BEE Solutions May-2019

Q1.

(a) Explain the working principle of Single Phase Transformer. (4)

Ans: When an alternating voltage $V1$ is applied to a primary winding, an alternating current $I1$ flows in it producing an alternating flux in the core. As per Faraday's laws of electromagnetic induction, an emf $e1$ is induced in the primary winding.

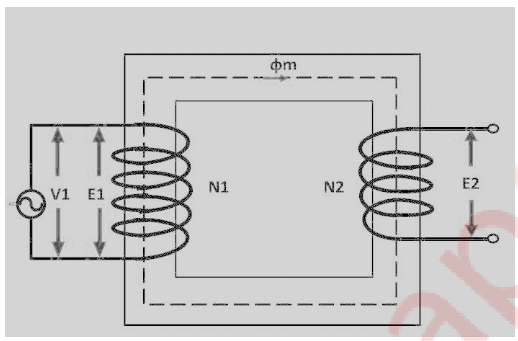


Fig.(a)

$e1 = -N1 \dfrac{d\varphi}{dt}$

Where $N1$ is the number of turns in the primary winding. The induced emf in the primary winding is nearly equal and opposite to the applied voltage $V1$.

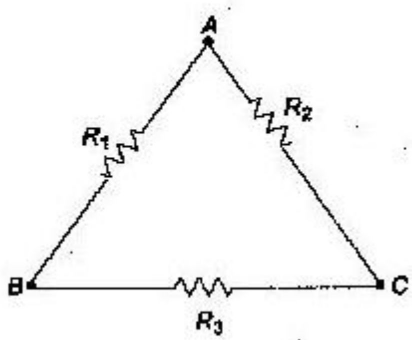
Assuming leakage flux to be negligible, almost the flux produced in primary winding links with the secondary winding. Hence, an emf $e2$ is induced in the secondary winding.

$e2 = -N2 \dfrac{d\varphi}{dt}$

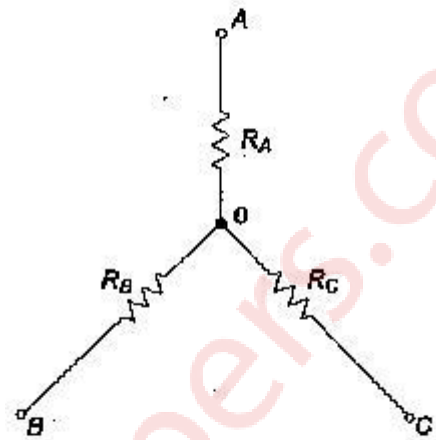
Where $N2$ is the number of turns in the secondary winding. If the secondary circuit is closed through the load, a current $I2$ flows in the secondary winding. Thus energy is transferred from the primary winding to the secondary winding.

(b) Derive the formulas to convert a Star circuit into equivalent Delta. (4)

Ans:



Fig(a)



Fig(b)

From the delta network shown,

The resistance between terminal 1 and 2 = $RC \mid\mid (RA + RB)$

$$= \frac{RC(RA+RB)}{RA+RB+RC} \quad \text{.................... (1)}$$

From the star network shown above,

The resistance between terminals 1 and 2 = $R1 + R2$ .... (2)

Since the two networks are electrically equivalent.

$$R1 + R2 = \frac{RC(RA+RB)}{RA+RB+RC} \quad \text{.................... (3)}$$

Similarly,

$$R2 + R3 = \frac{RA(RB+RC)}{RA+RB+RC} \quad \text{................ (4)}$$

And

$R1 + R3 = \frac{RB(RA+RC)}{RA+RB+RC}$ ………………. (5)

Subtracting Eq. (4) from Eq. (3)

$R1 - R3 = \frac{(RB*RC-RA*RB)}{RA+RB+RC}$ ……………… (6)

Adding Eq.(6) and Eq. (5)

$R1 = \frac{RB*RC}{RA+RB+RC}$

Similarly, $R2 = \frac{RA*RC}{RA+RB+RC}$

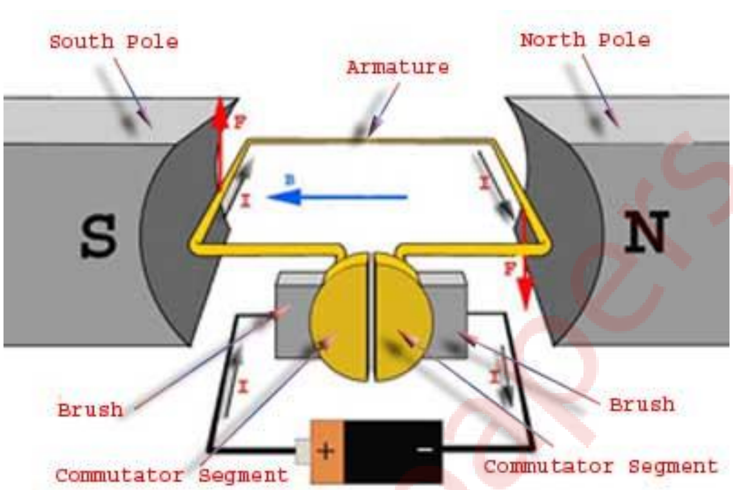
$R3 = \frac{RA*RB}{RA+RB+RC}$

Thus, star resistor connected to a terminal is equal to the product of the two delta resistors connected to the same terminal divided by the sum of the delta resistor.

(c) Explain the principle of operation of DC motor. (4)

Ans:

A machine that converts DC electrical power into mechanical power is known as a Direct Current motor.
DC motor working is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force.



The direction of this force is given by Fleming's left-hand rule and its magnitude is given by

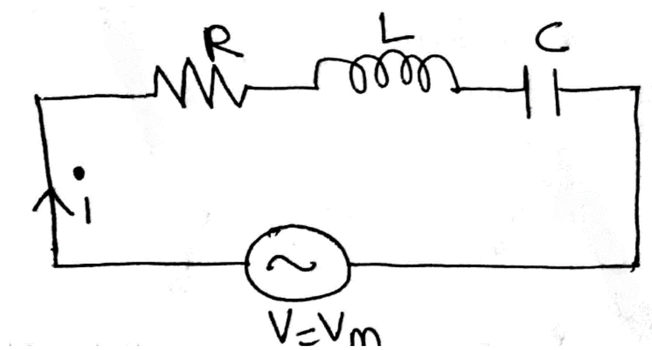
$$F = BIL$$

Where, B = magnetic flux density,
I = current
L = length of the conductor within the magnetic field.

(d) What is the necessary condition for resonance in series circuit? Derive the expression for resonance frequency. (4)

Ans:

A circuit containing reactance is said to be resonance if the voltage across the circuit is in phase with the current through it. At resonance the circuit thus behaves as a pure resistor and et reactance is zero.



Expression for Resonance is,

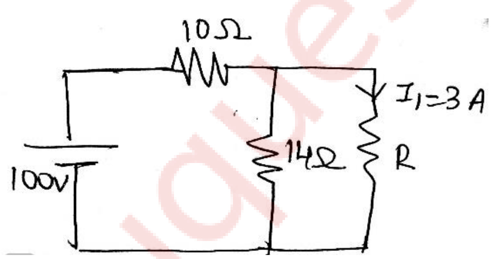
$$Z = R + jX_L - jX_C$$
$$= R + j\omega L - j\frac{1}{\omega C}$$
$$= R + j(\omega L - \frac{1}{\omega C})$$

At resonance,

$$\omega L - \frac{1}{\omega C} = 0$$
$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$
$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Where f_0= resonant frequency

(e) Find the value of R in the following circuit. (4)



Ans:

By KCl,

Assume Voltage across R is V,

$\frac{V-100}{10} + \frac{V}{14} + 3 = 0$

By LCM and shifting,

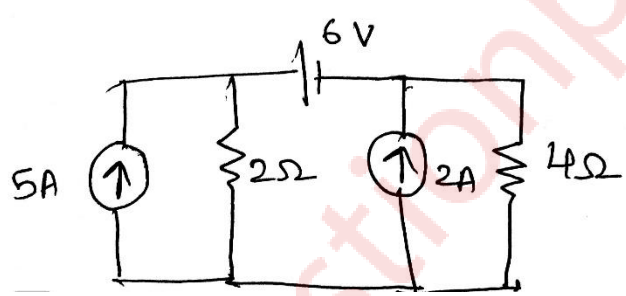
$\frac{6}{35}V = 7$

V = 40.33 v

So value R is

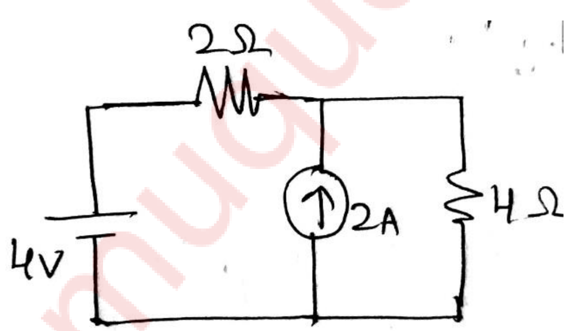
$$R = \frac{40.833}{3} = 13.611\ \Omega$$

(f) Find the current through 4Ω resistor by source transformation in the following circuit. (4)

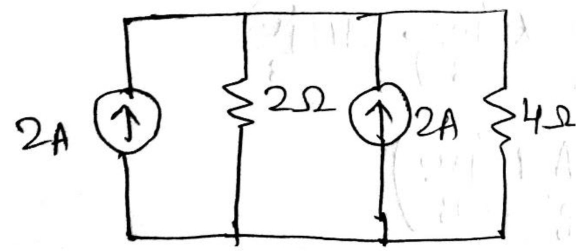


Ans:

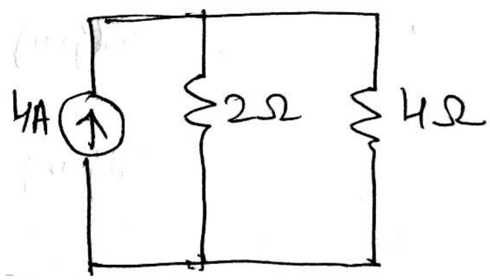
By Source Transformation,



Again trasnforming 4V and 2Ω,



Adding 2A with 2A as they are in parallel,

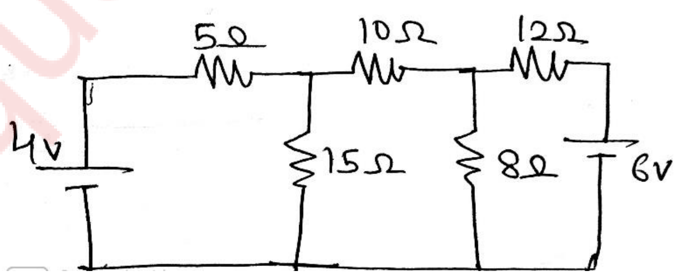


By Current Division Rule,

$I_{4\Omega} = 4\ X\ \frac{2}{2+4} = \frac{4}{3}\ A$

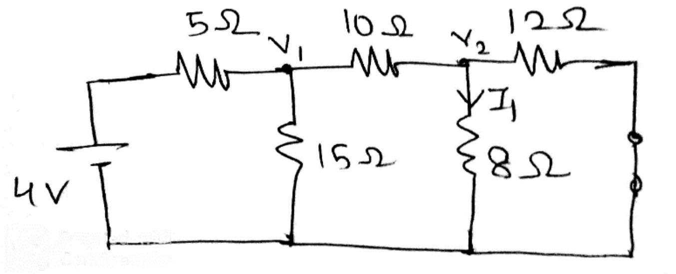
Q.2

(a) Determine the current through 8Ω resistor in the following network by superposition theorem: (8)



Ans:

When 4V is active and 6V is inactive



Applying KCl we get,

$$\frac{V1-4}{5}+\frac{V1-V2}{10}+\frac{V1}{15}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

And

$$\frac{V2-V1}{10}+\frac{V2}{8}+\frac{V2}{12}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

By solving eq (1),

$$\frac{11}{30}V1-\frac{V2}{10}=\frac{4}{5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

By soving eq (2),

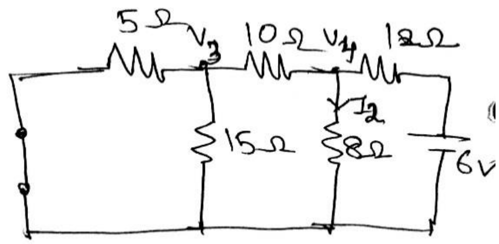
$$-\frac{V1}{10}+\frac{37}{120}V2=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Solving eq (3) and (4) we get,

V1 = 2.4v V2 = 0.7763v

$$I1=\frac{V2}{8}=0.097A$$

Similarly, when 6v is active,



By KCL,

$$\frac{V3}{5}+\frac{V3-V4}{10}+\frac{V3}{15}=0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5)$$

$$\frac{V4-V3}{10}+\frac{V4-6}{12}+\frac{V4}{8}=0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (6)$$

By solving eq (5) ,

$$\frac{11}{30}V3=\frac{1}{10}V4 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (7)$$

By solving eq. (6) ,

$$-\frac{1}{10}V3+\frac{37}{120}V4=\frac{1}{2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (8)$$

Solving (7) and (8), we get

V3 = 0.485v V4 = 1.78v

$$I2=\frac{V4}{8}=\frac{1.78}{8}=0.225A$$

By Superposition Theorem,

$I_{8\Omega}$ = I1 + I2 = 0.097 + 0.225 = 0.322 A

(b) An inductive coil having inductance of 0.04H and resistance 25 Ω has been connected in series with another inductive coil of inductance 0.2H and resistance 15 Ω. The whole circuit is powered with 230V, 50Hz mains. Calculate the power dissipation in each coil and total power factor. (8)

Ans:

Coil 1 : $R_1 = 25\Omega$

$X_{L1} = 2\pi fL = 2 \times \pi \times 50 \times 0.04 = 12.566\ \Omega$

$Z_1 = \sqrt{R_1^2 + X_{L1}^2} = 27.98\ \Omega$

$\cos\phi_1 = \frac{R1}{Z1} = \frac{25}{27.98} = 0.8935$

$V1 = 230 \times \frac{Z1}{Z1+Z2} = 230\ x\ \frac{27.98}{27.98+64.6} = 69.511V$

Power Dissipated = $V_1 I \cos\phi_1 = \frac{V1xV1}{Z1}\ x\ cos\phi1 = 152.3\ W$

Coil 2 : $R_2 = 15\Omega$

$X_{L2} = 2\pi fL = 2 \times \pi \times 50 \times 0.2 = 62.83\ \Omega$

$Z_2 = \sqrt{R_2^2 + X_{L2}^2} = 64.6\ \Omega$

$\cos\phi_2 = \frac{R2}{Z2} = \frac{15}{62.83} = 0.2387$

$V2 = 230 \times \frac{Z2}{Z1+Z2} = 230 \times \frac{64.6}{27.98+64.6} = 160.5V$

Power Dissipated = $V_2 I \cos\phi_2 = \frac{V2\ x\ V2}{Z2}\ x\ cos\phi2 = 95.2W$

And Total PF,

Req = R1 + R2 = 25 + 15 = 40 Ω

XLeq = XL1 + XL2 = 12.566 + 62.83 = 75.4 Ω

$Z = \sqrt{R_{eq}^2 + X_{Leq}^2} = 85.35\ \Omega$

Power Factor(total) = $\cos\phi_{tot} = \frac{Req}{Zeq} = \frac{40}{85.35} = 0.468$

(c) What are the losses in transformer? Explain why the ratings of transformer in KVA not in KW? (4)

Ans: There are two types of losses in a transformer:

1. Iron or core loss
2. Copper loss

IRON LOSS:

This loss is due to the reversal of flux in the core. The flux set-up in the core is nearly constant. Hence, iron loss is practically constant at all the loads, from no load to full load. The losses occurring under no-load condition are the iron losses because the copper losses in the primary winding due to no-load current are negligible. Iron losses can be subdivided into two losses:

1. Hysteresis loss
2. Eddy current loss

COPPER LOSS: This loss is due to the resistance of primary and secondary windings

$Wcu = I_1^2 R1 + I_2^2 R2$

Where, $R1$ = primary winding resistance

$R2$ = secondary winding resistance

Copper loss depends upon the load on the transformer and is proportional to square of load current of kVA rating of the transformer.
So this is why, the ratings of transformer is in KVA and not in KW.

Q.3

(a) With necessary diagrams prove that three phase power can be measured by only two wattmeter. Also prove that reactive power can be measured from the wattmeter reading. (10)

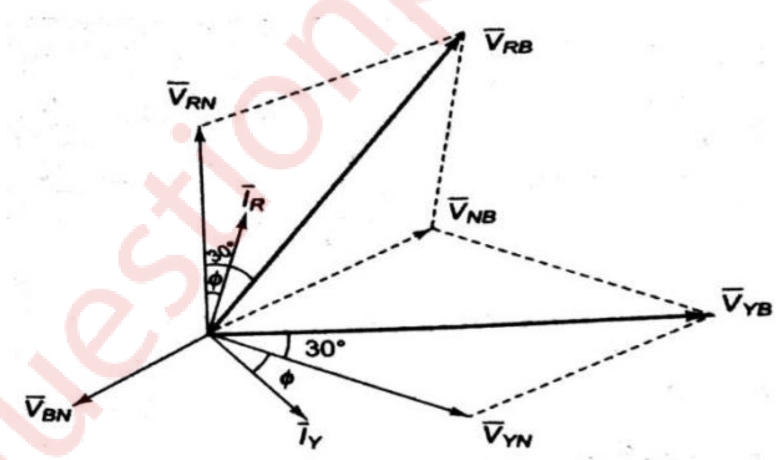
Ans:

Given figure shows a balanced star-connected load, the load may be assumed to be inductive. Let V_{RN}, V_{YN}, V_{BN} be the three phase voltages. I_R, I_Y, I_B be the phase currents. The phase currents will lag behind their respective phase voltages by angle φ. Current through current coil of $W1$ = I_R.

Voltages across voltage coil of $W1$ = V_{RB} = V_{RN} + V_{NB} = V_{RN} – V_{BN}

From the phasor diagram, it is clear that the phase angle between V_{RB} and I_R is (30° − φ).

$W1$ = V_{RB} $I_R$cos(30° − φ) Current through current coil of $W2$ = I_Y Voltage across voltage coil of $W2$ = V_{YB} = V_{YN} + V_{NB} = V_{YN} – V_{BN}



From phasor diagram, it is clear that phase angle between V_{YB} and I_Y is (30° + φ)

$W2$ = V_{YB} I_Y cos(30° + φ)

But I_R, = I_Y = I_L

V_{RB} = V_{YB} = V_L

$W1 = V_L I_L \cos(30° - \varphi)$

$W2 = V_L I_L \cos(30° + \varphi)$

$W1 + W2 = V_L I_L \cos(30° - \varphi) + V_L I_L \cos(30° + \varphi)$

$W1 + W2 = V_L I_L (2cos30°cos\varphi)$

P(active power) = $W1 + W2 = \sqrt{3}\, V_L I_L (cos\varphi)$

Thus the sum of two wattmeter reading gives three phase power

For calculating reactive power :-

$W1 - W2 = V_L I_L \cos(30° - \varphi) - V_L I_L \cos(30° + \varphi)$

$W1 - W2 = V_L I_L [-2sin\ [\frac{30-\varphi+30+\varphi}{2}]\ sin\ [\frac{30-\varphi-30-\varphi}{2}]]$

$W1 - W2 = V_L I_L [-2 \sin(30) \sin(-\varphi)]$

$W1 - W2 = V_L I_L (\sin\varphi)$

Q (reactive power) = $W1 - W2 = V_L I_L (\sin\varphi)$

(b) An alternating voltage is represented by v(t)=141.4sin(377t) V. Derive the RMS value of the voltage.

Find:

1) **Instantaneous voltage value at t=3ms**
2) **The time taken for voltage to reach 70.7 V for first time (10)**

Ans:

To calculate RMS value of this voltage

V(t) = 141.4sin(377t)

V = $V_m \, sin\theta$ for $0 < \theta < 2\pi$

Vm = 141.4 for $\theta = 377t$

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta d\theta = \frac{1}{2\pi}\int_0^{2\pi} 141.4^2 \sin^2\theta d\theta = \frac{141.42}{2\pi}\int_0^{2\pi} \sin^2\theta d\theta$$

$$\frac{141.42}{2\pi}\int_0^{2\pi} \frac{1-cos\theta}{2} d\theta = \frac{141.42}{2\pi}\left(\frac{\theta}{2} - \frac{sin2\theta}{2}\right)^{2\pi}{}_0 = \frac{141.42}{2\pi}\left[\frac{2\pi}{2}\right] = 9996.98$$

$Vrms$ = √9996.98 = 99.98

Vrms = 99. 98V

1) Instantaneous value at t = 3ms

   t = 3 × 10^{-3} = 0.003sec
   V = $V_{rms} \, sin\theta$
   V = 141.4sin(337 x 0.003)
   V = 2.4949V

   Instantaneous voltage at t = 3ms is v = 2.494V

2) Time taken to reach till 70.7V for first time

   V = $V_{rms} \, sin\theta$
   V = 70.7V

   70.7 = 141.4sin(377t)
   0.5 = sin(337t)
   Sin^{-1} (0.5) = 337t
   30 = 337t
   t = 0.089sec.

   Time required to reach till 70.7V is 0.089sec

Q.4

(a)State and Prove Maximum Power Transfer Theorem. (8)

Ans:

It states that “The maximum power is delivered from a source to a load when the load resistance is equal to the source resistance”.

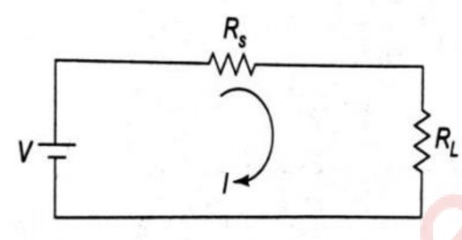


Fig: (a)

The maximum power will be transferred to the load when load resistance is equal to the source resistance.

Proof:

From the fig.(a)

$$I = \frac{V}{R_S + RL}$$

Power delivered to the load R_L = $P = I^2 R_L = \left(\frac{V}{(R_S + R_L)}\right)^2 R_L$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dRl} = 0$$

$$\frac{dP}{dRl} = \frac{d}{dRl}\left(\frac{V}{(R_S + R_L)}\right)^2 R_L$$

$$= \frac{V^2[(R_S + R_L)^2 - (2R_L)(R_S + R_L)]}{(R_S + R_L)^4}$$

$(R_S + R_L)^2 - 2R_L(R_S + R_L) = 0$

$R_S^2 + R_L^2 + 2R_L R_S - 2R_L R_S - 2R_L^2 = 0$

$R_S = R_L$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Hence Proved.

(b)

A 5KVA 1000/200V, 50 Hz Single phase transformer gave the following test result

OC TEST(hv side): 1000V 0.24A 90 W

SC TEST(hv side) : 50V 5 A 110 W

Calculate :

1. **Equivalent circuit for transformer with circuit constant**
2. **Regulation at full load at 0.8 lagging**
3. **kVA load for maximum efficiency** **(12)**

Ans:

1) Equivalent circuit of the transform and parameters

From OC test(meters are connected on LV side i.e. primary)

Wi = 90w $V1$ = 1000V I_0 = 0.24Am

$\text{Cos}\varphi_0 = \frac{\text{Wi}}{V_1 \ I_0} = \frac{90}{1000 \times 0.24} = 0.38$

$\sin\varphi 0_0 = (1 - 0.38^2)^{0.5} = 0.732$

$Iw = I_0 \cos\varphi_0 = 0.24 \times 0.38 = 0.0912A$

$RO = \frac{V1}{Iw} = \frac{1000}{0.0912} = 10.96K\Omega$

$I\mu = I o sin\varphi o = 0.24 \times 0.732 = 0.176Am$

$Xo = \frac{V1}{I\mu} = \frac{1000}{0.176} = 5.682K\Omega$

From SC test (meters are connected on HV side i.e. secondary)

Wsc = 110w Vsc = 50v Isc = 5A

$Z_{02} = \frac{\text{Vsc}}{\text{Isc}} = \frac{50}{5} = 10\Omega$

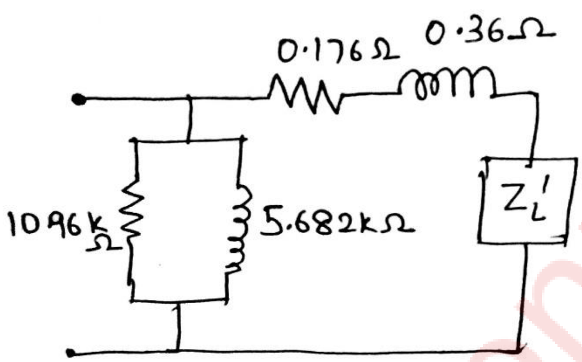
$R_{02} = \frac{\text{Wsc}}{\text{Isc}^2} = \frac{110}{25} = 4.4\Omega$

$X_{02} = (Z_{02}{}^2 - R_{02}{}^2)^{0.5} = (100 - 19.36)^{0.5} = 8.98\Omega$

$K = \frac{1000}{200} = 5$

$R_{01} = \frac{R_{02}}{K \times k} = 0.176\ \Omega$

$X_{01} = \frac{X02}{K^2} = 0.36\Omega$



2) <u>Efficiency at rated load and 0.8 pf lagging</u>

Wi = 90w = 0.09kw

Since meters are connected on secondary in SC test,

$I2$ = (5×1000)/400 = 12.5A

$WCu = {}_{I22}\ R_{02} = 12.5^2 \times 4.4 = 687.5W = 0.687kW$

x = 1 pf = 0.8

$$\%\eta = \frac{\text{x} \times \text{full load KVA} \times \text{pf}}{(\text{x} \times \text{full load KVA} \times \text{pf}) + \text{Wi} + \text{x2 Wcu}} \times 100$$

$$\%\eta = \frac{1 \times 5 \times 0.8}{1 \times 5 \times 0.8 + 0.09 + 1 \times 0.687} \times 100$$

$\%\eta = 83.73\%$

3) <u>Regulation at full load at 0.8 lagging is</u>

$$\%regulation = \frac{I_2(R_{02}\cos\phi + X_{02}\sin\phi)}{E_2} \times 100$$

$$= \frac{12.5(4.4\times0.8+8.98\times0.38)}{400} \times 100 = 21.66\ \%$$

Q.5

(a) Three similar coils having a resistance of 10Ω and inductance of 0.04H are connected in star across 3-phase 50Hz, 200v supply. Calculate the line current, total power absorbed, reactive volt amperes and total volt amperes. (8)

Ans:

R= 10Ω
L= 0.04H
V_L = 200V
F= 50Hz

For a star connected load,

$$V_{ph} = \frac{VL}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115.47$$

$X_L = 2\pi fL = 2\pi x50x0.04 = 12.6\ \Omega$

$Z_{ph} = R + jX_L$
$= 10+j12.6$
$= 16.08 < 51.56^o\ \Omega$

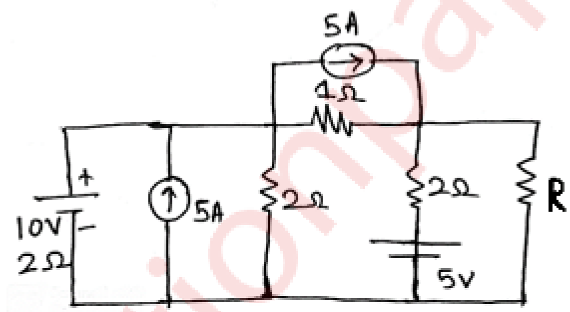
Power Factor = cos(51.56) = 0.273(lagging)

$I_{ph} = \frac{Vph}{Zph} = \frac{115.47}{16.08}$ = 7.2A

$I_L = I_{ph}$ = 7.2A

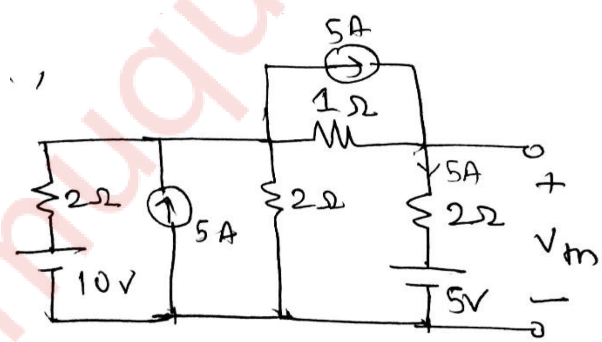
P = $\sqrt{3}V_LI_L\cos\phi$ = 1.73 x 200 x 7.2 x 0.273 = 680 W
Q = $\sqrt{3}V_LI_L\sin\phi$ = 1.73 x 200 x 7.2 x sin(51.56) = 2.4 KVAR
S = $\sqrt{3}U_L I_L$ = 1.73 x 200 x 7.2 = 2.5KVA

(b) In the following circuit find R for maximum power delivered to it. Also find maximum power delivered Pmax. (8)



Ans:

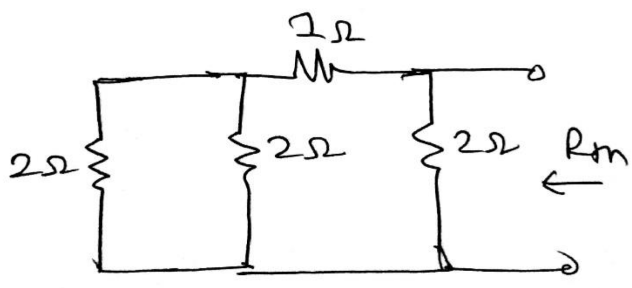
Calculate Vth ,



As 5A current of flowing through 2Ω branch, by applying KVL at outer loop, Vth – 5x2 – 5 = 0

Vth = 15V

Now to Calculate Rth, Short all voltage sources and open all current sources,



Solving above, we get

Rth = 1Ω

Now according to Maximum Power Transfer Theorem,

Rth = R

Thus, R=1Ω

And maximum power transferred is,

$$Pmax = \frac{Vth^2}{4 \times Rth} = \frac{15 \, x \, 15}{4 \, x \, 1} = \frac{255}{4} W$$

(c) Two impedance 12 + j16 Ω and 10-j20 Ω are connected in parallel across 230V, 50Hz. Single phase ac supply. Find kW, kVA and kVAR and power factor. (4)

Ans:

Z1 = 12 + j16 Ω Z2 = 10-j20 Ω

Admittance in each branch ,

$$Y1 = \frac{1}{Z1} = \frac{1}{12 + j16\,\Omega} = 0.03 - j\,0.04 = 0.05\angle - 53.13°$$

$Y2 = \frac{1}{Z2} = \frac{1}{10 - j20\,\Omega} = 0.02 + 0.04j = 0.045\angle\ 63.43°$

Current in each branch,

$I_1 = V \times Y1 = 200 \times Y1 = 10\angle -\ 53.13°$

$I_2 = V \times Y2 = 200 \times Y2 = 9\angle\ 63.43°$

Power Factor,

$\cos\varphi 1 = \cos(-53.13°) = 0.6$

$\cos\varphi 2 = \cos(63.43°) = 0.45$

Active Power = $P_1 = V \times I_1 = 200 \times 10 = 2KW$
$P_2 = V \times I_2 = 200 \times 9 = 1.8KW$

Reactive Power = $Q_1 = V \times I_1 \times \cos\varphi 1 = 200 \times 10 \times 0.6 = 1.2KVAR$
$Q_2 = V \times I_2 \times \cos\varphi 2 = 200 \times 9 \times 0.45 = 0.81KVAR$

Apparent Power = $S_1 = \sqrt{(P_1^2 + Q_1^2)} = 2.33$ KVA
$S_2 = \sqrt{(P_2^2 + Q_2^2)} = 1.97$ KVA

Q.6

(a)Draw and explain the phasor diagram for the practical transformer connected to lagging power factor. (6)

Ans :

When the transformer secondary is connected to an inductive load, the current flowing in the secondary winding is lagging w.r.t secondary terminal voltage. Let us assume that the current is lagging by an angle of θ_2.

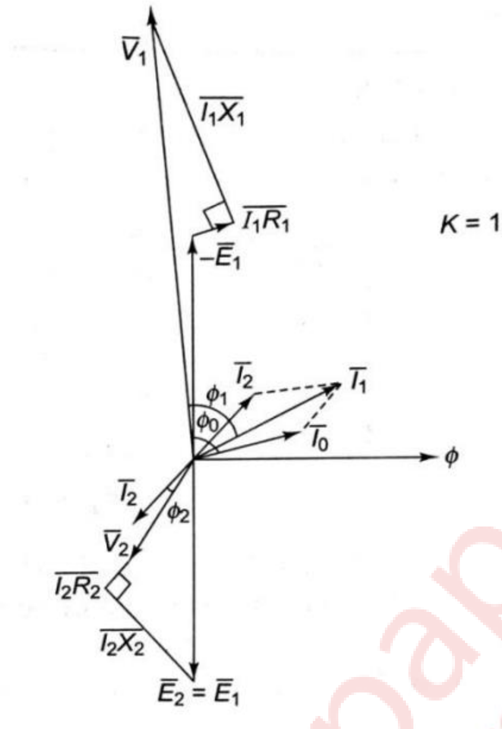
Let, R_1 = Primary winding Resistance

X_1 = Primary winding leakage Reactance

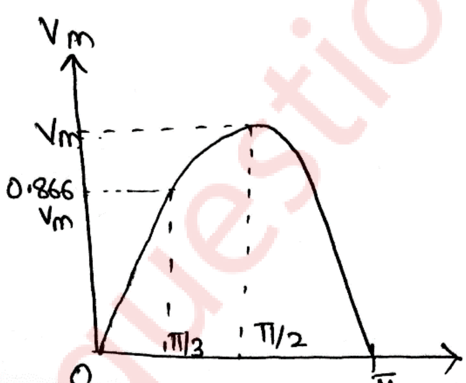
R_2 = Secondary winding Resistance

X_2 = Secondary winding leakage Reactance

The Phasor Diagram is as follows :



(b) Find 1) average value 2) rms value



(10)

Ans:

1) Average Value :

$$V_{DC} = \frac{Vm}{2\pi}\int_0^{\pi} \sin\omega t \, d\omega t$$

$= \frac{Vm}{2\pi}\ [-\cos\omega t]_0^{\pi}$

$= \frac{Vm}{2\pi}\ [-\cos\pi + \cos 0]$

$= \frac{Vm}{2\pi}\ [1+1]$

$= 2\ x\ \frac{Vm}{2\pi}$

$= \frac{Vm}{\pi}$

2) RMS Value

The RMS voltage is

$V_{RMS} = \sqrt{\frac{V_m^2}{2\pi}} \int_0^{\pi} \sin 2\omega t \, d\omega t$

$= \sqrt{\frac{V_m^2}{2\pi}} \int_0^{\pi} \frac{(1 - \cos 2\omega t)}{2}\ )\ d\omega t$

$= \sqrt{\frac{V_m^2}{4\pi}} [\omega t - \frac{\sin 2\omega t}{2}]_0^{\pi}$

$= \sqrt{\frac{V_m^2}{4\pi}} [\ \pi - \frac{(\sin\pi)}{2}\ - (0 - \frac{(\sin 0)}{2})]$

$= \sqrt{\frac{V_m^2}{4\pi}}\ x\ (\ \pi\ )$

$= \sqrt{\frac{V_m^2}{4}}$

$= \frac{Vm}{2}$

(c) State and explain Thevenins theorem and Nortons theorem. (4)

Ans:

Thevenin's Theorem states that "Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance or impedance connected across the load".

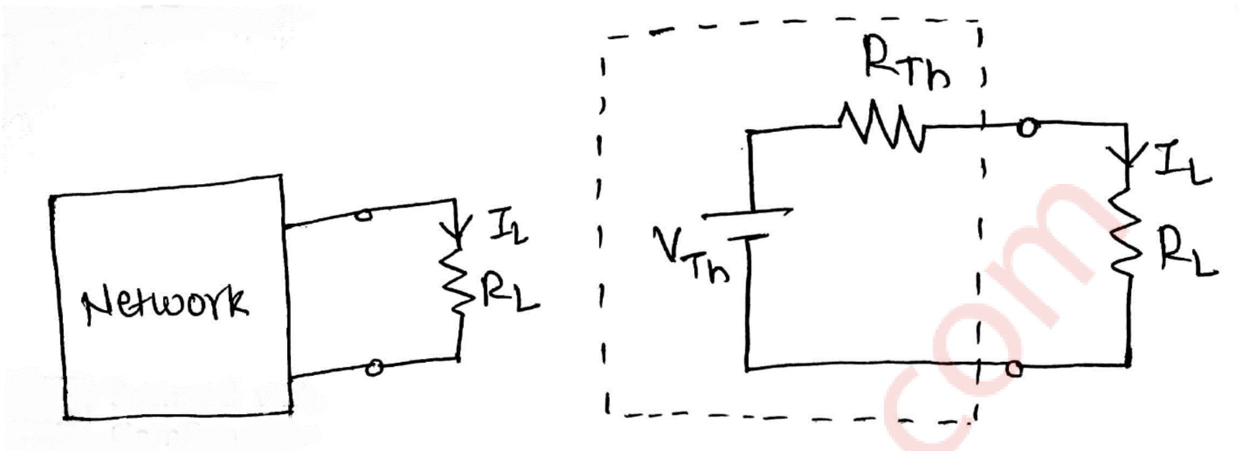


Fig (b) Thevenins Equivalent

Norton's Theorem states that – A linear active network consisting of independent or dependent voltage source and current sources and the various circuit elements can be substituted by an equivalent circuit consisting of a current source in parallel with a resistance. The current source being the short-circuited current across the load terminal and the resistance being the internal resistance of the source network.

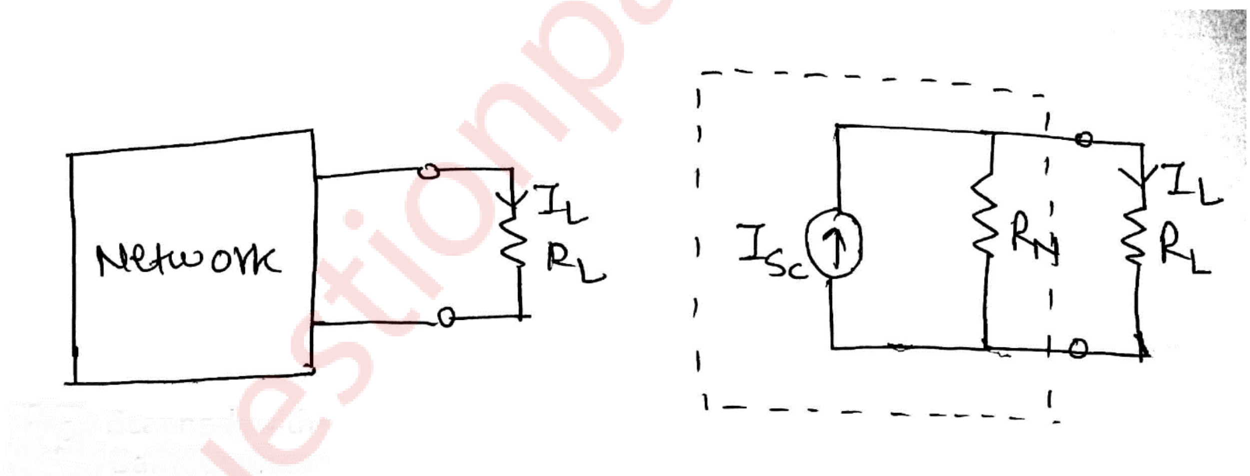


Fig (b) Norton Equivalent Circuit