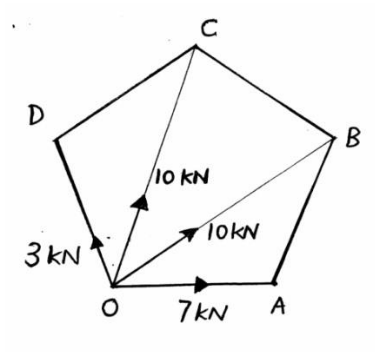
Mumbai University Engineering Mechanics May 2019 Question Paper Solution

Q1. Attempt Any Four:

(a) Find the resultant of forces as shown in fig. **(05 marks)**



Solution:

$$\sum F_x = \left[(7) + (10\cos(36°)) + (10\cos(72°)) + (-3\cos(72°))\right] \text{ kN}$$

$$\therefore \sum F_x = 17.25 \text{ kN} (\rightarrow)$$

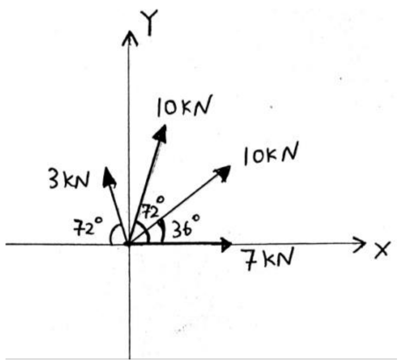
$$\sum F_y = \left[(10\sin(36°)) + (10\sin(72°)) + (3\sin(72°))\right] \text{ kN}$$

$$\therefore \sum F_y = 18.24 \text{ kN} (\uparrow)$$

$$\text{Resultant} = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} \text{ kN}$$

$$= \sqrt{(17.25)^2 + (18.24)^2} \text{ kN}$$

$$= 25.10 \text{ kN } (\quad)$$

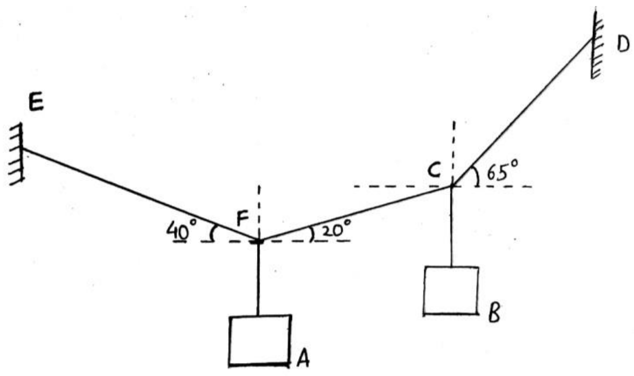


$$\therefore \boxed{\text{Resultant} = 25.10 \text{ kN } (\quad)}$$

$$\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right] = \tan^{-1}\left[\frac{18.24}{17.25}\right] = 46.6°$$

$$\therefore \boxed{\theta = 46.6°}$$

(b) If the cords suspended the two buckets in equilibrium position shown in Fig. Determine weight of bucket B if bucket A has a weight of 60 N. (05 marks)



Solution:

$W_A = 60 \text{ N} \quad ...\{\text{Given}\}$

Applying Lami's Theorem at point F,

$$\therefore \frac{W_A}{\sin(120°)} = \frac{F_{FC}}{\sin(130°)} = \frac{F_{FE}}{\sin(110°)}$$

$$\therefore F_{FC} = \frac{W_A}{\sin(120°)}(\sin(130°)) \text{ N} = \frac{60}{\sin(120°)}(\sin(130°)) \text{ N}$$

$$\therefore F_{FC} = 53.07 \text{ N}$$

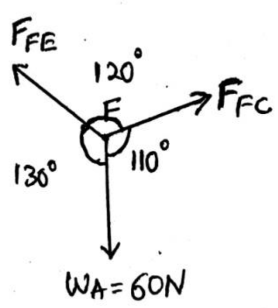
Applying Lami's Theorem at point C,

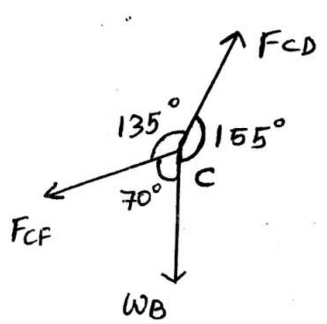
$$\therefore \frac{W_B}{\sin(135°)} = \frac{F_{CD}}{\sin(70°)} = \frac{F_{CF}}{\sin(155°)}$$

$$\therefore W_B = \frac{F_{CF}}{\sin(155°)}(\sin(135°)) \text{ N} = \frac{53.07}{\sin(155°)}(\sin(135°)) \text{ N}$$

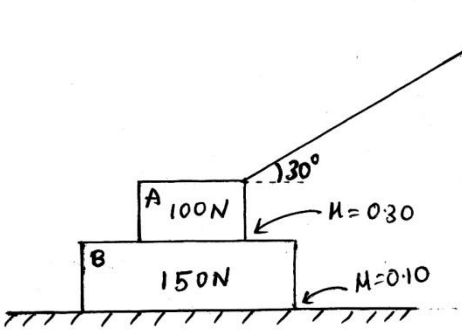
$$\therefore W_B = 88.79 \text{ N}$$

$\therefore$ $\boxed{\text{The weight of the bucket B = 88.79 N}}$



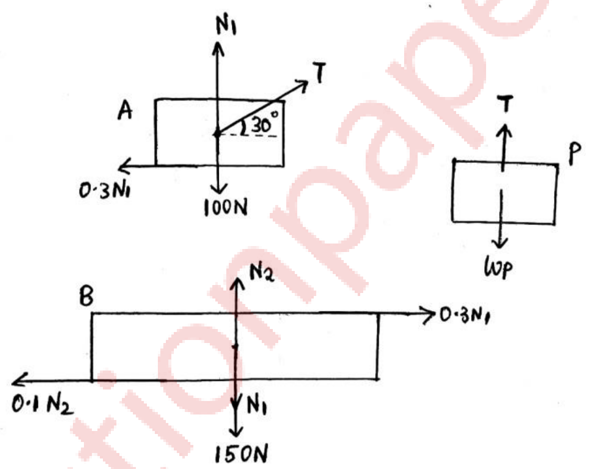


(c) Two blocks A=100 N and B=150 N are resting on the ground as shown in fig. Find the minimum weight P in the pan so that body A starts. Assume pulley to be mass less and frictionless. (05 marks)



Solution:

The FBD is shown below,



Let the tension in the string be 'T' N

Let the normal force between the two blocks A and B be N_1 N

Let the normal force between block B and ground be N_2 N

For block to just start to move, the friction force acting on block A will be backwards

And on block B the same force will be forwards.

The friction force between block B and ground will be backwards on block B.

$\therefore$ Applying equilibrium conditions on Block B,

$\sum F_x = 0$

$\therefore (0.3N_1) - (0.1N_2) = 0 \qquad ...(1)$

$$\sum F_y = 0$$

$$\therefore N_2 - 150 - N_1 = 0$$

$$\therefore -N_1 + N_2 = 150 \qquad ...(2)$$

From (1) and (2)

$N_1 = 75$ N and $N_2 = 225$ N

$\therefore$ Applying equilibrium conditions on Block A,

$$\sum F_x = 0$$

$$\therefore (T\cos(30°))\text{-}(0.3N_1) = 0$$

$$\therefore (T\cos(30°))\text{-}(0.3(75)) = 0$$

$\therefore$ T= 25.98 N

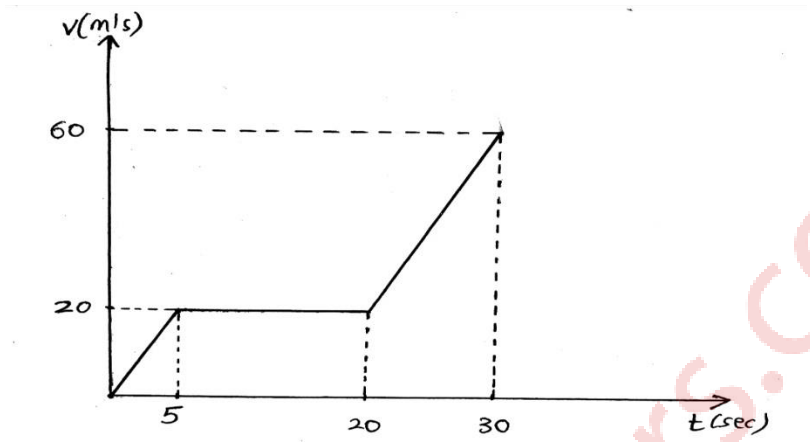
On block P,

$$W_P = T$$

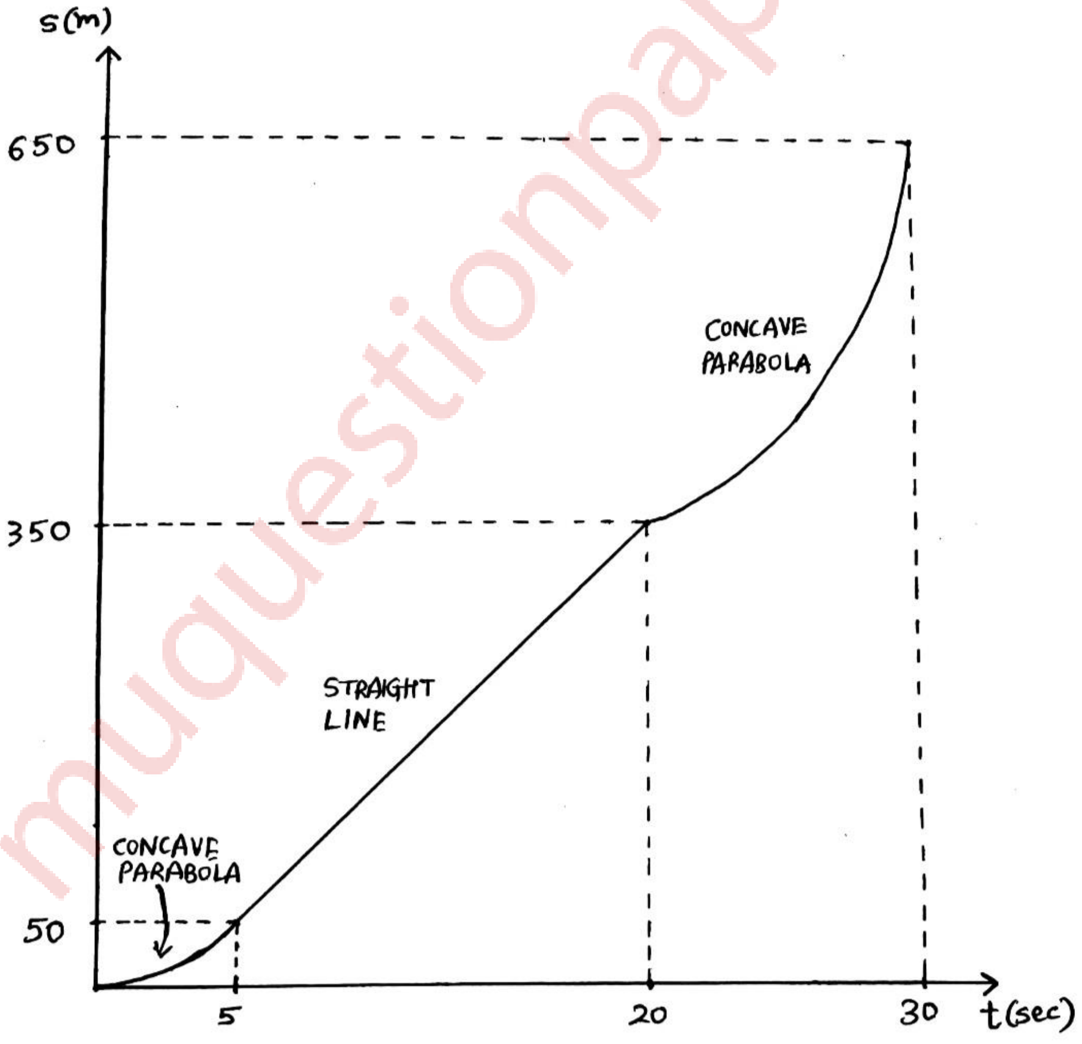
$\therefore$ $\boxed{\text{The minimum weight P in the pan so that block A just starts = 25.98 N}}$

(d) **The motion of jet plane while travelling along a runway is defined by the v-t graph as shown in Fig. Construct the s-t graph for the motion. The plane starts from rest.**

(05 marks)



Solution: The required graph is:



Explanation :

For S-T graph

$\int ds = \int v.dt$ =Area under the graph under time interval

Since the object is at rest initially, $S_0 = 0$ m

For time 0 to 5 seconds

$$\int_0^{S_5} ds = \int_0^5 v.dt = \text{Area under graph from 0 to 5 seconds} = \frac{1}{2}(5)(20)\ \text{m}$$

$$\therefore S_5 - 0 = 50\ \text{m} \quad \therefore \boxed{S_5 = 50\ \text{m}}$$

$$\int_{S_5}^{S_{20}} ds = \int_5^{20} v.dt = \text{Area under graph from 5 to 20 seconds} = (20-5)(20)\ \text{m}$$

$$\therefore S_{20} - S_5 = 300\ \text{m} \quad \therefore \boxed{S_{20} = 350\ \text{m}}$$

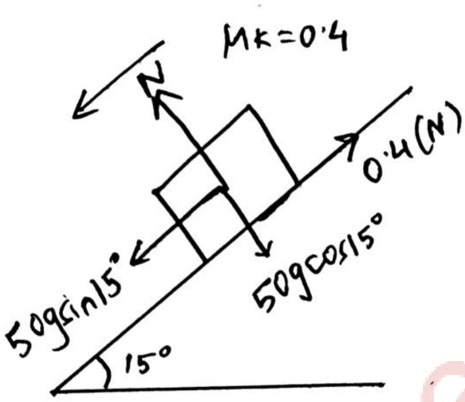
$$\int_{S_{20}}^{S_{30}} ds = \int_{20}^{30} v.dt = \text{Area under graph from 20 to 30 seconds} = \left[(30-20)(20) + \frac{1}{2}(30-20)(60-20)\right]\ \text{m}$$

$$\therefore S_{30} - S_{20} = 400\ \text{m} \quad \therefore \boxed{S_{30} = 750\ \text{m}}$$

(e) A 50 kg block is kept on the top of a $15°$ slopping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_k = 0.4$, determine the acceleration of the block.

(05 marks)

Solution: The FBD is,



On the block,

N=50gsin(15°)=50(9.8)sin(15°)

$\therefore N$=126.82 N

$50\text{gsin}(15°) - 0.4(N) = 50(a)$ where a is acceleration of the block

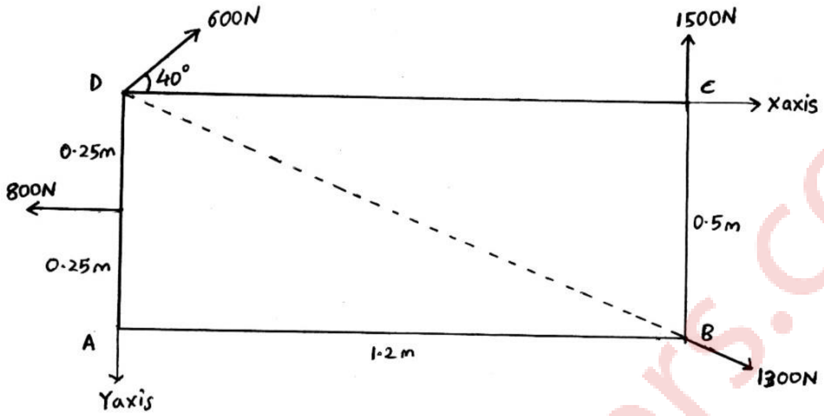
$50(9.8)\sin(15°) - 0.4(126.82) = 50(a)$

$\therefore a = 1.52 \text{ m/s}^2$

$\therefore$ **The acceleration of the block =** 1.52 m/s^2

Q2. Attempt:

(a) Four forces acting on a rectangle in the same plane as shown in fig. below. Find magnitude and direction of resultant force. Also find intersection of line of action of resultant with X and Y axes, assuming D as origin. (06 marks)



Solution:

Finding $\angle ABD$

$$\angle ABD = \tan^{-1}\left[\frac{AD}{AB}\right] = \tan^{-1}\left(\frac{0.5}{1.2}\right)$$

$\therefore \angle ABD = 22.62°$

$\therefore$ The angle made by 1300 N force with horizontal is 22.62°

$$\sum F_x = [1300\cos(22.62°) + 600\cos(40°) - 800]\ \text{N}$$

$$\sum F_x = 859.63\ \text{N}\ (\rightarrow)$$

$$\sum F_y = [600\sin(40°) + 1500 - 1300\sin(22.62°)]\ \text{N}$$

$$\sum F_y = 1385.67\ \text{N}\ (\uparrow)$$

$$\text{Resultant}=\sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}\ \text{N}$$

$$=\sqrt{859.63^2 + 1385.67^2}\ \text{N}$$

$$=1630.66\ \text{N}\ (\square)$$

$$\therefore \boxed{\text{Resultant}=1630.66\ \text{N}\ (\square)}$$

$$\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right] = \tan^{-1}\left(\frac{1385.67}{859.63}\right) = 58.19°$$

$$\therefore \boxed{\theta = 58.19°}$$

$$\sum M_0 = (1500)(1.2) + (1300\cos(22.62°)(0.5) - \left(1300\sin(22.62°)(1.2)\right) - (800)(0.25)$$

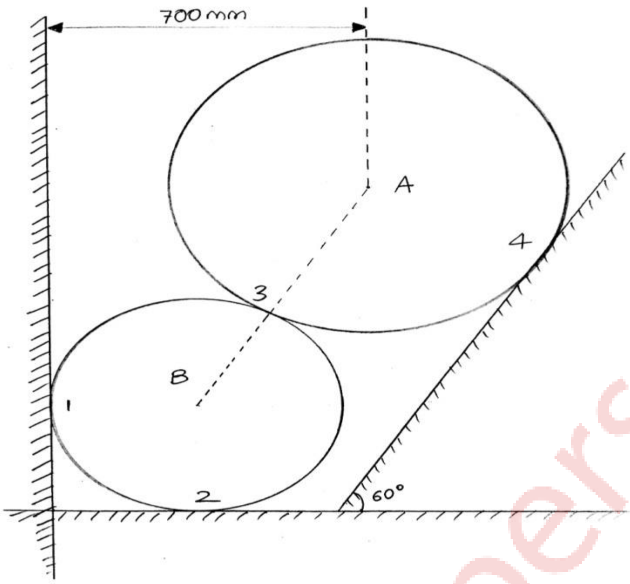
$$\therefore \sum M_0 = 1600 \text{ N-m}$$

$$X = \frac{\sum M_0}{\sum F_y} = \frac{1600}{1385.67} = 1.15 \text{ m}$$

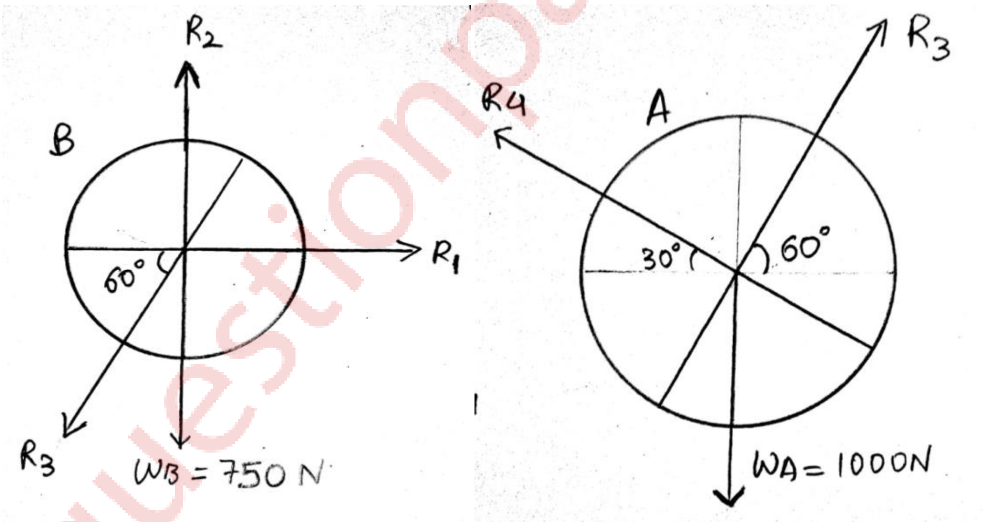
$$Y = \frac{\sum M_0}{\sum F_x} = \frac{1600}{859.63} = 1.86 \text{ m}$$

$$\therefore \boxed{X = 1.15 \text{ m and } Y = 1.86 \text{ m}}$$

(b) Two spheres A and B of weight 1000 N and 750 N respectively are kept as shown in fig. Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A=400 mm and B=300 mm. (08 marks)



Solution:



Applying equilibrium conditions on ball A

$\sum F_x = 0$

$\therefore R_3 \cos(60°) - R_4 \cos(30°) = 0 \quad ...(1)$

$\sum F_y = 0$

$\therefore R_3 \sin(60°) + R_4 \sin(30°) - 1000 = 0$

$\therefore R_3 \sin(60°) + R_4 \sin(30°) = 1000 \quad ...(2)$

From (1) and (2)

$\boxed{R_3 = 866.03 \text{ N and } R_4 = 500 \text{ N}}$

Applying equilibrium conditions on ball B

$\sum F_x = 0$

$\therefore R_1 - R_3 \cos(60°) = 0$

$\therefore R_1 = R_3 \cos(60°) = (866.03)\cos(60°)$

$\therefore \boxed{R_1 = 433.02\ \text{N}}$

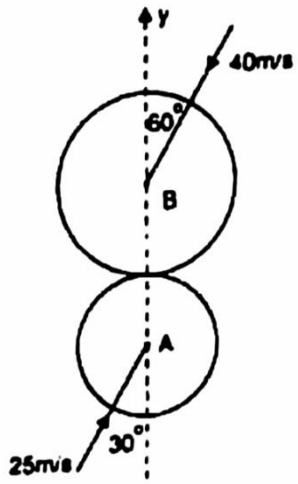
$\sum F_y = 0$

$\therefore R_2 - R_3 \sin(60°) - 750 = 0$

$R_2 - (866.03)\sin(60°) - 750 = 0$

$\therefore \boxed{R_2 = 1500\ \text{N}}$

(c) Two smooth balls A (mass 3 kg) and B (mass 4 kg) are moving with velocities 25 m/s and 40 m/s respectively. Before impact, the directions of velocity of two balls are 30° and 60° with the line joining the centres as shown in Fig. If e=0.8, find magnitude and direction of velocities of the balls after impact. (06 marks)



Solution:

Let u_A and u_B be the initial velocities of balls A and B respectively,

Let v_A and v_B be the final velocities of balls A and B respectively,

$\therefore u_A = 25\sin(30°)i + 25\cos(30°)j$

$\therefore u_B = -40\sin(60°)i - 40\cos(30°)j$

Let v_{Ax} and v_{Ay} be the x and y components of velocity of ball A resp,

Let v_{Bx} and v_{By} be the x and y components of velocity of ball B resp,

Applying Law of conservation of linear momentum along y direction,

$m_A u_{Ay} + m_B u_{By} = m_A v_{Ay} + m_B v_{By}$

$\therefore 3(25\cos(30°)) + 4(-40\cos(60°)) = 3v_{Ay} + 4v_{By} \quad ...(1)$

$$e = \frac{v_{By} - v_{Ay}}{u_{Ay} - u_{By}} = 0.8 = \frac{v_{By} - v_{Ay}}{25\cos(30°) - (-40\cos(60°))}$$

$0.8\left[25\cos(30°) - (-40\cos(60°))\right] = -v_{Ay} + v_{By} \quad ...(2)$

From (1) and (2)

$v_{Ay} = -21.19$ m/s $v_{By} = 12.13$ m/s

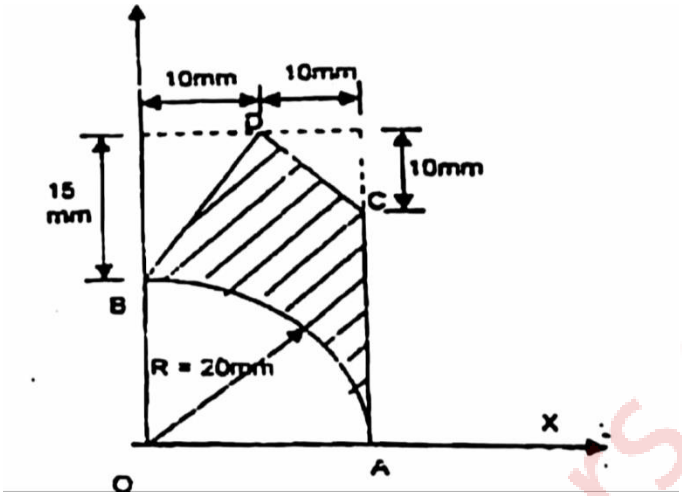
The velocities along the perpendicular to line of action remain unchanged,

$\therefore v_{Ax} = 25\sin(30°) = 12.5$ m/s and $v_{Bx} = -40\sin(60°) = -34.64$ m/s

$$\therefore \boxed{\begin{aligned} v_A &= [12.5i - 21.19j] \text{ m/s} \\ v_B &= [-34.64i + 12.13j] \text{ m/s} \end{aligned}}$$

Q3. Attempt:

(a) Find the centroid of shaded area as shown in fig. **(08 marks)**



Solution:

	Shape	Area(in mm^2)	X-Coordinate	Y-Coordinate	AX	AY
1.	Rectangle	$=(35)(20)$ $=700$	$=10$	$=17.5$	$=7000$	$=12250$
2.	Quarter Circle	$=-\dfrac{\pi(20)^2}{4}$ $=-314.16$	$=\dfrac{4(20)}{3\pi}$ $=8.49$	$=\dfrac{4(20)}{3\pi}$ $=8.49$	$=-2667.22$	$=-2667.22$
3.	Triangle (ht=15 mm, bs=10 mm)	$=-\dfrac{1}{2}(10)(15)$ $=-75$	$=\dfrac{10}{3}$ $=3.33$	$=35-\dfrac{15}{3}$ $=30$	$=-250$	$=-2250$
4.	Triangle (ht=10 mm, bs=10 mm)	$=-\dfrac{1}{2}(10)(10)$ $=-50$	$=20-\dfrac{10}{3}$ $=16.67$	$=35-\dfrac{10}{3}$ $=31.67$	$=-833.5$	$=-1583.5$

$$\sum A = 700 - 314.16 - 75 - 50 \text{ mm}^2$$
$$\therefore \sum A = 260.84 \text{ mm}^2$$

$$\sum AX = 7000 - 2667.22 - 250 - 833.5 \text{ mm}^2$$
$$\therefore \sum AX = 3249.28 \text{ mm}^2$$

$$\sum AY = 12250 - 2667.22 - 2250 - 1583.5 \text{ mm}^2$$
$$\therefore \sum AY = 5749.28 \text{ mm}^2$$

$$X=\frac{\sum AX}{\sum A}=\frac{3249.28}{260.84}=12.46 \text{ mm}$$

$$Y=\frac{\sum AY}{\sum A}=\frac{5749.28}{260.84}=22.04 \text{ mm}$$

Centroid is,

$\therefore \boxed{X=12.46 \text{ mm and } Y=22.04 \text{ mm}}$

(b) Three forces F_1, F_2 and F_3 act at origin O. $F_1 = 70\ \text{N}$ acting along OA, where A (2, 1, 3). $F_2 = 80\ \text{N}$ acting along OB, where B(-1, 2, 0). $F_3 = 100\ \text{N}$ acting along OC, where C(4, -1, 5). Find the resultant of these concurrent forces. **(06 marks)**

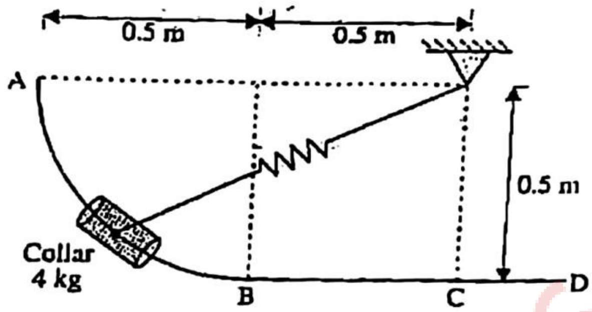
Solution:

$$\overline{F_1} = 70[2i + j + 3k]\ \text{N}$$
$$\overline{F_2} = 80[-i + 2j]\ \text{N}$$
$$\overline{F_3} = 100[4i - j + 5k]\ \text{N}$$

$$\overline{F_{net}} = \overline{F_1} + \overline{F_2} + \overline{F_3}\ \text{N}$$
$$\therefore \overline{F_{net}} = \left[70[2i + j + 3k] + 80[-i + 2j] + 100[4i - j + 5k]\right]\ \text{N}$$

$$\therefore \overline{F_{net}} = [460i + 130j + 710k]\ \text{N}$$

(c) A 4 kg collar is attached to a spring, slides on a smooth bent rod ABCD. The spring has constant k=500 N/m and is undeformed when the collar is at 'C'. If the collar is released from rest at A. Determine the velocity of collar, when it passes through 'B' and 'C'. Also find the distance moved by collar beyond 'C' before it comes to rest again. Refer fig. (06 marks)



Solution:

$$l(OB)=\sqrt{0.5^2+0.5^2}=0.5\sqrt{2}=0.707\ \text{m}$$

Natural Length (l_0) =0.5 m

$$x_A = OA - l_0 = 1 - 0.5 = 0.5\ \text{m}$$
$$x_B = OB - l_0 = 0.707 - 0.5 = 0.207\ \text{m}$$

Applying work energy theorem from A to B

$$W_g + W_{sp} = \Delta K$$

$$mgh + \frac{1}{2}k\left(x_A{}^2 - x_B{}^2\right) = \frac{1}{2}m\left(v_B{}^2 - v_A{}^2\right)$$

$$4(9.8)(0.5) + \frac{1}{2}(500)(0.5^2 - 0.207^2) = \frac{1}{2}(4)(v_B{}^2 - 0^2)$$

$$\therefore \boxed{v_B = 5.97\ \text{m/s}}$$

Applying work energy theorem from B to C,

$$W_g + W_{sp} = \Delta K$$

$$mgh + \frac{1}{2}k\left(x_B{}^2 - x_C{}^2\right) = \frac{1}{2}m\left(v_C{}^2 - v_B{}^2\right)$$

$$0 + \frac{1}{2}(500)(0.207^2 - 0^2) = \frac{1}{2}(4)(v_C{}^2 - 5.97^2)$$

$$\therefore \boxed{v_C = 6.40\ \text{m/s}}$$

Applying work energy theorem from C to D,

$$W_g + W_{sp} = \Delta K$$

$$mgh + \frac{1}{2}k\left(x_C{}^2 - x_D{}^2\right) = \frac{1}{2}m\left(v_D{}^2 - v_C{}^2\right)$$

$$0 + \frac{1}{2}(500)(0^2 - x_D{}^2) = \frac{1}{2}(4)(0^2 - 6.4^2)$$

$$\therefore x_D = 0.572 \text{ m}$$

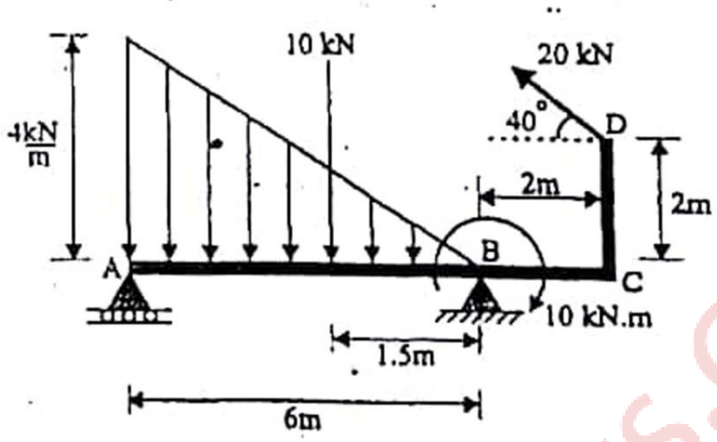
$$l(OD) = l_0 + x_D = 0.5 + 0.572 = 1.07 \text{ m}$$

$$\therefore \text{CD}=\sqrt{OD^2 - OC^2} = \sqrt{1.07^2 - 0.5^2} = 0.946 \text{ m}$$

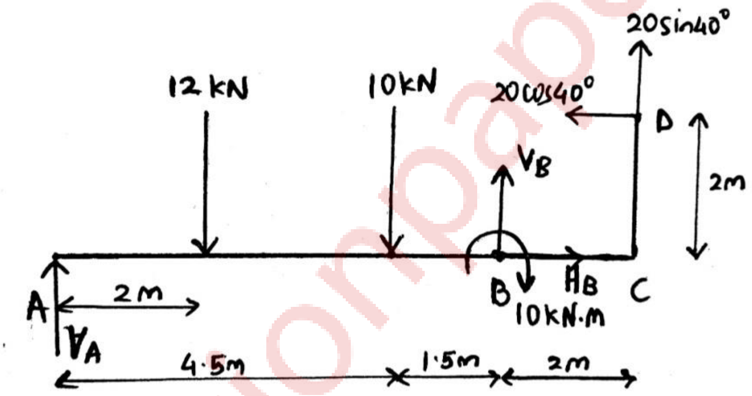
$$\therefore \boxed{\text{CD}=0.946 \text{ m}}$$

Q4. Attempt:

(a) Find the support reactions of beam loaded as shown in fig. **(08 marks)**



Solution: The FBD is**,**



$$\sum M_B = 0$$

$$\therefore -V_A(6) + 12(4) + 10(1.5) + 20\cos(40°)(2) + 20\sin(40°)(2) = 0$$

$$\therefore \boxed{V_A = 19.89 \text{ kN } (\uparrow)}$$

$$\sum F_X = 0$$

$$\therefore H_B - 20\cos(40°) = 0$$

$$\therefore \boxed{H_B = 15.32 \text{ kN } (\rightarrow)}$$

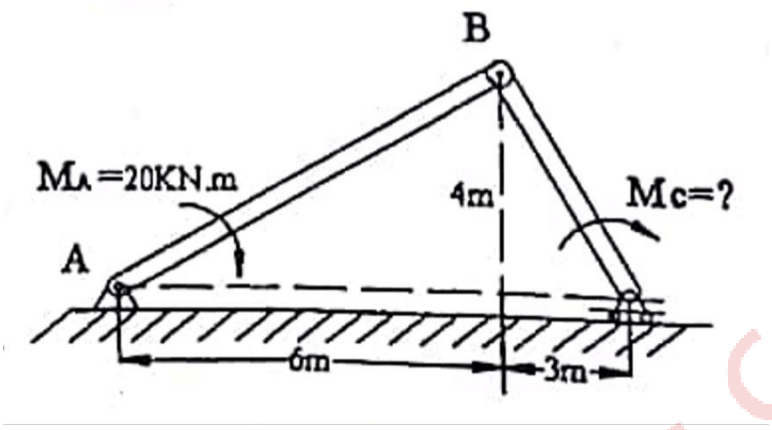
$$\sum F_Y = 0$$

$$\therefore V_A - 12 - 10 + V_B + 20\sin(40°) = 0$$

$$19.89 - 12 - 10 + V_B + 20\sin(40°) = 0$$

$$\therefore \boxed{V_B = -10.75 \text{ kN} = 10.75 \text{ kN} (\downarrow)}$$

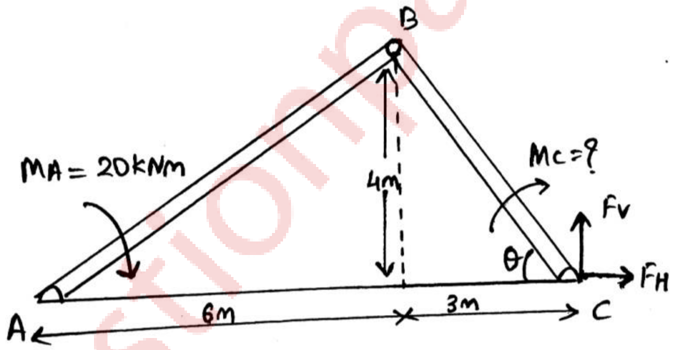
(b) Determine the moment to be applied at C for equilibrium of pin jointed mechanism. Use virtual work method. Refer Fig. (06 marks)



Solution:

From line BD:

Active Forces	Coordinates	Virtual Displacement
F_H	3	$5\cos\theta$
F_V	0	0



For maintaining equilibrium,

By Principal of virtual work,

$$\sum V.W = 0$$

$$\therefore F_V(0) + F_H(5\cos\theta) - 20 = 0$$

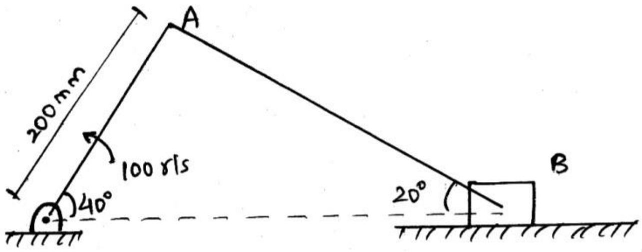
$$\therefore F_H = \frac{20}{5\cos\theta} = 4\sec\theta = \frac{20}{3}\ \text{kN}$$

Hence the moment to be applied at point C is

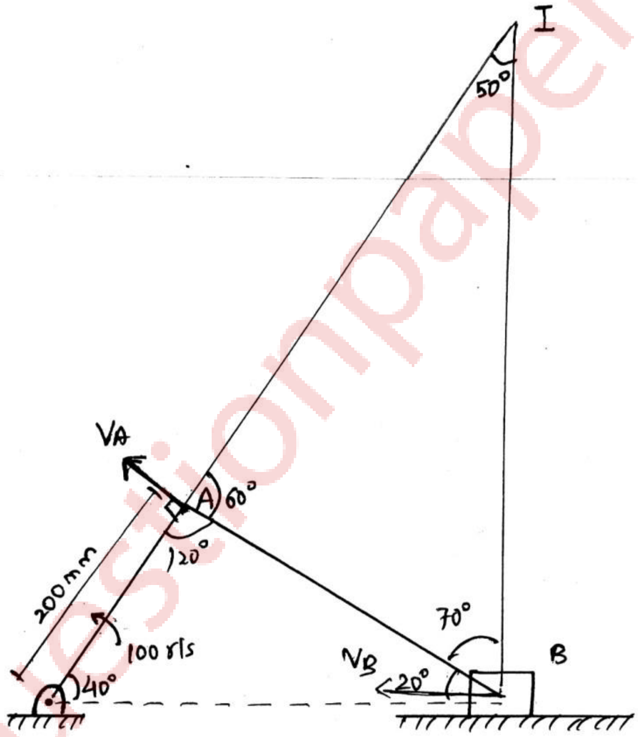
$$\boxed{M = \frac{20}{3}(3) = 20\ \text{kNm}}$$

(c) A slider crank mechanism is shown in Fig. The crank OA rotates anticlockwise at 100 rad/s. Find the angular velocity of rod AB and the velocity of the slider at B.

(06 marks)



Solution: Finding ICR,



By sine rule,

$$\frac{200}{\sin(20°)} = \frac{AB}{\sin(40°)} = \frac{OB}{\sin(120°)}$$

$\therefore$ AB=375.88 mm and OB=506.42 mm

By sine rule,

$$\frac{AB}{\sin(50°)} = \frac{IA}{\sin(70°)} = \frac{IB}{\sin(60°)}$$

$\therefore$ IA=461.09 mm and IB=424.94 mm

$$V_A = W_{OA}.OA$$

$$\therefore V_A = (100)\frac{200}{1000} = 20 \text{ m/s}$$

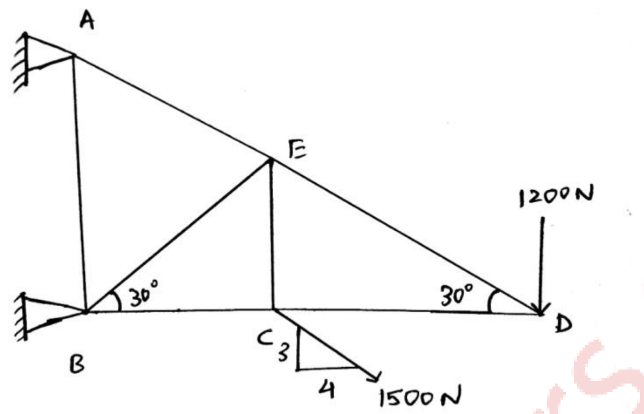
$$W_I = \frac{V_A}{IA} = \frac{20}{0.46} = 43.48 \text{ rad/s}$$

$$V_B = W_I \times IB = 43.48(0.42) = 18.26 \text{ m/s}$$

$$\boxed{\begin{aligned} W_I &= 43.48 \text{ rad/s} \\ V_B &= 18.26 \text{ m/s} \end{aligned}}$$

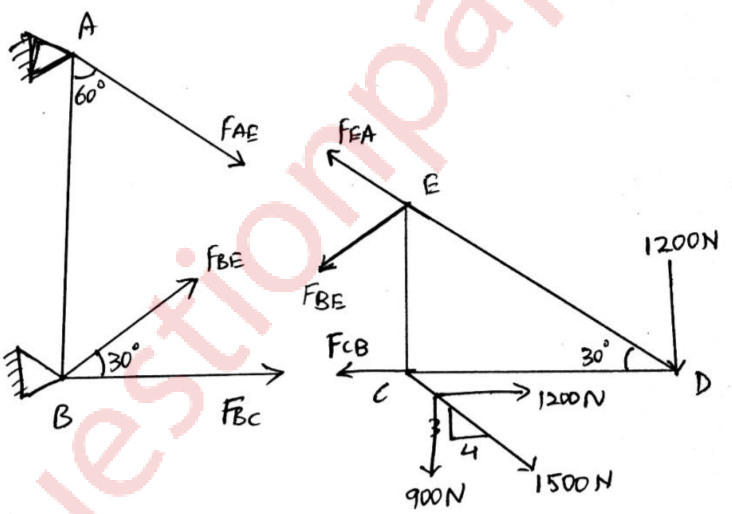
Q5. Attempt:

(a) Find the forces in the members BC, BE and AE by method of sections and remaining members by method of joints. (08 marks)



Solution:

By method of section, cutting the given truss along AE, BE and BC



Consider the right part,

$$\sum F_X = 0$$

$$\therefore \; -F_{EA}\cos(30°) - F_{BC} - F_{BE}\sin(60°) + 1200 = 0 \quad ....(1)$$

$$\sum F_Y = 0$$

$$\therefore \; F_{EA}\sin(30°) - F_{BE}\cos(60°) - 1200 - 900 = 0 \quad ....(2)$$

Let the length of hypotenuse be l

$$\sum M_E{}^F = 0$$

$$-F_{BC}(l\sin(30°)) + 1200(l\sin(30°)) - 1200(l\cos(30°)) = 0$$

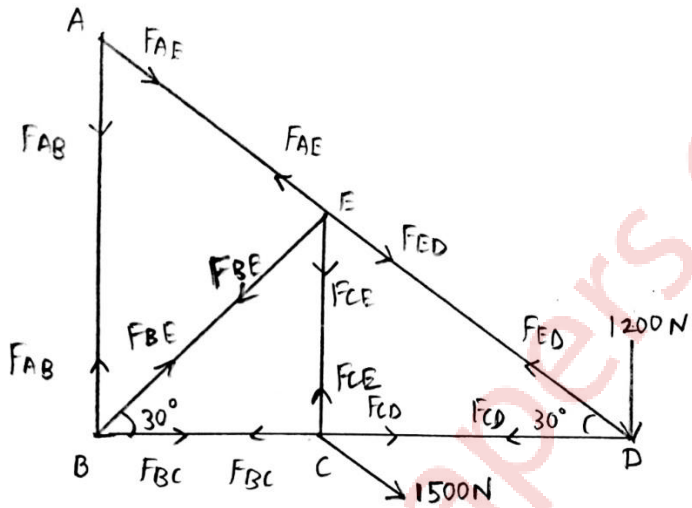
$$\therefore \boxed{F_{BC} = -878.46 \text{ N}}$$

$\therefore$ Put F_{BC} in (1)

$-F_{EA}\cos(30°)-(-878.46)-F_{BE}\sin(60°)+1200=0$

$\boxed{\begin{array}{l}F_{EA}=3300\text{ N}\\F_{BE}=-900\text{ N}\end{array}}$

By method of joints,



At point D

$\sum F_Y=0$

$\therefore -1200+F_{ED}\sin(30°)=0$

$\therefore \boxed{F_{ED}=2400\text{ N}}$

$\sum F_X=0$

$\therefore -F_{CD}-F_{ED}\cos(30°)=0$

$\therefore \boxed{F_{CD}=-2078.46\text{ N}}$

At point C

$\sum F_X=0$

$\therefore F_{CD}-F_{BC}+1200=0$

$\therefore -2078.46-F_{BC}+1200=0$

$\therefore \boxed{F_{BC}=-878.46\text{ N}}$

$\sum F_Y=0$

$\therefore -900+F_{CE}=0$

$\therefore \boxed{F_{CE}=900\text{ N}}$

At point E

$$\sum F_X = 0$$

$$\therefore -F_{AE}\cos(30°) - F_{BE}\sin(60°) + F_{ED}\cos(30°) = 0$$

$$\therefore -F_{AE}\cos(30°) - F_{BE}\sin(60°) + 2400\cos(30°) = 0$$

$$\sum F_Y = 0$$

$$\therefore F_{AE}\sin(30°) - F_{BE}\cos(60°) - F_{CE} - F_{ED}\sin(30°) = 0$$

$$\therefore F_{AE}\sin(30°) - F_{BE}\cos(60°) - 900 - 2400\sin(30°) = 0$$

$$\boxed{\begin{array}{l} F_{AE} = 3300\text{ N} \\ F_{BE} = 900\text{ N} \end{array}}$$

At point A

$$\sum F_Y = 0$$

$$\therefore -F_{AB} - F_{AE}\cos(60°) = 0$$

$$\therefore F_{AB} = -3300\cos(60°)$$

$$\therefore \boxed{F_{AB} = -1650\text{ N}}$$

Member	**Force Magnitude (in N)**	**Nature of Force**
AB	1650	Compressive
BE	900	Tensile
BC	878.46	Compressive
AE	3300	Tensile
CE	900	Tensile
ED	2400	Tensile
CD	2078.46	Compressive

(b) A particle moves in x-y plane and it's is given by $r=(3t)i+(4t-3t^2)j$, where r is the position vector of particle in metres at time t sec. Find the radius of curvature of the path and normal and tangential components of acceleration when it crosses X-axis region. (06 marks)

Solution:

$$\bar{r}=\left[(3t)i+(4t-3t^2)j\right] \text{ m}$$

When it crosses the x axis, the y coordinate is 0

$$\therefore (4t-3t^2)=0$$

$$\therefore t=0s \text{ or } t=\frac{4}{3}s$$

$$\bar{v}=\frac{d\bar{r}}{dt}=\frac{d}{dt}\left[(3t)i+(4t-3t^2)j\right]$$

$$\therefore \bar{v}=\left[3i+(4-6t)j\right] \text{ m/s}$$

$$\therefore \text{At t=}\frac{4}{3}s,$$

$$\bar{v}=\left[3i-4j\right] \text{ m/s=}5\angle-53.13^\circ \text{ m/s}$$

$$\bar{a}=\frac{d\bar{v}}{dt}=\frac{d}{dt}\left[3i+(4-6t)j\right] \text{ m/s}^2$$

$$\therefore \bar{a}=-6j \text{ m/s}^2$$

$$\text{Radius of curvature}(\rho)=\frac{v^3}{\left|v_x a_y - v_y a_x\right|} \text{ m}$$

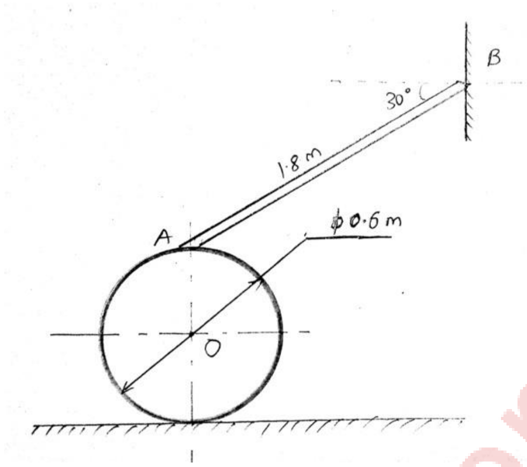
$$\therefore \rho=\frac{5^3}{\left|(3)(-6)-(-4)(0)\right|}=\frac{125}{\left|-18\right|}=6.94 \text{ m}$$

$$\text{a}_N=\frac{v^2}{\rho}=\frac{5^2}{6.94}=\frac{25}{6.94}=3.6 \text{ m/s}^2$$

$$\text{a}_T=\sqrt{a^2-a_N{}^2}=\sqrt{6^2-3.6^2}=4.8 \text{ m/s}^2$$

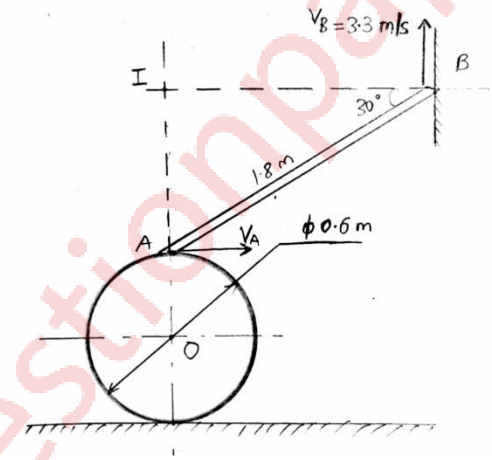
$$\therefore \boxed{\begin{array}{l}\text{Radius of curvature=6.94 m and}\\ \text{Normal component of acceleration=3.6 m/s}^2\text{ and}\\ \text{Tangential component of acceleration=4.8 m/s}^2\end{array}}$$

(c) C is a uniform cylinder to which a rod AB is pinned at A and the other end of the rod is moving along a vertical wall as shown in fig. If the end B of the rod is moving upwards along the wall with a speed of 3.3 m/s find the angular velocity of wheel and rod assuming that cylinder is rolling without slipping. (06 marks)



Solution:

Locating the ICR,



$\text{IB}=1.8\cos\left(30°\right)=1.56 \text{ m and IA}=1.8\sin\left(30°\right)=0.9 \text{ m}$

$V_B = 3.3 \text{ m/s}$

$W_I = \dfrac{V_B}{IB} = \dfrac{3.3}{1.56} = 2.12 \text{ r/s}$

$V_A = W_I \times (IA) = (2.12)(0.9)$

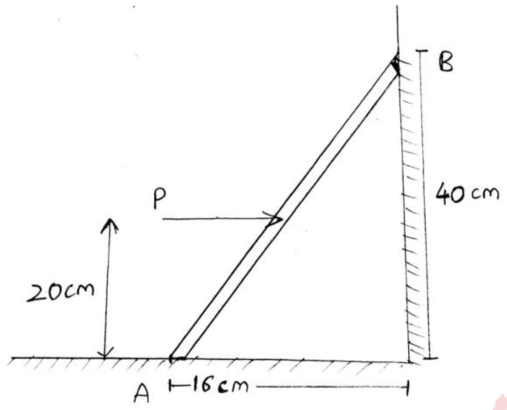
$\therefore V_A = 1.91 \text{ m/s}$

$\therefore W = \dfrac{V_A}{R} = \dfrac{1.91}{0.3} = 6.37 \text{ r/s}$

$\therefore$ The angular velocity of wheel =6.37 r/s and the velocity of the rod =1.91 m/s

Q6.Attempt:

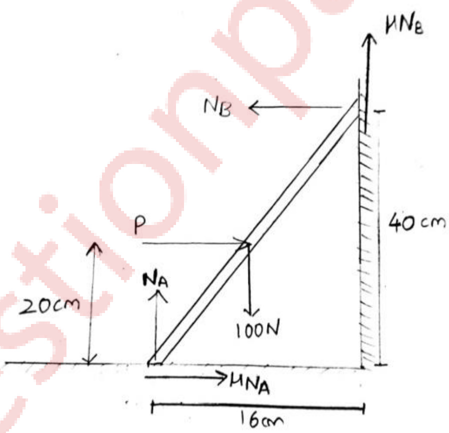
(a) A 100 N uniform rod AB is held in position as shown. If $\mu = 0.15$ at A and B calculate range of value of P for which equilibrium is maintained. (08 marks)



Solution:

Let N_A and N_B be the normal reactions at A and B respectively

For minimum value of P, FBD is,



For equilibrium,

$$\sum F_x = 0$$

$$\therefore \mu N_A - N_B + P = 0$$

$$\therefore 0.15 N_A - N_B + P = 0 \quad ...(1)$$

$$\sum F_y = 0$$

$$\therefore N_A + \mu N_B - 100 = 0$$

$$\therefore N_A + 0.15 N_B + 0P = 100 \quad ...(2)$$

$$\sum M_A = 0$$

$$\therefore P\left(\frac{20}{100}\right) + 100\left(\frac{8}{100}\right) - N_B\left(\frac{40}{100}\right) - \mu N_B\left(\frac{16}{100}\right) = 0$$

$\therefore 0N_A - 42.4N_B + 20P = -800 \quad ...(3)$

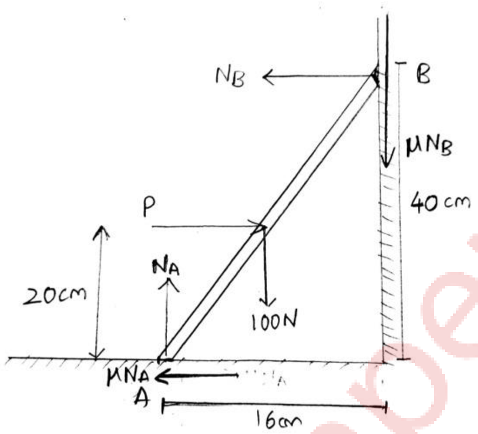
From (1), (2) and (3),

$N_A = 96.58$ N

$N_A = 22.78$ N

P=8.29 N

For maximum value of P, FBD is,



For equilibrium,

$\sum F_x = 0$

$\therefore -\mu N_A - N_B + P = 0$

$\therefore -0.15N_A - N_B + P = 0 \quad ...(4)$

$\sum F_y = 0$

$\therefore N_A - \mu N_B - 100 = 0$

$\therefore N_A - 0.15N_B + 0P = 100 \quad ...(5)$

$\sum M_A = 0$

$\therefore P\left(\frac{20}{100}\right) + 100\left(\frac{8}{100}\right) - N_B\left(\frac{40}{100}\right) + \mu N_B\left(\frac{16}{100}\right) = 0$

$\therefore 0N_A - 37.6N_B + 20P = -800 \quad ...(6)$

From (4), (5) and (6),

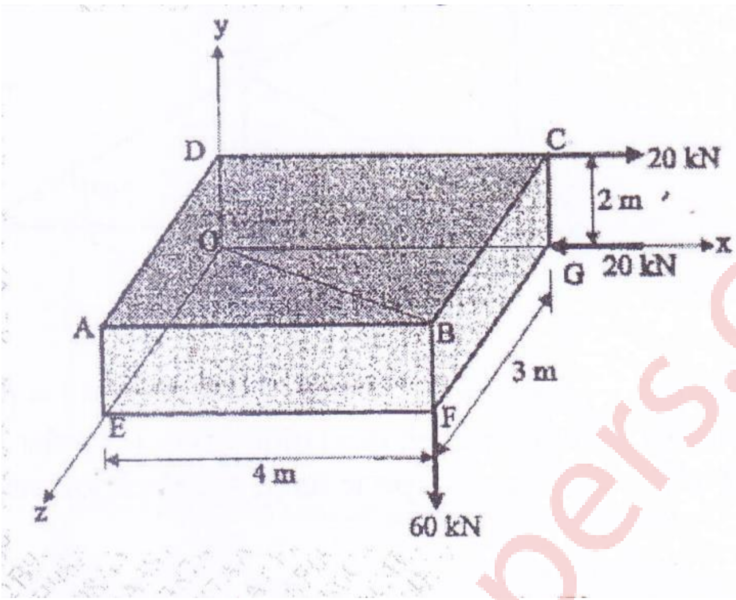
$N_A = 109.62$ N

$N_A = 64.14$ N

P=80.58 N

$\therefore$ $\boxed{\text{The range of value of P for which equilibrium is maintained is from 8.29 N to 80.58 N}}$

(b) A box of size $3\times4\times2$ m is subjected to three forces as shown in fig. Find in vector form the sum of moments of the three forces about diagonal OB. (06 marks)



Solution:

The three forces are given as

$$\overline{F_{DC}} = 20\hat{i}\ \text{kN}$$
$$\overline{F_{GO}} = -20\hat{i}\ \text{kN}$$
$$\overline{F_{BF}} = -60\hat{j}\ \text{kN}$$

The unit vector along the direction OB is

$$\hat{OB} = \frac{(4-0)\hat{i} + (2-0)\hat{j} + (3-0)\hat{k}}{\sqrt{4^2+2^2+3^2}}$$
$$= \frac{4\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{29}}$$

The vector moment of force F_{DC} along OB is

$$\overline{M} = \left[\overline{OC}\quad F_{DC}\quad \hat{OB}\right]\hat{OB}$$

$$\overline{M} = \begin{vmatrix} 4 & 2 & 0 \\ 20 & 0 & 0 \\ \frac{4}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{3}{\sqrt{29}} \end{vmatrix}\hat{OB}$$

$$\therefore \overline{M} = -20\left(\frac{6}{\sqrt{29}}\right)\overset{\wedge}{OB}$$

$$\overline{M} = \frac{-120}{\sqrt{29}}\left[\frac{4\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{29}}\right]$$

$$\therefore \boxed{\overline{M} = -16.55\hat{i} - 8.28\hat{j} - 12.41\hat{k}}$$

The vector moment of force F_{GO} along OB is

$$\overline{M} = \left[\overline{OG} \quad F_{GO} \quad \overset{\wedge}{OB}\right]\overset{\wedge}{OB}$$

$$\overline{M} = \begin{vmatrix} 4 & 0 & 0 \\ -20 & 0 & 0 \\ \frac{4}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{3}{\sqrt{29}} \end{vmatrix}\overset{\wedge}{OB}$$

$$\therefore \overline{M} = 20\left(\frac{6}{\sqrt{29}}\right)\overset{\wedge}{OB}$$

$$\overline{M} = \frac{120}{\sqrt{29}}\left[\frac{4\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{29}}\right]$$

$$\therefore \boxed{\overline{M} = 16.55\hat{i} + 8.28\hat{j} + 12.41\hat{k}}$$

The vector moment of force F_{BF} along OB is

$$\overline{M} = \left[\overline{OB} \quad F_{BF} \quad \overset{\wedge}{OB}\right]\overset{\wedge}{OB}$$

$$\overline{M} = \begin{vmatrix} 4 & 2 & 3 \\ -20 & 0 & 0 \\ \frac{4}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{3}{\sqrt{29}} \end{vmatrix}\overset{\wedge}{OB}$$

$$\therefore \overline{M} = 0\overset{\wedge}{OB} \qquad \text{....}\{\text{Since 2 rows of the matrix are equal}\}$$

$$\therefore \boxed{\overline{M} = 0\hat{i} + 0\hat{j} + 0\hat{k}}$$

(c) Two blocks A and B are separated by 10 m as shown in fig on $20°$ incline plane. If the blocks start moving, find the time t when the blocks collide and distance travelled by each block. Assume $\mu_k = 0.3$ for block A and block A and plane and $\mu_k = 0.10$ for block B and plane. **(06 marks)**

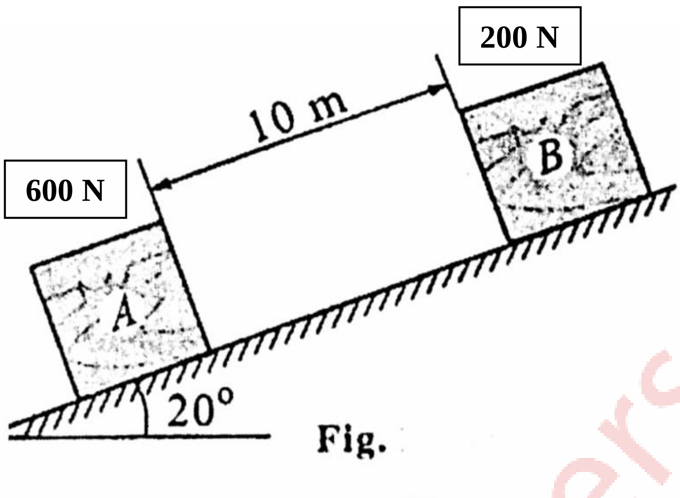
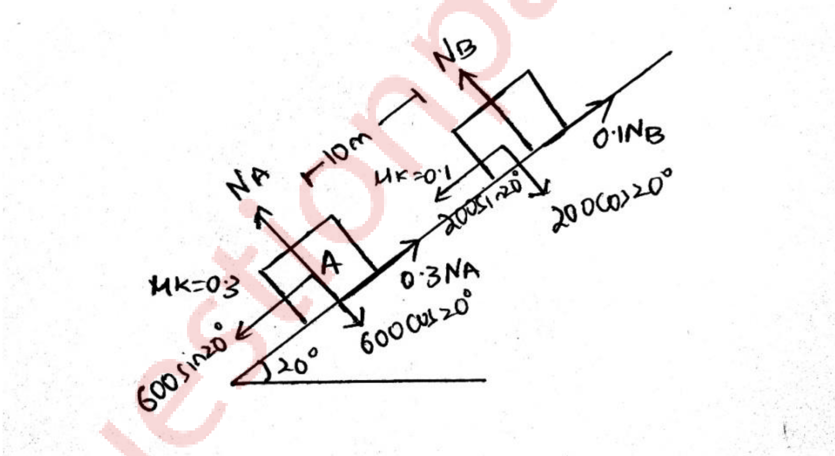


Fig.

Solution:

The FBD is:



Let the time taken by the blocks to meet be t seconds

Let the distance travelled by block A be x m

$\therefore x = 0 + \frac{1}{2}(a_A)t^2$ .....(1), a_A = Acceleration of block A

Hence, distance travelled by block B is

$x + 10 = 0 + \frac{1}{2}(a_B)t^2$ .....(2), a_B = Acceleration of block B

From the FBD,

On block A,

mass of block A=$m_A = \frac{600}{g} = \frac{600}{9.8} = 61.22$ kg

$600\cos(20°) = (\mathrm{N_A})$

$\therefore \mathrm{N_A} = 563.82$ N

$600\sin(20°) - 0.3\mathrm{N_A} = m_\mathrm{A} a_A$

$\therefore \ a_A = 0.589 \text{ m/s}^2$

On block B,

mass of block B=$m_B = \frac{200}{g} = \frac{200}{9.8} = 20.41$ kg

$200\cos(20°) = (\mathrm{N_B})$

$\therefore \mathrm{N_B} = 187.94$ N

$200\sin(20°) - 0.1\mathrm{N_B} = m_\mathrm{B} a_B$

$\therefore \ a_B = 2.43 \text{ m/s}^2$

By putting the values of a_A and a_B in equations (1) and (2),

$x = \frac{1}{2}(0.589)t^2$ .....(3)   and $x + 10 = \frac{1}{2}(2.43)t^2$ .....(4)

Dividing equation (3) by (4),

$$\frac{x}{x+10} = \frac{\frac{1}{2}(0.589)t^2}{\frac{1}{2}(2.43)t^2}$$

$\therefore x = 3.2$ m

From (3)

$$t=\sqrt{\frac{2(3.2)}{0.589}} = 3.3 \text{ s}$$

The blocks collide after time=3.3 seconds and the distance travelled by block A is 3.2 m and that by block B is (3.2+10) m=13.2 m.