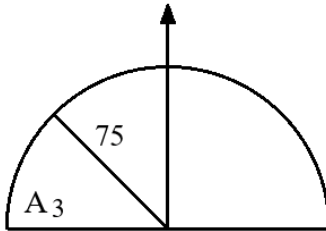
$$A_2 = \frac{1}{2} \times 173.2 \times 50$$

$$A_2 = 4330 \text{ mm}^2$$

and

$$Y_2 = -\frac{2}{3} \times 50 = -33.33 \text{ mm}$$

iii) Consider Semicircle PQR :



$$-A_3 = -\frac{\pi \times 75^2}{2} = -8835.73 \text{ mm}^2$$

and

$$Y_3 = \frac{4 \times 75}{3\pi} \times 50 = 31.83 \text{ mm}$$

v) Coordinates of the centroid of given shaded area can be calculated as

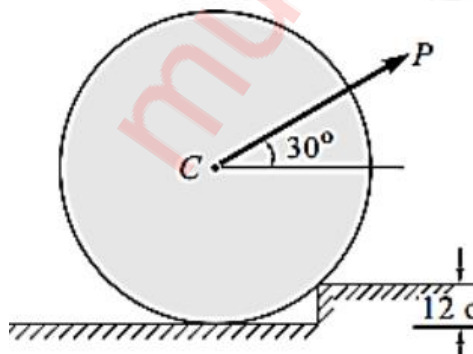
$$\bar{x} = \frac{20944 \times 27.57 + 4330 \times (-33.33) + (-8835.73) \times 31.83}{20944 + 4330 - 8835.73}$$

$$\bar{x} = 9.239 \text{ mm.}$$

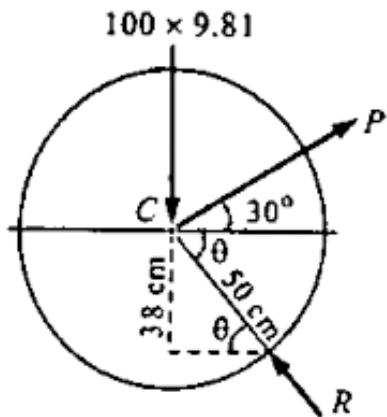
∴ Coordinates of the centroid w.r.t. origin O are G(0,9.239) mm.

b. Determine force P applied at 45° to the horizontal just necessary to start a roller of 100 cm diameter and weighing 100 kg over a block of 12 cm high.

(6)



Solution:



FBD of the diagram

$$\sin \theta = \frac{38}{50}$$

$$\therefore \theta = 49.46^\circ$$

(iii) By Lami's theorem,

$$\frac{981}{\sin 79.46^\circ} = \frac{-P}{\sin 220.54^\circ}$$

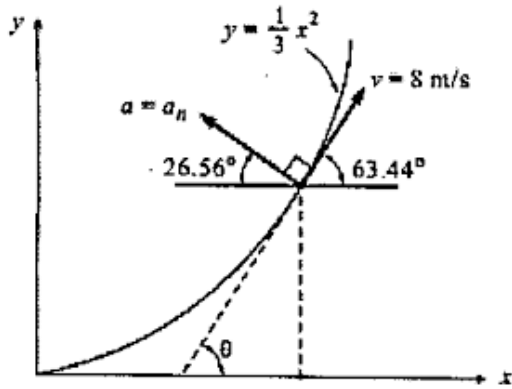
$$P = \frac{-981 \times \sin 220.54^\circ}{\sin 79.46^\circ}$$

$$\therefore P = 648.57 \text{ N}$$

c. A point moving along a path $y=x^2/3$ with a constant speed of 8m/s. What are the x and y components of its velocity when $x=3\text{m}$? Also, find the radius of curvature and acceleration.

(6)

Solution:



Given : $v = 8 \text{ m/s}$ is constant;

$$a_t = 0$$

$$a_t = \frac{dv}{dt} = 0$$

$$\therefore a_n = a$$

$$y = \frac{1}{3} x^2$$

$$\frac{dy}{dx} = \frac{2}{3} x$$

$$\left(\frac{dy}{dx}\right)_{x=3} = \frac{2}{3} \times 3 = 2$$

$$\frac{d^2y}{dx^2} = \frac{2}{3}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=3} = \frac{2}{3}$$

$$P = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{[1 + (2)^2]^{3/2}}{2/3} \right|$$

$$P = 16.77 \text{ m}$$

$$a_n = \frac{v^2}{p} = \frac{8^2}{16.77}$$

$$a_n = 3.82 \text{ m/s}^2$$

$$\tan \theta = \left(\frac{dy}{dx} \right)_{x=3} = 2$$

$$\theta = 63.44^\circ$$

$$\therefore v_x = v \cos \theta = 8 \cos 63.44 = 3.58 \text{ m/s}$$

$$\therefore v_y = v \sin \theta = 8 \sin 63.44 = 7.15 \text{ m/s}$$

$$a = \sqrt{a_t^2 + a_n^2}$$

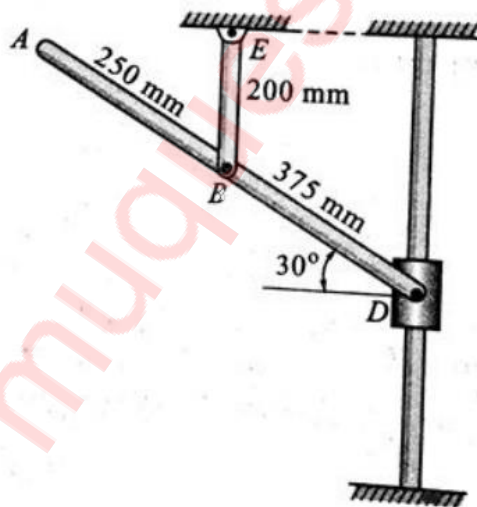
$$a = \sqrt{0 + 3.82^2}$$

$$\therefore a = 3.82 \text{ m/s}^2$$

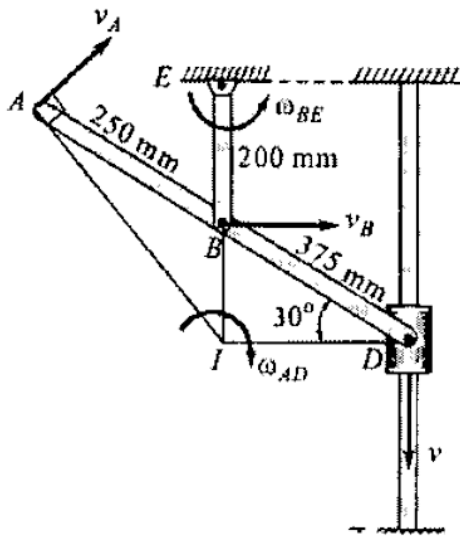
Q.6

a. Knowing that at the instant the angular velocity of rod BE is 4 rad/sec counterclockwise determine the angular velocity of rod AD and velocity of collar D.

(6)



Solution:



FBD of the diagram

(i) Rod BE (Performs rotational motion about point E)

$$V_B = (BE) \omega_{BE} = 0.2 \times 4$$

$$V_B = 0.8 \text{ m/s } (\rightarrow)$$

$$IB = BD \sin 30^\circ = 0.375 \times \sin 30^\circ$$

$$IB = 0.1875 \text{ m}$$

$$ID = (BD) \cos 30^\circ = 0.375 \times \cos 30^\circ$$

$$ID = 0.325 \text{ m}$$

$$(IA)^2 = (AD)^2 + (ID)^2 - 2(AD)(ID) \cos 30^\circ$$

$$(IA)^2 = (0.625)^2 + (0.325)^2 - 2(0.625)(0.325) \cos 30^\circ$$

$$IA = 0.38 \text{ m}$$

(ii) Rod AD (Performs general plane motion)

At the given instant, point I is the ICR.

$$V_B = (IB) \omega_{AD}$$

$$\omega_{AD} = \frac{0.8}{0.1875}$$

$$\omega_{AD} = 4.267 \text{ rad/s } (\curvearrowright)$$

$$V_A = (IA) (\omega_{AD})$$

$$V_A = 0.38 \times 4.267$$

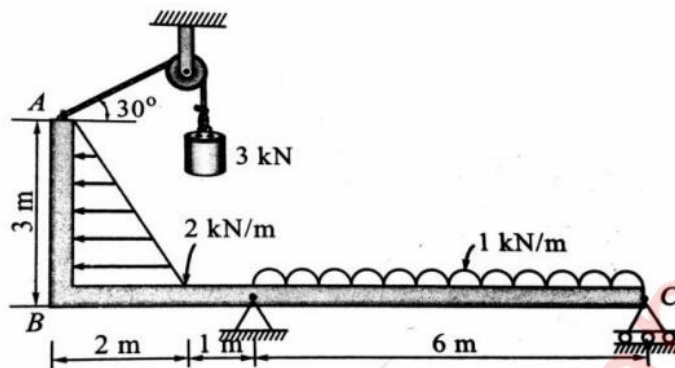
$$V_A = 1.62 \text{ m/s}$$

$$V_D = (ID) (W_{AD}) = 0.325 \times 4.267$$

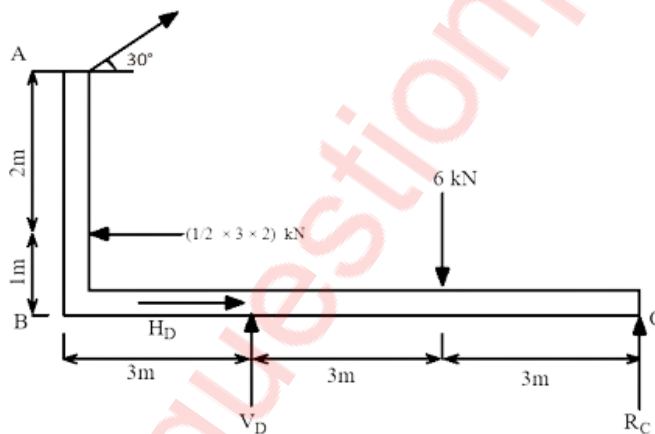
$$V_D = 1.3867 \text{ m/s } (\downarrow)$$

b. Find the support reactions for the beam loaded as shown in fig.

(6)



Solution:



FBD of the diagram

$$(ii) \sum M_D = 0$$

$$R_C \times 6 - 3 \cos 30^\circ \times 3 - 3 \sin 30^\circ \times 3 - 6 \times 3 + \left(\frac{1}{2} \times 3 \times 2\right) \times 1 = 0$$

$$R_C = 4.55 \text{ kN } (\uparrow)$$

$$(iii) \sum F_x = 0$$

$$3 \cos 30^\circ - \left(\frac{1}{2} \times 3 \times 2\right) + HD = 0$$

$$H_D = 0.4 \text{ kN } (\rightarrow)$$

$$(iv) \sum F_Y = 0$$

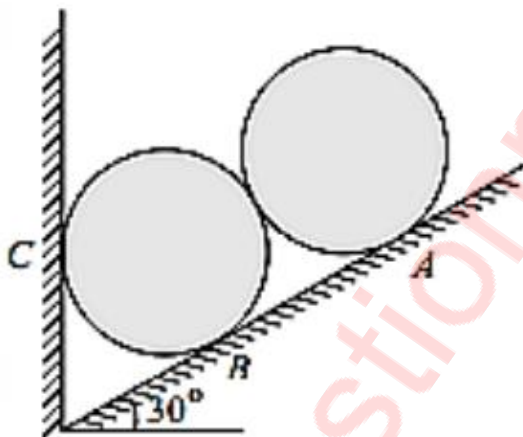
$$V_D + 3 \sin 30 + R_C - 6 = 0$$

$$V_D = - 0.05 \text{ (Wrong assumed direction)}$$

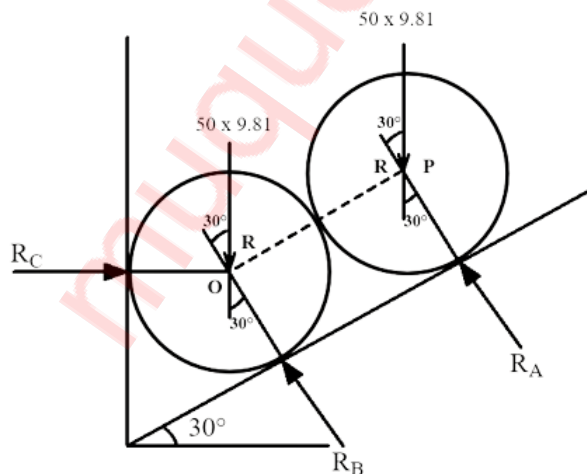
$$V_D = 0.05 \text{ kN } (\downarrow)$$

c. Two identical rollers of mass 50kg are supported as shown in figure. To maintain the equilibrium, Determine the support reactions assuming all smooth surfaces.

(6)



Solution:



FBD of the diagram

(i) Consider F.B.D. of both rollers together and let A be the radius of rollers.

(ii) $\Sigma M_O = 0$

$$R_X \times 2R - 50 \times 9.81 \cos 30^\circ \times 2R = 0$$

$$R_A = 424.79 \text{ N}$$

(iii) $\Sigma F_Y = 0$

$$R_B \cos 30^\circ + R_A \cos 30^\circ - 50 \times 9.81 - 50 \times 9.81 = 0$$

$$R_B = 707.97 \text{ N}$$

(iv) $\Sigma F_X = 0$

$$R_C - R_A \sin 30^\circ - R_B \sin 30^\circ = 0$$

$$R_C = 424.79 \sin 30^\circ + 707.97 \sin 30^\circ$$

$$R_C = 566.38 \text{ N } (\rightarrow) .$$