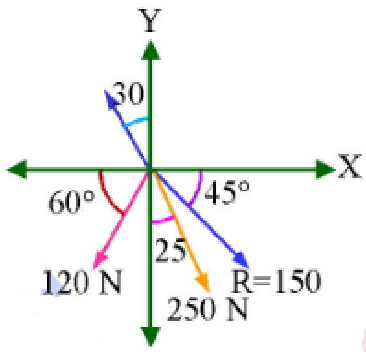
ENGINEERING MECHANICS – SEMESTER 1

CBCGS MAY 18

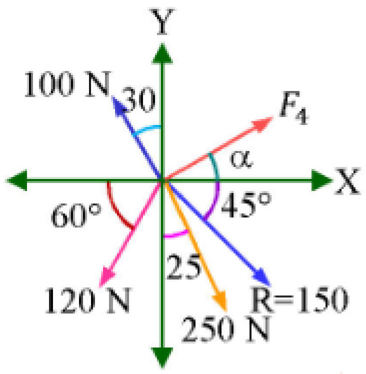
Q1] a) Find fourth force(F_4) completely so as to give the resultant of the system force as shown in figure. (4)



Solution:-

Let F_4 act as an angle α as shown in the figure.

Given, resultant of forces F_1, F_2, F_3 and F_4 is R = 150N



Resolving the forces along Y-axis ,

100cos30-120sin60-250cos25+$F_4\sin\alpha$ = -150sin45

$F_4\sin\alpha = -150sin45 - 100cos30 + 120sin60 + 250cos25$

$F_4\sin\alpha = 137.8314$ ……………….(1)

Resolving the forces along X-axis,

-100sin30-120cos60+250sin25+$F_4\cos\alpha$ = 150cos45

$F_4\cos\alpha = 110.4115$N ……………..(2)

Squaring and adding (1) and (2),

$(F_4 \sin\alpha)^2 + (F_4 \cos\alpha)^2 = (137.8314)^2 + (110.4115)^2$

$F_4{}^2(sin^2\alpha + cos^2\alpha) = 18997.5052 + 12190.6887$

$F_4{}^2 = 31188.1939$

$F_4 = 176.6018$N

Dividing (1) by (2), $\frac{F_4 \sin\alpha}{F_4 \cos\alpha} = \frac{137.8314}{110.4115}$

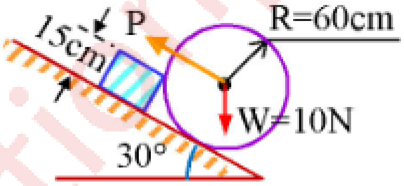
$\tan\alpha = 1.2483$

$\alpha = 51.3031°$

Hence,

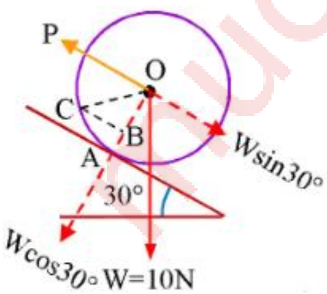
$F_4 = 176.6018$N , $\alpha = 51.3031°$

Q1] b) Determine the magnitude and direction of the smallest force P required to start the wheel W= 10N over the block. (4)



Solution:-

The simplified figure is as shown



Let point C is the tip of rectangle block from figure

OC = OA = 60cm ....(1)

AB = 15 cm ....(height of the block)

Hence OB = 60-15 = 45cm ....(2)

By Pythagoras theorem

BC = $\sqrt{OC^2 - OB^2} = \sqrt{60^2 - 45^2} = 39.6863$cm

BC = 39.6863cm

When the wheel is about to start. Normal reaction at the point A is zero and

ΣM_C = 0.

$\therefore P \times OB - W cos30 \times BC - W sin30 \times OB = 0$

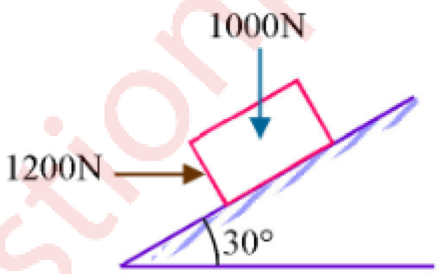
45P = $10cos30 \times 39.6863 + 10sin30 \times 45$

45P = 568.6932.

P = 12.6376N

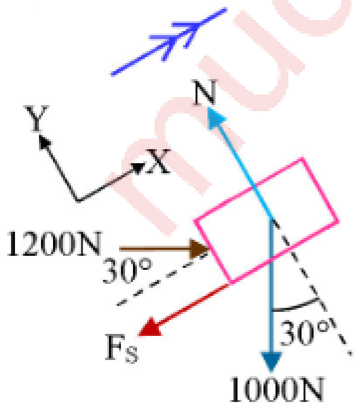
Hence the force P required to start the wheel is 12.6376N.

Q1] c) If a horizontal force of 1200N is applied to the block of 1000N then block will be held in equilibrium or slide down or move up? μ = 0.3. (4)



Solution:-

Let N be normal reaction and F_s be the frictional force



At the instant of impending motion ΣF_Y = 0

Therefore N- 1000cos30 – 1200sin30 = 0

N = 1000cos30 + 1200sin30.

N = 1466.0254N

F_s = µ x N = 0.3(1466.0254)

F_s = 439.8076N. ....(1)

Neglecting friction, net upward up the plane

-1000sin30 + 1200cos30 = 539.2305N. ....(2)

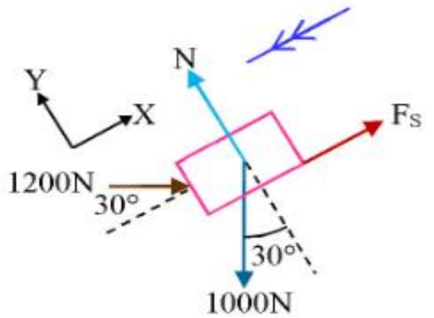
CASE 1:- Block is impending to move up the plane

ΣF_X = -F_s – 1000sin30 + 1200cos30

= -439.8076 + 539.2305. ....(From 1 & 2)

ΣF_X = 99.4229N

Therefore a net force of 99.4229N acts up the plane so the block moves up the plane.



CASE 2:- Block is impending to move down the plane

ΣF_X = F_s – 1000sin30 + 1200cos30

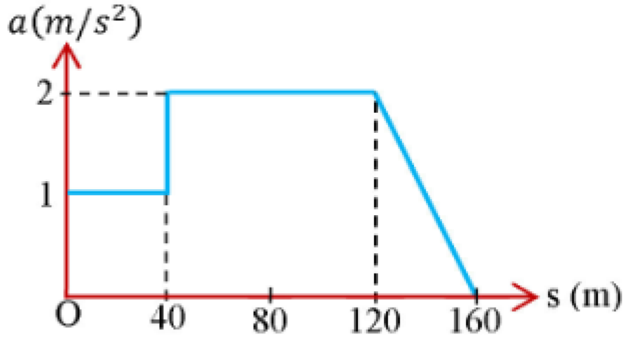
= 439.8076 - 539.2305. ....(From 1 & 2)

ΣF_X = 979.0381N

Therefore a net force of 979.0381N acts up the plane so the block moves up the plane.

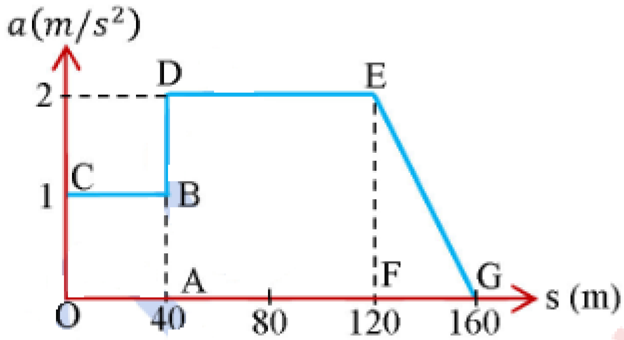
Q1] d) Starting from rest at S = 0 a car travels in a straight line with an acceleration as shown by they a-s graph. Determine the car's speed when S = 20m ,S = 100m,

S =150m. **(4)**



Solution:-

Part 1:-



For first 40m of journey

a = 1m/s^2

V$\frac{dv}{ds}$ = 1. ....(a = V$\frac{dv}{ds}$)

Vdv = ds

On integration, $\frac{V^2}{2}$= s + c_1 ....(1)

When s = 0, v= 0

c_1= 0 ....(2)

From (1) and (2) $\frac{V^2}{2}$ = s

Therefore V^2 = 2s

When s = 20m, V^2 = 40

Therefore v = 6.3246 m/s

When s = 40m, V^2 = 80

Therefore v = 8.9443 m/s ....(3)

Part 2:-

Motion of car from 40m to 120m

a = 2 m/s^2

V$\frac{dv}{ds}$ = 2.

Vdv = ds

On integration , $\frac{V^2}{2}$ = s +c_1.

V^2 = 4s +2c_1. ....(4)

When s = 40 , = 80 ....from (3)

80 = 160 + 2c_2

-80 = 2c_2. ....(5)

From (4) and (5)

V^2= 4s – 80

When s = 100m , V^2 = 400-80 = 320

v = 17.8885 m/s

When s = 120m , = 480-80 = 400

V = 20m/s. ....(6)

Part 3:-

Motion of car from 120m to 160m

E(120,2) and F(160,0)

Using two-point from equation of EF is

$\frac{a-2}{2-0} = \frac{s-120}{120-160}$.

-20a + 40 = s- 120

160-s =20a

160-s = 20 V

(160-s)ds = 20vdv

On integration, 160s – = 20 + C_3 ....(7)

When s = 120, v = 20m/s. ....from (6)

160 x 120 – (0.5) x 120 x 120 = 10 x 20 x20 + C_3

C_3 = 8000. ....(8)

From (7) and (8)

$160s - \frac{s^2}{2} = 20 \times \frac{v^2}{2} + c^3$

When s = 150m, 160(150) $-\frac{1}{2}s^2$ = $10v^2$ -8000

v^2= 475

V = 21.7945m/s

Hence when s = 20m , v = 6.3246m/s

When s = 100m, v = 17.8885m/s

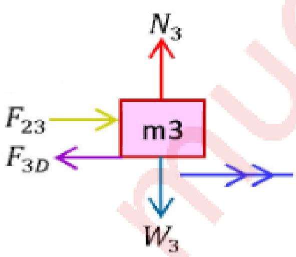
When s = 150m, v = 21.7945m/s.

Q1] e) Three m1, m2, m3 of masses 1.5kg, 2kg and1kg respectively are placed on a rough surface with coefficient of friction 0.20 as shown. If a force F is applied to accelerate the blocks at 3m/s^2 . What will be the force that 1.5kg block exerts on 2kg block? (4)

Solution:-

m_1= 1.5kg, m_2= 2kg, m_3 = 1kg , µ=0.20 , a= 3m/s^2

For m3:-



Weight w_3 = m_3 x g = 1g

Normal reaction N_3 = W_3 = g

Dynamic friction force= F_3 = µN_3 = 0.2g

Let the force exerted by m2 on m3 be F23

By Newton 2nd law

ΣF = ma

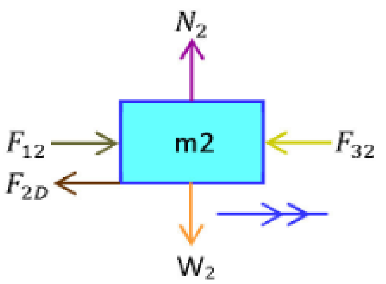
F_{23} - F_3 = m_3 x a

F_{23} =F_3 D + m_3 x a

F_{23} = 0.2g + 1a ....(1)

Similarly,

<u>For m_2:-</u>



Weight W_2= m_2g

Normal reaction N_2 = W_2 = 2g

Dynamic friction force =F_2 D = μ N_2= 0.2 x 2g = 0.4g.

Let the force exerted by m_1 on $m_2$be F_{12}

By Newton's 2nd law

ΣF = ma

F_{12} - F_{32} - F_2 = m_2x a

F_{12} = F_{32} + F_2 + = m_2 x a

= (0.2g + 1a) + 0.4g + 2a. ....from (1)

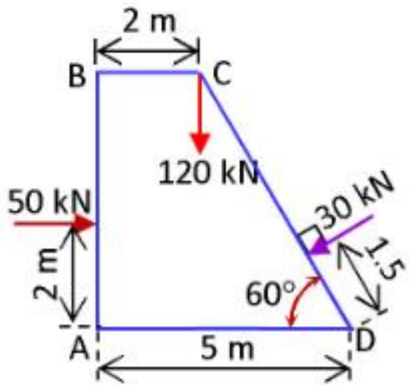
= 0.6g + 3a

= 0.6 x 9.81 + 3 x 3

= 14.886 N

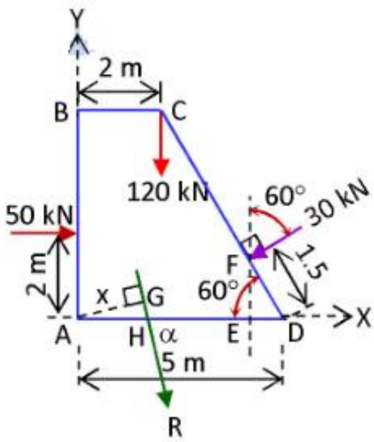
The force that 1.5 kg block exerts on 2kg block = 14.886N.

Q2] a) A dam is subjected to three forces as shown in fig. determine the single equivalent force and locate its point of intersection with base AD (6)



Solution:-

Let R be the resultant and let it act at an angle α to the horizontal



In ΔFED , FD = 1.5cm

∴ FE = FDsin60° = 1.2990 and ED = FDcos60° = 0.75

∴ AE = AD-AE = 5-0.75 = 4.25

Resolving the forces along X-axis, $R_X = 50 - 30sin60$ =24.0192N

Resolving the forces along Y-axis, $R_Y = -120 - 30cos60$ = -135

$\therefore R = \sqrt{{R_X}^2 + {R_Y}^2} = \sqrt{(24.0192)^2 + (-135)^2}$ = 137.1201N

And , $\alpha = \tan^{-1}\left(\frac{R_Y}{R_X}\right) = \tan^{-1}\left(\frac{-135}{24.0192}\right) = -79.9115°$

Resultant moment at A = -50$\times$ $2 - 120 \times 2 - 30sin60° \times 4.25 + 30sin60° \times 1.2990$

A = -370N m

By Varignon's Theorem,

$\therefore 370 = 137.1201 \times X$

X = 2.6984m

In ΔAGH, sin$\alpha = \frac{x}{AH}$

AH = $\frac{x}{\sin\alpha} = \frac{2.6984}{\sin(79.9115)} = 2.7407$m

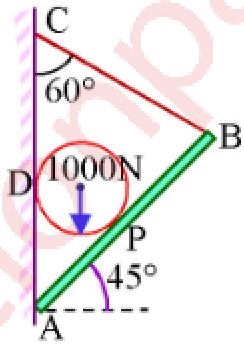
Hence,

Resultant force = 137.1201N (79.9115°)

Resultant moment = 370N m

Resultant cuts base AD at a distance of 2.7407m right of A

Q2] b) A cylinder weighing , 1000N and 1.5m diameter is supported by a beam AB of length 6m and weight 400N as shown. Neglecting friction at the surface of contact of the cylinder. Determine (1) wall reaction at 'D'; (2) hinged reaction at support 'A'; (3) tension in the cable BC (8)

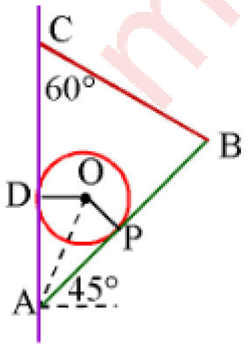


Solution:-

Diameter of cylinder = 1.5m

Radius OD = OP =0.75m

Also, AB =6



Now, ∠DAP = 90° $- 45° = 45°$

Also, $\Delta AOP \cong \Delta AOD$

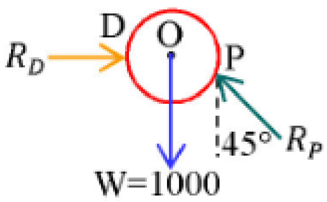
∠OAP = $\frac{1}{2} \times 45 = 22.5°$

In ΔAOP, ∠OPA =90°

tan∠OAP = $\frac{OP}{AP}$

tan22.5 = $\frac{0.75}{AP}$

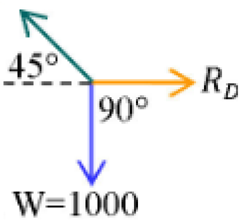
AP = = $\frac{0.75}{tan22.5} = 1.8107$m



FBD of cylinder is as shown

Since the cylinder is in equilibrium,

By Lami's theorem.



$\frac{W}{\sin(180-45)} = \frac{R_D}{\sin(90+45)} = \frac{R_P}{sin90}$

$\frac{1000}{sin135} = \frac{R_D}{sin135}$ and $\frac{1000}{sin135} = \frac{R_P}{1}$

$R_D = 1000$N and $R_P = 1414.2136$N

FBD of beam AB is as shown

Let Q be mid-point of AB

AQ= 3m

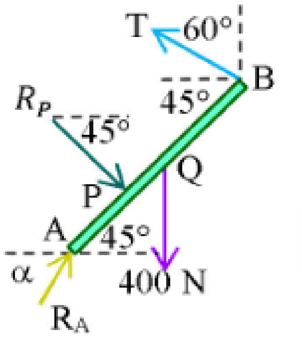
Since beam AB is in equilibrium,

$\sum M_A = 0$

$-R_P \times AP - 400 \times AQcos45 + Tsin(30 + 45) \times AB = 0$

$-1414.2136 \times 1.8107 - 400 \times 3 \times 0.7071 + T \times 0.9659 \times 6 = 0$

T = 588.266N



Also, $\sum F_X = 0$

$R_A cos\alpha + R_P cos45 - Tsin60 = 0$

$R_A cos\alpha + 1414.2136 \times 0.7071 - 588.266 \times 0.866 = 0$

$R_A cos\alpha = -490.5676$ ………………….(1)

And $\sum F_Y = 0$

$R_A sin\alpha - 400 - R_P sin45 + Tcos60 = 0$

$R_A sin\alpha - 400 - 1414.2136 \times 0.7071 + 588.266 \times 0.5 = 0$

$R_A sin\alpha = 1105.8790$ ……………………(2)

Squaring and adding (1) and (2)

$R_A{}^2 cos^2\alpha + R_A{}^2 sin^2\alpha = (-490.5676)^2 + (1105.8790)^2$

$R_A = 1209.8037$N

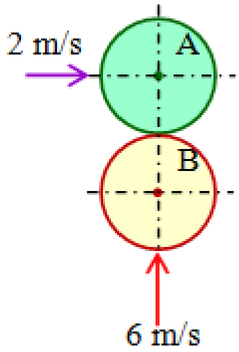
Dividing, (2) by (1) , $\frac{R_A sin\alpha}{R_A cos\alpha} = \frac{1105.8790}{490.5676}$

$\alpha = \tan^{-1} 2.25431 = 66.0779°$

Hence,

1. Wall reaction at D = R_D= 1000N
2. Hinged reaction at support A = 1209.8037

Q2] c) Two balls of 0.12kg collide when they are moving with velocities 2m/sec and 6m/sec perpendicular to each other as shown in fig. if the coefficient of restitution between 'A' and 'B' is 0.8 determine the velocity of 'A' and 'B' after impact (6)



Solution:-

$m_1 = m_2 = 0.12kg \;\; u_{1x} = 2m/s \;\; u_{1y} = 0m/s \;\; u_{2x} = 0m/s \;\; u_{2y} = 6m/s$ c =0.8

Case1 :- line of impact is X-axis

Velocities along Y-axis remains constant

$v_{1y} = u_{1y} = 0m/s$ and $v_{2y} = u_{2y} = 6m/s$ ....(1)

By law of conservation of momentum,

$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$

$0.12\times 2 + 0.12 \times 0 = 0.12 \times v_{1x} + 0.12 \times v_{2x}$

Dividing by 0.12 , 2 = $v_{2x} + v_{1x}$ ....(2)

Also, coefficient of restitution = e = $\frac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$

0.8 = $\frac{v_{2x} - v_{1x}}{2-0}$

1.6 = $v_{2x} - v_{1x}$ ....(3)

Solving (2) and (3)

$v_{2x} = 1.8m/s$ and $v_{1x} = 0.2m/s$ ....(4)

From (1) and (4)

$v_1 = \sqrt{{v_{1x}}^2 + {v_{1y}}^2}$ and $v_2 = \sqrt{{v_{2x}}^2 + {v_{2y}}^2}$

$v_1 = \sqrt{0.2^2 + 0^2}$ and $v_2 = \sqrt{1.8^2 + 6^2}$

$v_1 = 0.2m/s$ and $v_2 = 6.2642m/s$

Also after impact

Velocity of ball A = 0.2m/s

Velocity of ball B = 6.2642 m/s

Case 2:- line of impact is y-axis

Velocities along x-axis remains constant

$v_{1x} = u_{1x} = 2m/s$ and $v_{2x} = u_{2x} = 0m/s$ ………….(5)

By law of conservation of momentum,

$m_1u_{1y} + m_2u_{2y} = m_1v_{1y} + m_2v_{2y}$

$0.12\times 0 + 0.12 \times 6 = 0.12 \times v_{1y} + 0.12 \times v_{2y}$

Dividing by 0.12 , 6 = $v_{2y} + v_{1y}$ ……………(6)

Also, coefficient of restitution = e = $\frac{v_{2y} - v_{1y}}{u_{1y} - u_{2y}}$

0.8 = $\frac{v_{2y} - v_{1y}}{0-6}$

-4.8 = $v_{2y} - v_{1y}$ ……………(7)

Solving (6) and (7)

$v_{2y} = 0.6m/s$ and $v_{1y} = 5.4m/s$ …………………(8)

From (4) and (8)

$v_1 = \sqrt{{v_{1x}}^2 + {v_{1y}}^2}$ and $v_2 = \sqrt{{v_{2x}}^2 + {v_{2y}}^2}$

$v_1 = \sqrt{2^2 + 5.4}$ and $v_2 = \sqrt{0^2 + 0.6^2}$

$v_1 = 5.7585m/s$ and $v_2 = 0.6m/s$

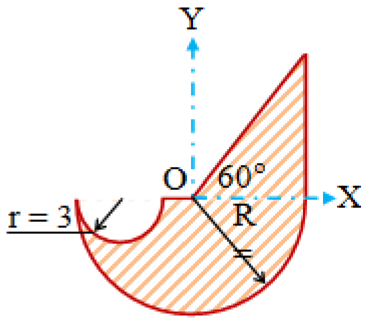
Also, $\alpha_1 = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right)$ = $\tan^{-1}\left(\frac{5.4}{2}\right)$ = 69.67°

Hence, after impact

Velocity of ball A = 5.7585 m/s

Velocity of ball B = 0.6m/s

Q3] a) Find the centroid of the shaded portion of the given area shown in figure (8)



(All Dimensions in cm)

Solution:-

In ΔAOB

OB = 8cm

tan60 = $\frac{AB}{BO}$ $\therefore \sqrt{3} = \frac{AB}{8}$ $\therefore$ AB = $8\sqrt{3}$

Also OD = 8cm and CD = 3cm

OC = 8-3=5cm

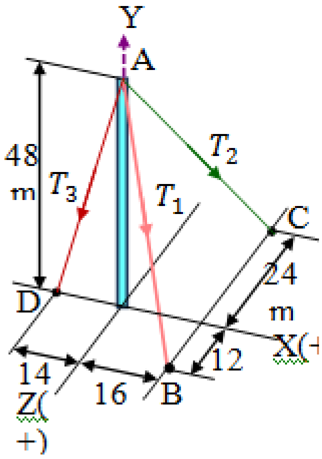
SR NO	PART	Area(in cm^2)	X-co-ord of C.G.(x_1)	Y-co-ord of C.G.(y_1)	Ax_1	Ay_1
1)	Triangle AOB B =8, H= $8\sqrt{3}$	= 0.5BH = 0.5× 8 × $8\sqrt{3}$ =55.4256	$8 - \frac{B}{3} = 8 - \frac{8}{3}$ = 5.3333	$\frac{H}{3} = \frac{8\sqrt{3}}{3}$ =4.6188	295.6033	256.000
2)	Semicircle radius(R)=8	$0.5\pi R^2$ =0.5× $8^2\pi$ =100.5372	0	$-\frac{4R}{3\pi} = \frac{-4\times 8}{3}$ = -3.3955	0.0000	-341.333
3)	Cut semicircle radius(r) = 3	-$0.5\pi r^2$ =-0.5× $3^2\pi$ =-14.1372	-5	$-\frac{4r}{3\pi} = \frac{-4\times 3}{3}$ = -1.2732	70.6858	18.000
	Total	141.8193			366.2891	-67.3333

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{366.2891}{141.8193} = 2.5828$$

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{-67.3333}{141.8193} = -0.4748$$

Hence centroid is equals to (2.5828,0.4748)

Q3] b) Knowing that the tension in AC is $T_2 = 20kN$ determine required values $T_1$and T_3so that the resultant of the three forces are 'A' is vertical. Also, calculate this resultant. (6)



Solution:-

From figure we observe,

A = (0,48,0) ; B =(16,0,12); C = (16,0,-24) ; D = (-14,0,0);

Let $\bar{a}, \bar{b}, \bar{c}$ *and* $\bar{d}$ be the position vectors of points A,B,C and D respectively w.r.t. to origin 0.

$\therefore \overline{OA} = \bar{a} = 48\vec{j}$; $\overline{OB} = \bar{b} = 16\vec{j} + 12\vec{k};$

$\overline{OC} = \bar{c} = 16\vec{\imath} - 24\vec{k};$ $\overline{OD} = \bar{d} = -14\vec{\imath};$

Now,

$\overline{AB} = \bar{b} - \bar{c} = 16\vec{j} + 12\vec{k} - 48\vec{j}$

$\overline{AC} = \bar{c} - \bar{a} = 16\vec{\imath} - 24\vec{k} - 48\vec{j}$

$\overline{AD} = \bar{d} - \bar{a} = -14\vec{\imath} - 48\vec{j}$

Magnitude,

$|\overline{AB}| = \sqrt{(16)^2 + (-48)^2 + (12)^2} = 52$

$|\overline{AC}| = \sqrt{(16)^2 + (-48)^2 + (-24)^2} = 56$

$|\overline{AD}| = \sqrt{(-14)^2 + (-48)^2} = 50$

Unit vector,

$\widehat{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{16\vec{\imath} - 48\vec{j} + 12\vec{k}}{52} = \frac{4}{13}\vec{\imath} - \frac{12}{13}\vec{j} + \frac{3}{13}\vec{k}$

$\widehat{AC} = \frac{\overline{AC}}{|\overline{AC}|} = \frac{16\vec{\imath} - 48\vec{j} - 24\vec{k}}{56} = \frac{2}{7}\vec{\imath} - \frac{6}{7}\vec{j} - \frac{3}{7}\vec{k}$

$\widehat{AD} = \frac{\overline{AD}}{|\overline{AD}|} = \frac{-14\vec{\imath} - 48\vec{j}}{50} = \frac{-7}{25}\vec{\imath} - \frac{24}{25}\vec{j}$

Tension in AB = $\overline{T_1} = T_1\left(\frac{4}{13}\vec{\imath} - \frac{12}{13}\vec{\jmath} + \frac{3}{13}\vec{k}\right)$

Tension in AC = $\overline{T_2} = 20\left(\frac{2}{7}\vec{\imath} - \frac{6}{7}\vec{\jmath} - \frac{3}{7}\vec{k}\right)$

Tension in AD = $\overline{T_3} = T_3\left(\frac{-7}{25}\vec{\imath} - \frac{24}{25}\vec{\jmath}\right)$

Net force = $\overline{T_1} + \overline{T_2} + \overline{T_3}$ = $T_1\left(\frac{4}{13}\vec{\imath} - \frac{12}{13}\vec{\jmath} + \frac{3}{13}\vec{k}\right) + 20\left(\frac{2}{7}\vec{\imath} - \frac{6}{7}\vec{\jmath} - \frac{3}{7}\vec{k}\right) + T_3\left(\frac{-7}{25}\vec{\imath} - \frac{24}{25}\vec{\jmath}\right)$

= $\left(\frac{4}{13}T_1 + 20\times\frac{2}{7} - \frac{7}{25}T_3\right)\vec{\imath} - \left(\frac{12}{13}T_1 + 20\times\frac{6}{7} + \frac{24}{25}T_3\right)\vec{\jmath} + \left(\frac{3}{13}T_1 - 20\times\frac{3}{7}\right)\vec{k}$ ....(1)

Given, the resultant at 'A' is vertical i.e., along y-axis

$\frac{3}{13}T_1 - 20\times\frac{3}{7}$ & $\frac{4}{13}T_1 + 20\times\frac{2}{7} - \frac{7}{25}\ T_3$ ....(2)

$\frac{3}{13}T_1 = 20\times\frac{3}{7}$

$T_1 = \frac{260}{7} = 37.1429kN$ ....(3)

From (2) and (3), $\frac{4}{13}\times\frac{260}{7} + 20\times\frac{2}{7} - \frac{7}{25}\ T_3 = 0$

$\frac{120}{7} = \frac{7}{25}\ T_3$

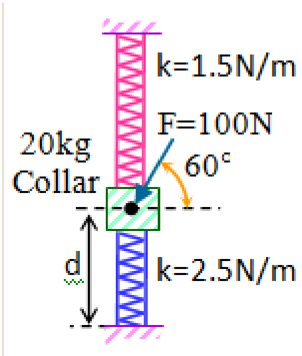
$T_3 = \frac{3000}{49} = 61.2245kN$ ....(4)

From (1), (3) and (4),

Resultant = $-\left(\frac{12}{13}\times\frac{260}{7} + 20\times\frac{6}{7} + \frac{24}{25}\times\frac{3000}{49}\right)$

Resultant = $-\frac{5400}{49} = -110.2041kN$

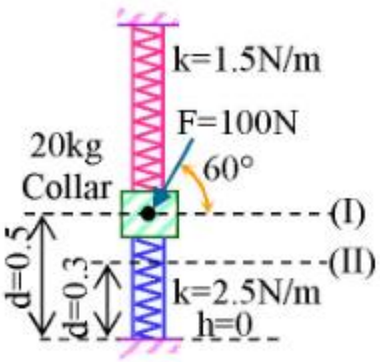
Q3] c) Fig shows a collar of mass 20kg which is supported on the smooth rod. The attached springs are both compressed 0.4m when d =0.5m. determine the speed of the collar after the applied force F=100N causes it to be displaced so that d =0.3m. knowing that collar is at rest when d=0.5m (6)



Solution:-

m =20kg,

position 1: when d =0.5m



v =0

$KE_1 = \frac{1}{2}mv_1^2 = 0$ and $PE_1 = mgh = 20g \times 0.5 = 10g$

Compression of Top spring $(x_{T1}) = 0.4$m

Compression of B spring $(x_{B1}) = 0.4$m

Spring energy of top spring $= E_{S_{T1}} = \frac{1}{2}K_{T1}x_{T1}^2 = \frac{1}{2} \times 1.5 \times 0.4^2 = 0.12$J

Spring energy of bottom spring $= E_{S_{B1}} = \frac{1}{2}K_{B1}x_{B1}^2 = \frac{1}{2} \times 2.5 \times 0.4^2 = 0.2$J

Position 2: when d = 0.3m

Let v be the velocity of the block

$KE_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \times v^2 = 10v^2$ and $PE_2 = mgh = 20g \times 0.3 = 6g$

Compression of top spring (x_{T2})= 0.4-0.2 = 0.2m

Compression of bottom spring (x_{B2})= 0.4+0.2 = 0.6m

Spring energy of top spring = $E_{S_{B2}} = \frac{1}{2}K_{T2}x_{T2}^2 = \frac{1}{2} \times 1.5 \times 0.2^2 = 0.03$J

Spring energy of bottom spring = $E_{S_{B2}} = \frac{1}{2}K_{B2}x_{B2}^2 = \frac{1}{2} \times 2.5 \times 0.6^2 = 0.45$J

Work done by the vertical component of the force = W =100sin60× 0.2 = 17.32015J

Applying work energy principle for the position (1) and (2) , $U_{1-2} = KE_2 - KE_1$

W + $PE_1 - PE_2 + E_{S_{T1}} + E_{S_{B1}} - E_{S_{B2}} - E_{S_{B2}} = KE_2 - KE_1$

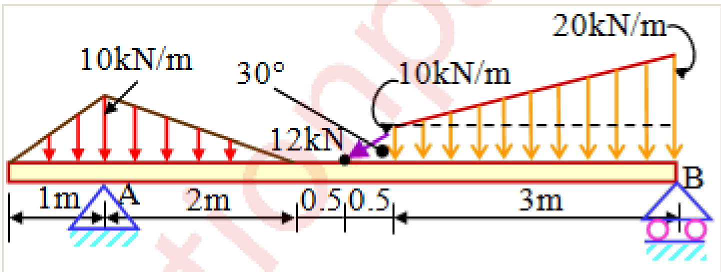
17.3205+10g-6g + 0.12+0.2-0.03-0.45 = $10v^2$

56.3871 = $10v^2$

V = 2.3746 m/s

The speed of the collar after the force F is applied = 2.3746 m/s.

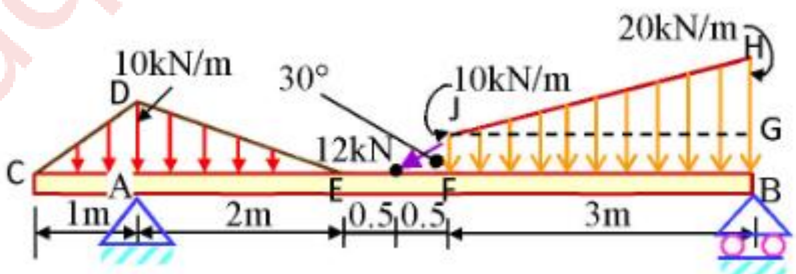
Q4] a) Find the support reactions at point 'A' and 'B' of the given beam (8)



Solution:-

Effective forces of distributed load CAD = $\frac{1}{2} \times 1 \times 10 = 5kN$

It acts as $\frac{1}{3}$m from A



Effective force of distributed load EAD = $\frac{1}{2} \times 2 \times 10 = 10kN$

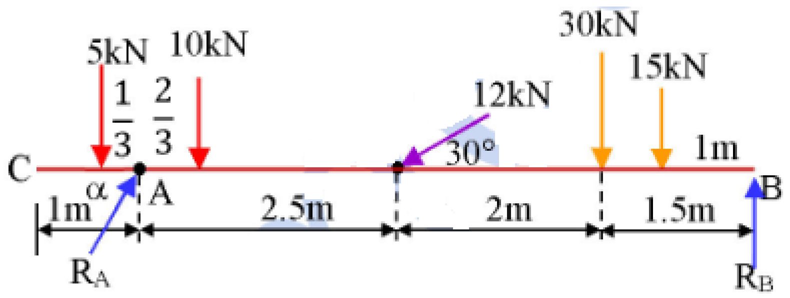
It acts at $\frac{2}{3}$m from A

Effective force of distributed load JFBGJ = 3 × 10 = 30kN

If acts at 1.5m from B

Effective force of distributed load JGH = $\frac{1}{2} \times 3 \times (20 - 10) = 15$kN

It acts at 1m from B



Since the beam is in equilibrium $\sum M_A = 0$

$5 \times \frac{1}{3} - 10 \times \frac{2}{3} - 12sin30 \times 2.5 - 30 \times 4.5 - 15 \times 5 + R_B \times 6 = 0$

$-230 + R_B \times 6 = 0$

$R_B = 38.333$kN

Also , $\sum F_X = 0$

$R_A cos\alpha - 12cos30 = 0$

$R_A cos\alpha = 10.3923$kN ....(1)

And, $\sum F_Y = 0$

$R_A sin\alpha - 5 - 10 - 30 - 12sin30 - 15 + R_B = 0$

$R_A sin\alpha - 66 + 38.333 = 0$

$R_A sin\alpha = 27.6667$kN ....(2)

Squaring and adding (1) and (2) ,

$R_A{}^2 cos^2\alpha + R_A{}^2 sin^2\alpha = 10.3923^2 + 27.6667^2$

$R_A = 29.5541$kN

Dividing, (2) by (1), $\frac{R_A sin\alpha}{R_A cos\alpha} = \frac{27.6667}{10.3923}$

$\alpha = \tan^{-1}(2.6622) = 69.4125°$

Hence,

Reaction at A = 29.5541

Reaction at B = 38.3333kN

Q4] b) The motion of the particle is defined by the relation a= 0.8t m/s^2 where 't' is measured in sec. It is found that at X = 5cm,V = 12m/sec when t = 2sec find the position and velocity at t = 6sec. (6)

Solution:-

Given :- a = 0.8t $\frac{dv}{dt} = 0.8t$

dv = 0.8tdt

On integration

V = 0.8 x $\frac{t^2}{2}$ + c

Therefore , V = 0.4t^2+ c. ....(1)

Given ,when t = 2 then V = 12

12 = 0.4 x 2^2 + c

12 = 1.6 + c

c = 10.4 ....(2)

From (1) & (2)

V = 0.4t^2 + 10. ....(3)

Therefore $\frac{dX}{dt}$ = 0.4t^2 + 10.

dX = (0.4t^2 + 10)dt

On integration, X = 0.4 x $\frac{t^3}{3}$ + 10.4t + k ....(4)

Given , when t = 2 then X = 5

5 = 0.4 x $\frac{2^3}{3}$ + 10.4 x 2 + k

k = -253/15. ....(5)

From (4) and (5)

X = 0.4 x $\frac{t^3}{3}$ + 10.4t + k

X = 0.4 x $\frac{t^3}{3}$ + 10.4t – $\frac{253}{15}$

When t = 6,

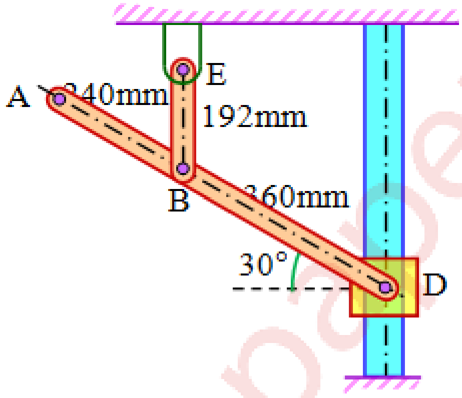
From (3) V = 0.4 x 6 x 6 + 10.4 = 24.8m/s

From (6) X = $0.4 \times \frac{6^3}{3} + 10.4t - \frac{253}{15}$

X = 74.333 from the initial position

Q 4] c) Rod EB in the mechanism shown in the figure has angular velocity of 4 rad/sec at the instant shown in counter clockwise direction. Calculate

1) Angular velocity of AD 2) velocity of collar 'D'. 3) velocity or point 'A' (6)



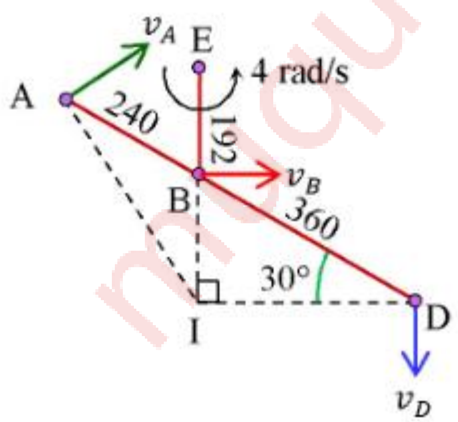
Solution:-

ω_{EB} = 4rad/sec , EB = 192mm = 0.192 m

AB = 240mm =0.24m , DB =360mm = 0.36m

Instantaneous center of rotation is the point of intersection of $\overrightarrow{v_A}$ and $\overrightarrow{v_B}$.

Let I be the ICR as shown in the figure



In Δ BID

∠BDI = 30° , ∠BID = 90°

∠IBD = 180-30° − 90° = 60°

IB =0.36sin30 = 0.18 …………………..(1) and

ID = 0.36cos30 = 0.3117m ………………(2)

Also ∠IBA =180- 60° = 120° ………………(3)

In Δ IBA, by cosine rule

$IA^2 = IB^2 + AB^2 - 2IA \times AB \times cos\angle \text{IBA}$

$IA^2 = 0.18^2 + 0.24^2 - 2 \times 0.18 \times 0.24 \times cos120°$ ……………..(from 1)

IA = 0.3650m ……………..(4)

By sine rule, $\frac{AB}{sinI} = \frac{IB}{sinA} = \frac{IA}{sinB}$

$\frac{0.24}{sinI} = \frac{0.3650}{sin120}$ ……………..(from 3 & 4)

$\text{sinI} = \frac{0.24 \times sin120°}{0.3650} = 0.5694$

∠AIB = 34.7113°

Now, instantaneous velocity of point B = rω

$v_B = EB \times \omega_{EB} = 0.192 \times 4 = 0.768m/s$ …………………(5)

Angular velocity of the rod AD = $\omega_{AD} = \frac{v_B}{r} = \frac{v_B}{IB} = \frac{0.768}{0.18} = 4.2667rad/s$ ………..(from 1 & 5) ……(6)

Instantaneous velocity of point D = rω

= ID× ω_{AD} = 0.3117 × 4.2667 = 1.3302m/s ……….(from 2 & 6)

And instantaneous velocity of point A = rω

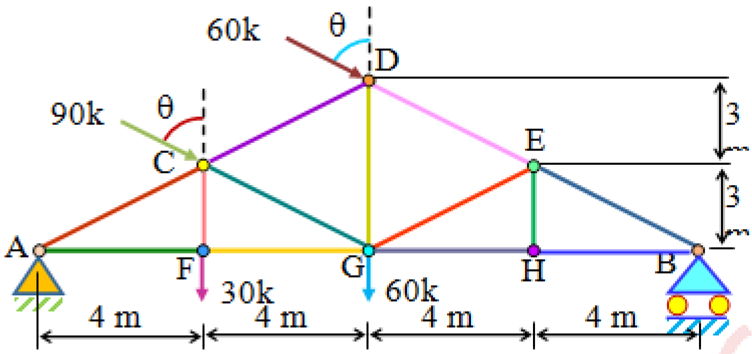
= IA× ω_{AD} = 0.3650 × 4.2667 = 1.5572m/s ……..(from 4 & 6)

Hence,

1. Angular velocity of AD = 4.2667 rad/sec
2. Velocity of collar 'D' = 1.3302 m/s

Velocity or point A = 1.5572 m/s

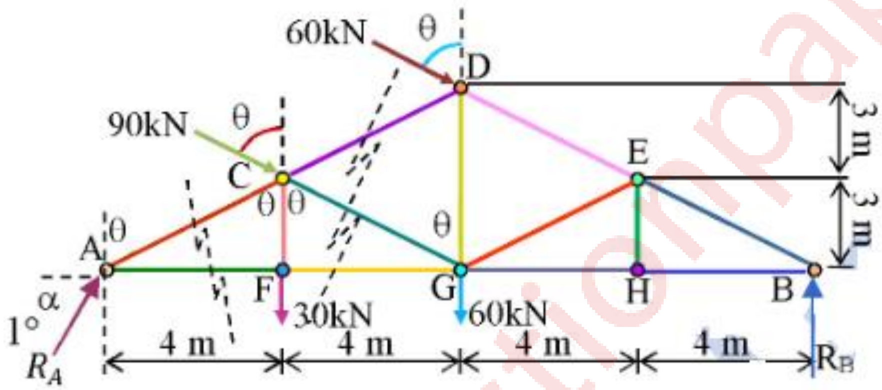
Q5] a) A simply supported pin jointed truss is loaded and supported as shown in fig, (1) identify the members carrying zero forces (2) find support reactions. (3) find forces in members CD, CG, FG and CF using method of section (8)



Solution:-

In ΔGFC, $\tan\theta = \frac{GF}{CF} = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1301°$



$\sin\theta = 0.8$ and $\cos\theta = 0.6$ ....(1)

zero force members:

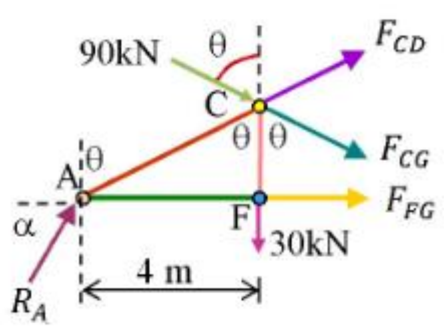
loading at joint H is as shown.

Member EH will have zero force.

Similarly, after EH is removed loading at joint E is as shown

Member EG will have zero force.

Support reactions:



As the truss is in equilibrium, $\sum M_A = 0$

$-30 \times 4 - 90cos\theta \times 3 - 60cos\theta \times 8 - 60sin\theta \times 6 - 60 \times 8 + R_B \times 16 = 0$

-120-360$\times 0.6 - 270 \times 0.8 - 480 \times 0.6 - 360 \times 0.8 - 480 + 16R_B = 0$ ....(from 1)

-1608+16$R_B = 0$

$R_B = 100.5kN$

Also, $\sum F_Y = 0$

$R_A sin\alpha - 30 - 60 - 90cos\theta - 60cos\theta + R_B = 0$

$R_A sin\alpha = 79.5kN$ ....(2)

And, $\sum F_X = 0$

$R_A cos\alpha + 90sin\theta + 60sin\theta = 0$

$R_A cos\alpha + 150 \times 0.8 = 0$

$R_A cos\alpha = -120kN$ ....(3)

Squaring and adding (2) and (3),

$(R_A sin\alpha)^2 + (R_A cos\alpha)^2 = (79.5)^2 + (-120)^2$

$R_A^2(sin^2\alpha + cos^2\alpha) = 6320.25 + 14400$

$R_A^2 = 20720.25$

$R_A = 143.9453kN$

Dividing, (2) and (3) , we get

tan$\alpha = 0.6625$

$\alpha = 33.5245°$

Method of sections:

Applying conditions of equilibrium to the section as shown $\sum M_A = 0$

-30$\times$ 4 $-$ $(90 + F_{CG})cos\theta \times 4 - (90 + F_{CG})sin\theta \times 3 = 0$

-120-(90+F_{CG}) $\times 0.6 \times 4 - (90 + F_{CG}) \times 0.8 \times 3 = 0$ ....(from 1)

-4.8(90+F_{CG}) $= 120$

$F_{CG} = -115kN$ ………………(4)

Also, $\sum F_Y = 0$

$R_A sin\alpha - 30 - F_{CG}cos\theta + F_{CD}cos\theta - 90cos\theta = 0$

79.5-30-115$\times 0.6 + F_{CD} \times 0.6 - 90 \times 0.6 = 0$

64.5+0.6$F_{CD} = 0$

$F_{CD} = -107.5kN$ ………………….(5)

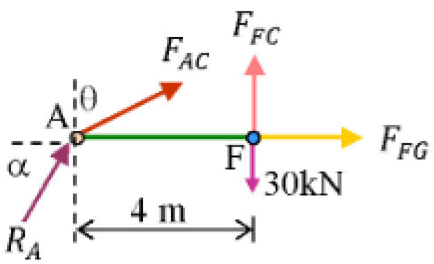
And, $\sum F_X = 0$

$R_A cos\alpha + F_{FG} + F_{CG}sin\theta + F_{CD}sin\theta + 90sin\theta = 0$

-120+$F_{FG} - 115 \times 0.8 - 107.5 \times 0.8 + 90 \times 0.8 = 0$

$F_{FG} = 226kN$

Applying conditions of equilibrium to the section shown below, $\sum M_A = 0$



$F_{FC} \times 4 - 30 \times 4 = 0$

$F_{FC} = 30kN$ …………………(6)

Also, $\sum F_Y = 0$

$R_A sin\alpha - 30 + F_{AC}cos\theta + F_{FC}$=0

79.5 – 30-$F_{AC} \times 0.6 + 30 = 0$

$F_{AC} = 132.5kN$

Members carrying zero forces are EH and EG

Support reactions:-

$R_A = 143.9453kN$

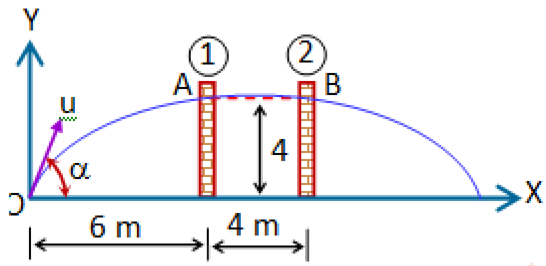
$R_B = 100.5kN$

Forces in members:-

CD = 107.5kN(C) CG = 115kN(C)

FG = 226kN(T) and CF = 30kN(T)

Q5] b) A jet of water discharging from nozzle hits a vertical screen placed at a distance of 6m from the nozzle at a height of 4m. when the screen is shifted by 4m away from the nozzle from its initial position the jet hits the screen again at the same point. Find the angle of projection and velocity of projection of the jet at the nozzle. (6)



Solution:-

The path of the projectile is given by y=xtan$\theta - \frac{gx^2}{2u^2} sec^2\theta$ ....(1)

The water jet passes through the point A(6,4) and B(10,4)

Substituting , x =6 and y = 4 in (1) we get 4 = 6tan$\theta - \frac{36g}{2u^2} sec^2\theta$ ....(2)

Substituting , x =10 and y = 4 in (1) we get 4 = 10tan$\theta - \frac{100g}{2u^2} sec^2\theta$ ....(3)

Multiplying equation (2) by 25 and equation (3) by 9 and then subtract

$\therefore 100 - 36 = \left(150tan\theta - \frac{900g}{2u^2} sec^2\theta\right) - \left(90tan\theta - \frac{900g}{2u^2} sec^2\theta\right)$

$\therefore 64 = 60tan\theta$

$\theta = \tan^{-1}\left(\frac{64}{60}\right)$ = 46.8476° ....(4)

From (3) and (4) ,

4 = 10tan$(46.8476) - \frac{100g}{2u^2} sec^2(46.8476)$

$\frac{1048.58}{u^2} = 6.6667$

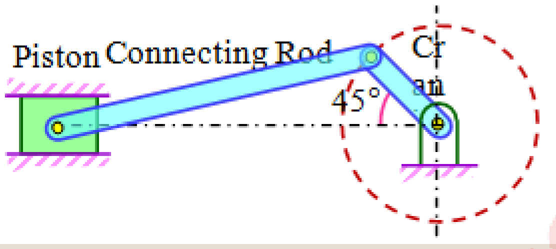
$u^2 = 157.2862$

U = 12.5414 m/s

Hence the angle of projection = 46.8476°

Velocity of projection = 12.5414 m/s

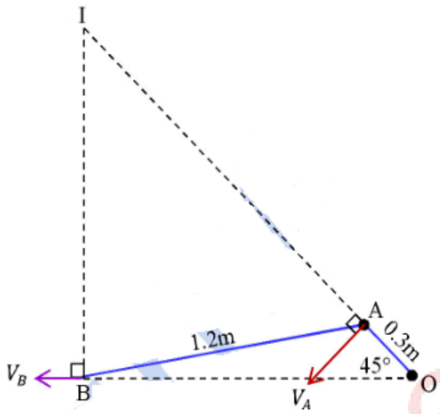
Q5] c) In a crack and connecting rod mechanism the length of crack and connecting rod are 300mm and 1200mm respectively. The crack is rotating at 180 rpm. Find the velocity of piston, when the crack is at an angle of 45° with the horizontal (6)



Solution:-

Let OA and AB be the crank and the connecting rod.

Frequency(n) = 180rpm = $\frac{180}{60}$ = 3rps



OA = 300nm = 0.3m , AB = 1200mm = 1.2m

We assume crank is rotating in anti-clockwise direction

∴ Angular velocity of the crank = $\omega_{AB} = 2\pi n = 2\pi \times 3 = 18.8496$ rad/s

∴ instantaneous velocity of point A = rω

$v_A = OA \times \omega_{OA} = 0.3 \times 18.8496 = 5.6549$ m/s

Instantaneous centre of rotation is the point of intersection of $\vec{v_A}$ and $\vec{v_B}$

Let I be the ICR of the connecting rod AB as shown in figure.

In ΔOAB by sine rule, $\frac{AB}{sinO} = \frac{OA}{sinB}$

∴ $\frac{1.2}{sin45} = \frac{0.3}{sin\angle ABO}$

$\therefore sin\angle ABO = \frac{0.3sin45}{1.2} = 0.1768$

$\therefore \angle ABO = 10.1821°$

In Δ IOB , $\angle BIO = 180° - 90° - 45° = 45°$

In Δ IAB, $\angle ABI = 90 - 10.1821° = 79.8179°$

$\therefore \angle IAB = 180° - 45° - 79.8179° = 55.1821°$

In Δ IAB by sine rule, $\frac{AB}{sinI} = \frac{IB}{sinA} = \frac{IA}{sinB}$

$\therefore \frac{1.2}{sin45} = \frac{IB}{sin55.1821} = \frac{IA}{sin79.8179}$

$\therefore \frac{1.2}{sin45} = \frac{IB}{sin55.1821}$ and $\therefore \frac{1.2}{sin45} = \frac{IA}{sin79.8179}$

IB =$\frac{1.2sin55.1821}{sin45} = 1.3931$ and

IA = $\frac{1.2sin79.8179}{sin45} = 1.6703$

Angular velocity of the rod AB = $\omega_{AB} = \frac{v_A}{r} = \frac{v_A}{IA} = \frac{5.6549}{1.6703} = 3.3855$ rad/s

Instantaneous velocity of B = $r\omega_{AB} = IB \times \omega_{AB} = 1.3932 \times 3.3855 = 4.7168$ m/s

Hence, velocity of piston = 4.7168 m/s

Q6] a) Force F = 80i+50j-60k passes through a point A(6,2,6). Compute its moment about a point B(8,1,4) (4)

Solution:-

$\bar{F} = 80\vec{i} + 50\vec{j} - 60\vec{k}$

Let $\bar{a}$ and $\bar{b}$ be the position vectors of point A and B respectively.

$\bar{a} = 6\vec{i} + 2\vec{j} + 6\vec{k}$ and $\bar{b} = 8\vec{i} + 1\vec{j} + 4\vec{k}$

$\overline{BA} = \bar{a} - \bar{b}$ = $(6\vec{i} + 2\vec{j} + 6\vec{k})$-$(8\vec{i} + 1\vec{j} + 4\vec{k})$ = $-2\vec{i} + 1\vec{j} + 2\vec{k}$

Moment of F about B = $\overline{BA} \times \bar{F}$

B = $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & 2 \\ 80 & 50 & -60 \end{vmatrix}$ = $\vec{i}(-60 - 100) - \vec{j}(120 - 160) + \vec{k}(-100 - 80)$ = $-160\vec{i} + 40\vec{j} - 180\vec{k}$

Hence , moment of F about B is -$160\vec{i} + 40\vec{j} - 180\vec{k}$ units

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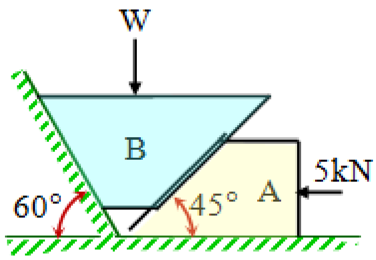
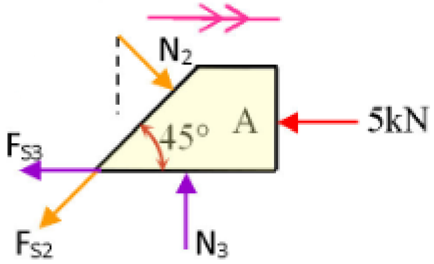
Q6] b) A force of 5KN is acting on the wedge as shown in fig. the coefficient of friction at all rubbing surfaces is 0.25. find the load 'W' which can be held in position. The weight of block 'B' may be neglected. (8)



Solution:-

Let N_1, N_2, N_3,be the normal reaction at the surface of contact



$\therefore\ F_{S1} = \mu_1 N_1 = 0.25N_1$, $F_{S2} = \mu_2 N_2 = 0.25N_2$, $F_{S3} = \mu_3 N_3 = 0.25N_3$ ....(1)

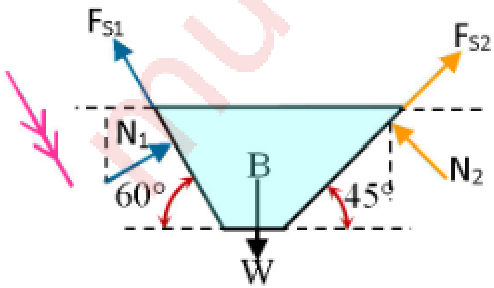
Block A is impending to move towards right.

Since the block A is under equilibrium , $\sum F_Y = 0$

$\therefore N_3 - F_{S2} sin45 - N_2 cos45 = 0$

$\therefore N_3 - 0.25N_2 \times 0.7071 - N_2 \times 7071 = 0$ ....(from 1)

$\therefore N_3 - 0.8839N_2 = 0$ ....(2)



Also $\sum F_X = 0$

$-5 - F_{S3} - F_{S2} cos45 + N_2 sin45 = 0$

$\therefore -5 - 0.25N_3 - 0.25N_2 \times 0.7071 + N_2 \times 0.7071 = 0$ ....(from 1)

$\therefore -0.25N_3 + 0.5303N_2 = 5$ ....(3)

Solving (2) and (3) simultaneously, we get $N_3 = 14.2876$kN and $N_2 = 16.1642$kN ....(4)

Block B is impending to move down

Since the block B is under equilibrium , $\sum F_X = 0$

$\therefore N_1 sin60 - F_{S1} cos60 + F_{S2} cos45 - N_2 sin45 = 0$

$\therefore 0.866N_1 - 0.25N_1 \times 0.5 + 0.25N_2 \times 0.7071 - N_2 \times 0.7071 = 0$ ....(from 1)

$\therefore 0.866N_1 - 0.125N_1 + 0.1768 \times 16.1642 - 16.1642 \times 0.7071 = 0$ ....(from 4)

$\therefore 0.741N_1 - 8.5719 = 0$

$N_1 = 11.4939$ kN ....(5)

Also $\sum F_Y = 0$

$\therefore -W + N_1 cos60 + F_{S1} sin60 + F_{S2} sin45 + N_2 cos45 = 0$

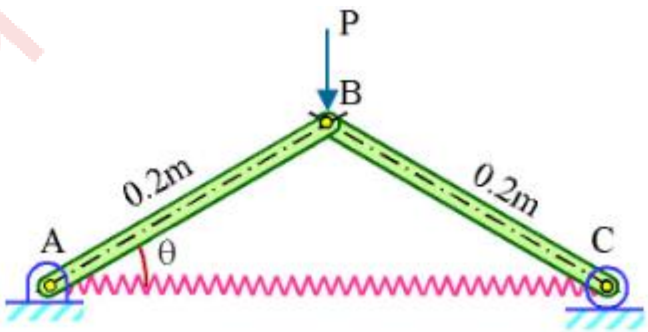
$\therefore N_1 \times 0.5 + 0.25N_1 \times 0.866 + 0.25N_2 \times 0.7071 + N_2 \times 0.7071 = W$ ....(from 1)

$\therefore$ 11.4939$\times$ $0.5 + 0.2165 \times 11.4939 + 0.1768 \times 16.1642 + 16.1642 \times 0.7071 = W$.....(from 4 and 5)

$\therefore$ W = 22.5225kN

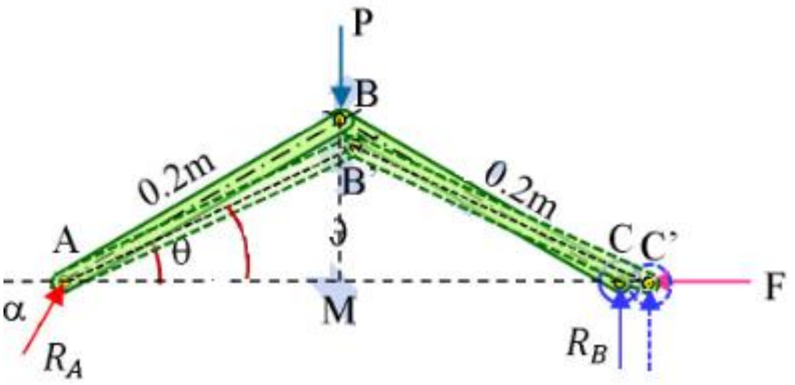
Hence a load of 22.5225kN can be held in the position.

Q6] c) The stiffness of the spring is 600 N/m. find the force 'P' required to maintain equilibrium such that $\theta = 30°$. The spring is unstretched when $\theta = 60°$.neglect weight of the rods. Use method of virtual work. (4)



Solution:-

Principle of virtual work:-



If a body is in equilibrium the total virtual work of forces acting on the body is zero for any virtual displacement.

K = 600N/m, $\theta = 30°$

When $\theta = 60°$

AM = ABcos60° $= 0.2cos60° = 0.1$ and BM = ABsin60° $= 0.2sin60° = 0.1732$m

Given, the spring is unstretched

Unstretched length of the spring = AC = 2AM =0.2m

When $\theta = 30°$

AM = ABcos30° $= 0.2cos30° = 0.1732$ and BM =ABsin30° $= 0.2sin30° = 0.1$m

AC = 2AM =0.4cos30° $= 0.3464$m

Extension in the spring(x) = 0.3464-0.2 = 0.1464m

Spring force(F) = Kx = 600$\times$ 0.1464 = 87.8461N

Let rod AB have a small virtual angular displacement $\delta\theta$ in the clockwise direction

The new position of rods AB' & B'C' is shown dotted

The reaction forces R_A and R_B are not active forces, so they do not perform any virtual work

Let A be the origin and dotted line through A be the X-axis of the system

Consider,

Active forces	Co-ordinate of the point of action along the forces	Virtual displacement
F = 87.8461	X Co-ordinate of C = x_c=0.4cosθ	δx_c= -0.4sinθ $\delta\theta$
P	Y Co-ordinate of C = y_B=0.2sinθ	δy_B= 0.2cosθ $\delta\theta$

By principle of virtual work, $\delta U = 0$

-F$\times$ $\delta x_c - P \times \delta y_B$= 0

-87.8461$\times$ $-0.4sin\theta\delta\theta - P \times 0.2cos\theta\delta\theta = 0$

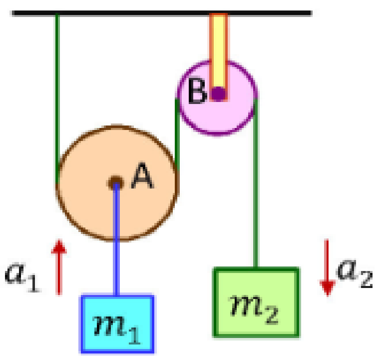
Dividing by $\delta\theta$ and put θ =30, 35.1384sin30° $- 0.2Pcos30° = 0$

$\frac{35.1384sin30°}{0.2\times cos30°} = P$

P= 101.4359N

Hence, the force P required to maintain equilibrium = 101.4359kN

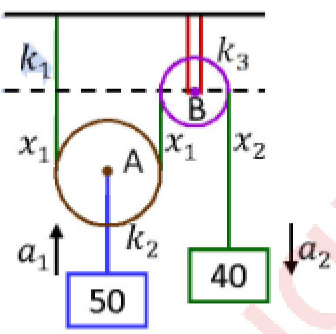
Q6] d) Two masses are interconnected with the pulley system. Neglecting frictional effect of pulleys and cord, determine the acceleration of masses m_1, take m_1 = 50kg and m_2= 40kg. (4)



Solution:-

$\boldsymbol{m_1}$ = 50kg and $\boldsymbol{m_2}$ = 40kg

Let x_1 and x_2 be displacement of pulleys A and B respectively.



The string around the pulley is of constant length.

k_1 + x_1 + k_2 + x_1 + k_3 + x_2= L

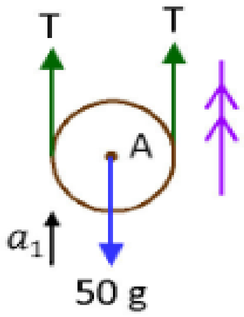
k_1 + k_2+ k_3 + 2x_1 = L - x_2

On differentiating w.r.t 't' we get 2v_1 = -v_2

Where $\frac{dx}{dt}$ = v the velocities of the two pulleys again differentiating w.r.t we get

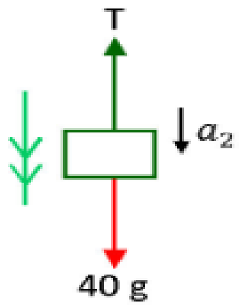
2a_1 = a_2

$a_1 = \frac{1}{2} a_2$ ....(1)



Let T be the Tension in the cord

Applying Newton's 2nd law , $\Sigma F_Y = m_1 a_1$



2T- 50g = 50 x (0.5) a_2 ....from (1)

2T = 50g + 25a_2.

T = 25g +12.5a_2. ....(2) (dividing by 2)

Applying Newton's 2nd law to 40kg block

$\Sigma F_Y = m_2 a_2$

40g-T = 40a_2

40g – (25g +12.5a_2) = 40a_2 ....from (2)

40g – 25g – 12.5a_2 = 40a_2

15g = 52.5a_2

$a_2 = \frac{15g}{52.5}$ = 2.8029 m/s^2

Acceleration of the mass m_2 = 2.8029 m/s^2