MUMBAI UNIVERSITY

SEMESTER-1

ENGINEERING MECHANICS SOLVED PAPER-DECEMBER 2016

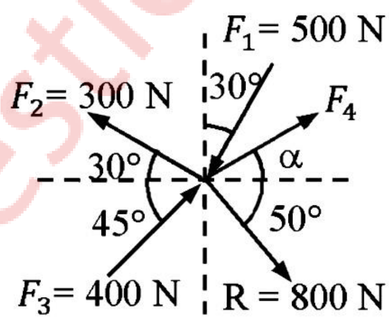
N.B:-(1)Question no.1 is compulsory.

(2)Attempt any 3 questions from remaining five questions.

(3)Assume suitable data if necessary,and mention the same clearly.

(4)Take g=9.81 m/s^2,unless otherwise specified.

Q.1(a) Find the force F_4, so as to give the resultant of the force as shown in the figure given below. (4 marks)

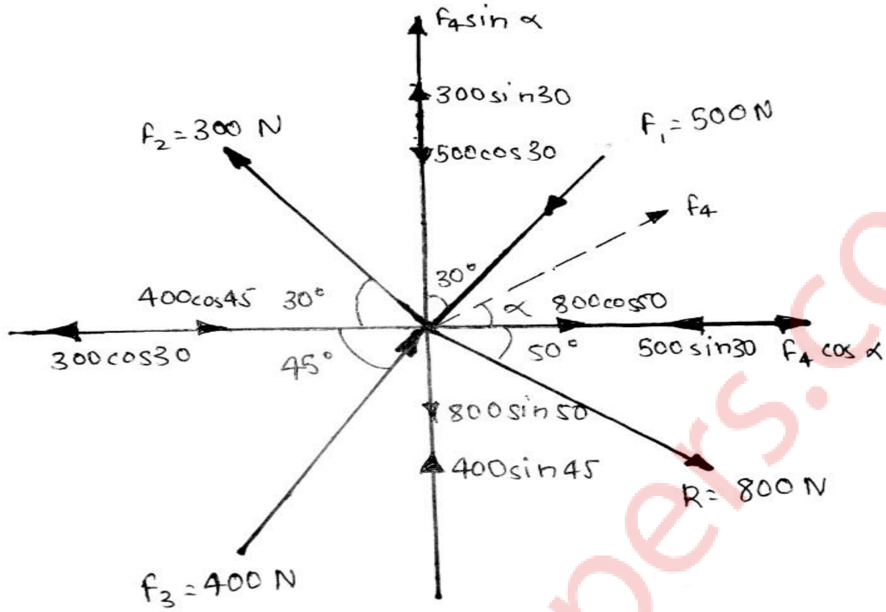


Solution :

Given : Forces and their resultant

To find : Force F_4

Solution :



Assume that force F_4 acts at an angle θ

Taking forces having direction towards right as positive and forces having direction upwards as positive.

Resolving forces along X direction :

$-F_1\sin 30 - F_2\cos 30 + F_3\cos 45 + F_4\cos\theta = R\cos 50$

$-500\sin 30 - 300\cos 30 + 400\cos 45 + F_4\cos\theta = 800\cos 50$

$F_4\cos\theta = 741.195$ …………(1)

Resolving forces along Y direction :

$-F_1\cos 30 + F_2\sin 30 + F_3\sin 45 + F_4\sin\theta = -R\sin 50$

$-500\cos 30 + 300\sin 30 + 400\sin 45 + F_4\sin\theta = -800\sin 50$

$F_4\sin\theta = -612.6656$ ……….(2)

Squaring and adding (1) and (2)

$(F_4\sin\theta)^2 + (F_4\cos\theta)^2 = (-612.6656)^2 + (741.195)^2$

$F_4^2(\sin^2\theta + \cos^2\theta) = 924729.1173$

$F_4 = 961.6284$ N

Dividing (2) by (1)

$\frac{F_4 sin\theta}{F_4 cos\theta} = \frac{-612.6656}{741.195}$

tan θ = -0.8266

θ = 39.5769° (in fourth quadrant)

F_4 = 961.6284 N (at an angle 39.5769° in fourth quadrant)

Q.1(b) A particle starts from rest from origin and it's acceleration is given by a = $\frac{k}{(x+4)^2}$ m/s².Knowing that v = 4 m/s when x = 8m,find :

(1)Value of k

(2)Position when v = 4.5 m/s (4 marks)

Solution :

Given : Particle starts from rest

a = $\frac{k}{(x+4)^2}$ m/s²

v = 4m/s at x = 8m

To find : Value of k and position when v= 4.5m/s

Solution:

We know that a = v$\frac{dv}{dx}$

v$\frac{dv}{dx}$=$\frac{k}{(x+4)^2}$

v dv = k$(x+4)^{-2}$ dx

Integrating both sides

$\int vdv = \int k(x+4)-2 \ dx$

$\frac{v^2}{2} = \frac{-k}{x+4} + c_1$ ………(1)

Putting x=0 and v=0

$c_1 = \frac{k}{4}$ ……….(2)

$\frac{v^2}{2} = \frac{-k}{x+4} + \frac{k}{4}$ …….(From 1 and 2) ……..(3)

k = 48

From (3)

$\frac{v^2}{2} = \frac{-48}{x+4} + \frac{48}{4}$

$v^2 = 24 - \frac{96}{x+4}$

Substituting v=4.5 m/s

$4.5^2 = 24 - \frac{96}{x+4}$

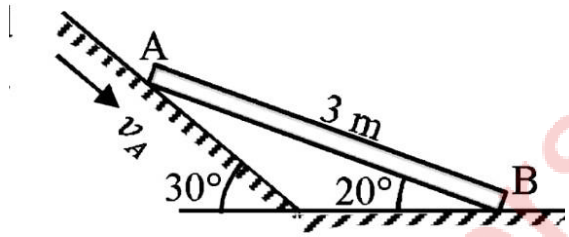
$\frac{96}{3.75} = x+4$

x = 21.6 m

Value of k = 48

The particle is at a distance of 21.6 m from origin when v = 4.5 m/s

Q.1(c)Rod AB of length 3 m is kept on a smooth plane as shown in the given figure.The velocity of end A is 5 m/s along the inclined plane.Locate the ICR and find velocity of end B. (4 marks)



Solution :

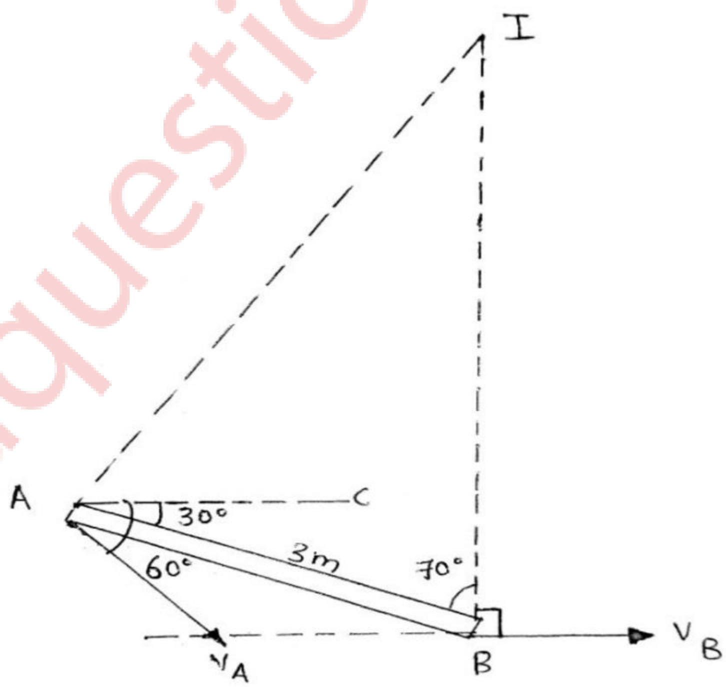
Given : Length of rod AB = 3m

v_a = 5 m/s

To find: ICR

Velocity of end B

Solution :



Solution:

Given : AB=3m

v_A=5m/s

To find : ICR

v_B

Solution:

ICR is shown in the diagram denoted by point I

Assume ω to be the angular velocity of rod AB

BY GEOMETRY:

∠CAD=30°, ∠ABD=20°

∠CAB= ∠ABD=20°

∠CAI=90°-30°

=60°

∠BAI = ∠CAI+ ∠CAB=60°+20°

=80°

In △IAB. ∠AIB = 180°-80°-70°

=30°

BY SINE RULE :

$$\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$$

$$\frac{3}{\sin 30} = \frac{IB}{\sin 80} = \frac{IA}{\sin 70}$$

IB=5.9088 m

IA=5.6382 m

$\omega = \frac{v_a}{r} = \frac{v_a}{IA} = \frac{5}{5.6382}$ = 0.8868 rad/s(anti-clockwise)

v_B = r ω

= IB x ω

= 5.9088 x 0.8868

=5.2401 m/s (Towards right)

Velocity of end B=5.2401 m/s(towards right)

Q.1(d)What is a zero force member in a truss? With examples state the conditions for a zero force member. (4 marks)

Solution:

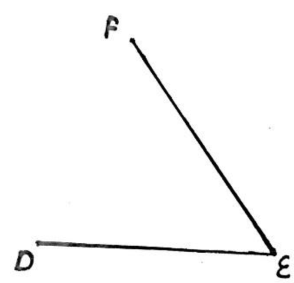
1. In engineering mechanics, a **zero force member** is a **member** (a single truss segment) in a truss which, given a specific load, is at rest that is it is neither in tension, nor in compression

2.The conditions for a zero force member are :

(a) In a truss,at a joint there are only three members of which two are collinear and if the joint has no external load then the non collinear members is a zero force member.

e.g.: DF is a zero force member in the given figure.

(b) In a truss,if at an unsupported joint there are only two members and if the joint has no external load then both the members are zero force members.



e.g.:DE and EF are zero force members in the given figure.

Q1(e)A glass ball is dropped on a smooth horizontal floor from which it bounces to a height of 9m.On the second bounce it rises to a height of 6 m.From what height was the ball dropped and find the coefficient of restitution between the glass and the floor. (4 marks)

Solution:

Given : First bounce height = 9 m

Second bounce height = 6 m

To find : Co-efficient of restitution

Solution :

Assume the ball fall from height h and then rebounds to height h_1

Before first bounce :

u = 0 , s = h , a = -g

Velocity after first bounce

$u_1 = ev = e\sqrt{2gh}$ ………(e is the co-efficient of restitution)

Using kinematical equation : $v_1^2 = u_1^2 + 2as_1$

$0^2 = e^2 \times 2gh - 2gh_1$

$2gh_1 = e^2 \times 2gh$

$h_1 = e^2h$ ……….(1)

Assume the ball rises to height of h_2 after the second bounce

$h_2 = e^2h_1$ …….(2)

Putting h_1 = 9 m and h_2 = 6 m

$6 = e^2 \times 9$

$e^2 = \frac{6}{9}$ ……..(3)

e = 0.8165

From (1) and (3)

$9 = \frac{6}{9} \times h$

h = 13.5 m

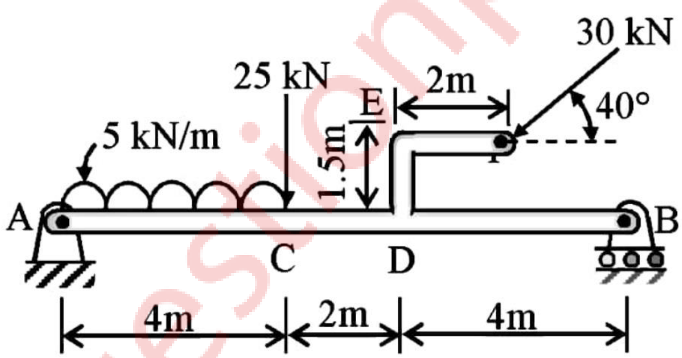
Co-efficient of restitution = 0.8165

Height from which ball was dropped = 13.5 m

Q2(a)The given figure shows a beam AB hinged at A and roller supported at B.

The L shaped portion is welded at D to the beam AB.

For the loading shown,find the support reactions. (8 marks)

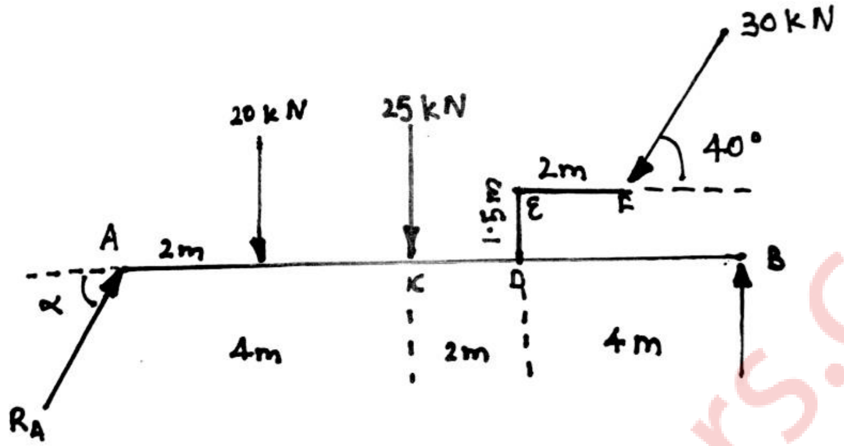


Solution :

Given : Beam AB hinged at A and roller supported at B and different forces acting on it.

To find : Support reactions

Solution :



Force of distributed load AC = 5 x 4

= 20 kN

Distance of force acting from point A = $\frac{4}{2}$ =2m

The beam is in equilibrium

Applying the conditions of equilibrium

$\Sigma M_A = 0$

$-20 \times 2 - 25 \times 4 - 30\sin 40 \times 8 + 30\cos 40 \times 1.5 + R_B \times 10 = 0$

$10R_B = 40+100+240\sin 40-45\cos 40$

$10R_B = 259.797$ kN

$R_B = 25.9797$ kN (Acting upwards)

Applying the conditions of equilibrium

$\Sigma F_X = 0$

$R_{AX} - 30\cos 40 = 0$

$R_{AX} = 22.9813$ kN ………(1)

$\Sigma F_Y = 0$

$R_{AY} - 20 - 25 - 30\sin 40 + R_B = 0$

$R_{AY} = 38.3039$ kN ………..(2)

$R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$

$R_A = \sqrt{22.9813^2 + 38.3039^2}$

R_A = 44.6691 kN

$\alpha = \tan^{-1}\left(\frac{R_{AY}}{R_{AX}}\right)$

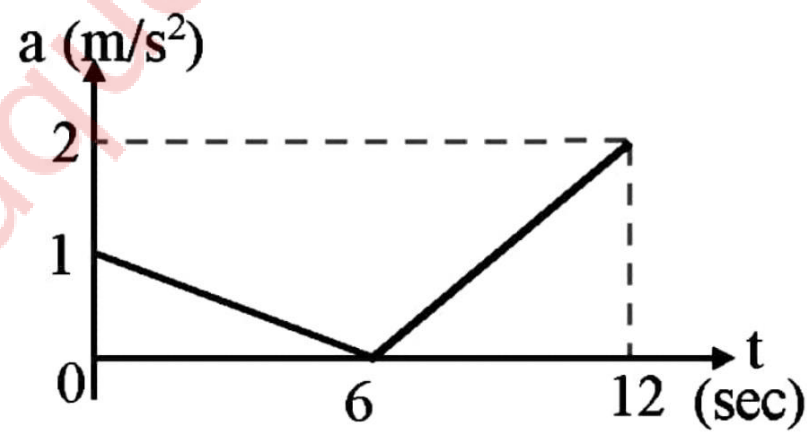
$= \tan^{-1}\frac{38.3039}{22.9813}$

$\alpha = 59.0374^o$

Reaction at hinge A = 44.6691 kN (59.0374° in first quadrant) Reaction at roller B = 25.9797 kN (Towards up)

Q.2(b)The acceleration time diagram for a linear motion is shown.

Construct velocity time diagram and displacement time diagram for the motion assuming that the motion starts with a initial velocity of 5 m/s from the starting point. (6 marks)



Solution :

Given : Acceleration time graph

To draw : Velocity time graph

Displacement time graph

Solution :

FOR VELOCITY TIME GRAPH :

We know that the area under a-t graph gives the velocity.

AB on a-t graph represents linearly varying deceleration

$v_0 = 5$ m/s

$v_1 = v_0 + A(\triangle\ OAB)$

$= 5 + \frac{1}{2} \times 6 \times 1$

$= 8$ m/s

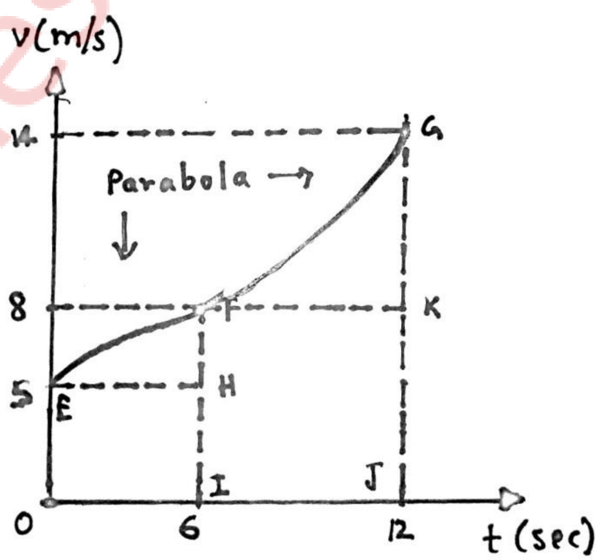
BC on a-t graph represents linearly varying acceleration

$v_2 = v_1 + A(\triangle BCD)$

$= 8 + \frac{1}{2} \times (12-6) \times 2$

$= 14$ m/s

The velocity time graph is drawn below :



FOR DISPLACEMENT TIME GRAPH :

Area under v-t graph gives the displacement

Area under EF = A(EFH) + A(□EHIO)

$= \frac{2}{3} \times 6 \times (8-5) + 6 \times 5$

$= 42$ m

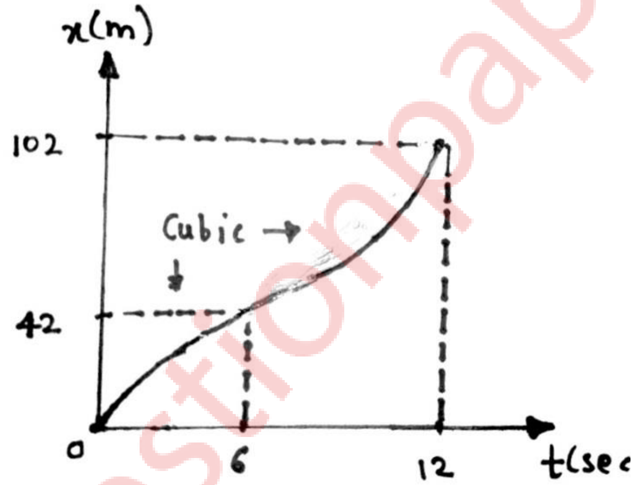
Area under FG = A(GFH) +A(□FIJK)

$= \frac{1}{3} \times (12 - 6) \times (14-8) + (12 - 6) \times 8$

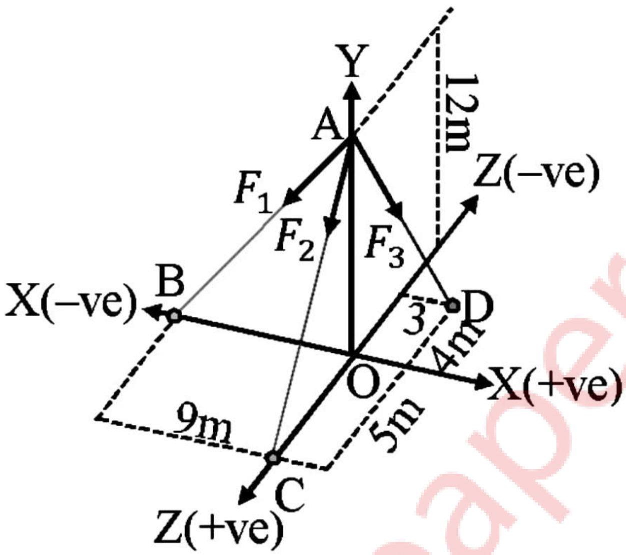
$= 12 + 48$

$= 60$ m

The displacement time graph is shown below :



Q.2(c)The resultant of the three concurrent space forces at A is $\bar{R} = (-788\bar{j})$ N. Find the magnitude of F1,F2 and F3 forces. (6 marks)



Solution :

Given : A=(0,12,0)

B=(-9,0,0)

C=(0,0,5)

D=(3,0,-4)

Resultant of forces = $(-788\bar{j})$ N

To find : Magnitude of forces F1,F2,F3

Solution:

Assume $\bar{a}$,$\bar{b}$,$\bar{c}$ and $\bar{d}$ be the position vectors of points A,B,C and D respectively w.r.t origin O

$\overline{OA} = \bar{a} = 12\bar{j}$

$\overline{OB} = \bar{b} = -9\bar{i}$

$\overline{OC} = \bar{c} = 5\bar{k}$

$\overline{OD} = \bar{d} = 3\bar{i} - 4\bar{k}$

$\overline{AB} = \bar{b} - \bar{a}$

$= -9\bar{i} - 12\bar{j}$

$\overline{AC} = \bar{c} - \bar{a} = 5\bar{k} - 12\bar{j}$

$\overline{AD} = \bar{d} - \bar{a} = 3\bar{i} - 12\bar{j} - 4\bar{k}$

Sr.no.	Vector	Magnitude
1.	$\overline{AB}$	15
2.	$\overline{AC}$	13
3.	$\overline{AD}$	13

Sr.no	Vector	Unit vector=$\frac{vector}{Magnitude\ of\ vector}$
1.	$\overline{AB}$	$\frac{-3}{5}\tilde{i} - \frac{4}{5}\bar{j}$
2.	$\overline{AC}$	$\frac{-12}{13}\bar{j} + \frac{5}{13}\bar{k}$
3.	$\overline{AD}$	$\frac{3}{13}\bar{i} - \frac{12}{13}\bar{j} - \frac{4}{13}\bar{k}$

Force along $\overline{AB} = \overline{F1} = F1(\frac{-3}{5}\bar{i} - \frac{4}{5}\bar{j})$

Force along $\overline{AC} = \overline{F2} = F2(\frac{-12}{13}\bar{j} + \frac{5}{13}\bar{k})$

Force along $\overline{AB} = \overline{F3} = F3(\frac{3}{13}\bar{i} - \frac{12}{13}\bar{j} - \frac{4}{13}\bar{k})$

Resultant force($\bar{R}$) = $\overline{F1}$+$\overline{F2}$+$\overline{F3}$

$-788\bar{j} = F1(\frac{-3}{5}\bar{i} - \frac{4}{5}\bar{j}) + \overline{F2}(\frac{-12}{13}\bar{j} + \frac{5}{13}\bar{k}) + \overline{F3}(\frac{3}{13}\bar{i} - \frac{12}{13}\bar{j} - \frac{4}{13}\bar{k})$

$0\bar{j} - 788\bar{j} + 0\bar{k} = \overline{F1}(\frac{-3}{5}\bar{i} - \frac{4}{5}\bar{j}) + \overline{F2}(\frac{-12}{13}\bar{j} + \frac{5}{13}\bar{k}) + \overline{F3}(\frac{3}{13}\bar{i} - \frac{12}{13}\bar{j} - \frac{4}{13}\bar{k})$

Comparing the equation on both sides

$\frac{-3F1}{5} + \frac{3F3}{13} = 0$ ………..(1)

$-\frac{4F1}{5} - -\frac{12F2}{13} - \frac{12F2}{13} = -788$ ………(2)

$\frac{5F2}{13}$ -$\frac{4F3}{13}$ = 0 ……………(3)

Solving (1),(2) and (3)

F1=153.9063 N

F2=320.125 N

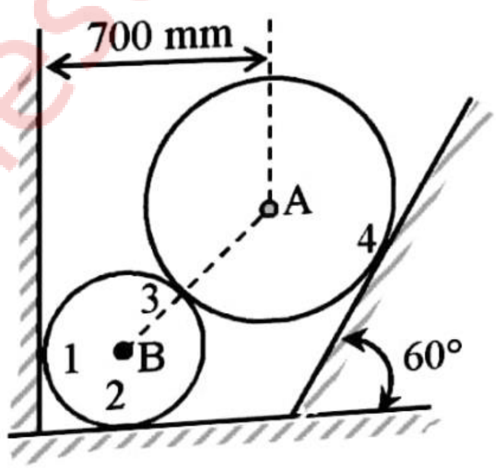
F3=400.1563 N

Answer

Sr.no.	Force	Magnitude
1.	F1	153.9063 N
2.	F2	320.125 N
3.	F3	400.1563 N

Q.3(a)Two spheres A and B of weight 1000N and 750N respectively are kept as shown in the figure..Determine reaction at all contact points 1,2,3 and 4.

Radius of A is 400 mm and radius of B is 300 mm. (8 marks)



Solution :

Given : Two spheres are in equilibrium

W_1=1000 N

W_2=750 N

r_A=400 mm

r_B=300 mm

To find : Reaction forces at contact points 1,2,3 and 4

Solution:

BC = BP = 300mm = 0.3m

AP = 400 mm = 0.4 m

AB = AP + BP

= 0.7m

CO = BC + BO

0.7 = 0.3 + BO

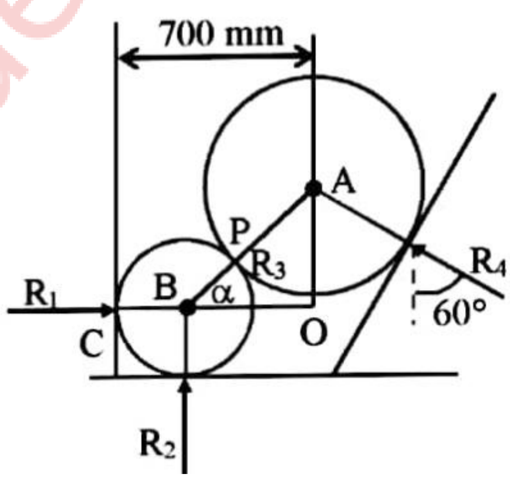
BO = 0.4m

In △AOB

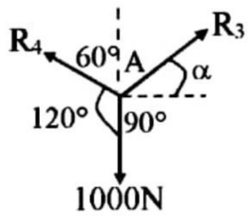
$\cos \alpha = \frac{BO}{AB} = \frac{0.4}{0.7}$

$\alpha = \cos^{-1}(0.5714)$

$\alpha = 55.1501^o$



Forces R_3,R_4 and 1000N are under equilibrium at point A



Applying Lami's theorem

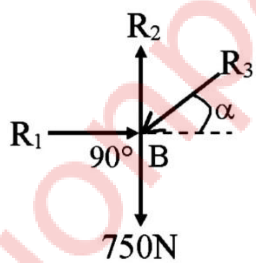
$$\frac{R3}{sin120} = \frac{1000}{\sin(150-\alpha)} = \frac{R4}{\sin(90+\alpha)}$$

$$\frac{R3}{sin120} = \frac{1000}{\sin(150-55.1501)} = \frac{R4}{\sin(90+55.1501)}$$

Solving the equations

R_3 = 869.1373 N

R_4 = 573.4819 N



Forces R_1,R_2,R_3 and 750N are under equilibrium at B

Applying conditions of equilibrium

$\Sigma F_Y=0$

$-R_3 \sin\alpha - 750 + R2 = 0$

$R_2 = 869.1373 \sin 55.1501 + 750$

R_2=1463.2591 N (Acting upwards)

Applying conditions of equilibrium

$\Sigma F_X=0$

$R_1 - R_3 \cos\alpha = 0$

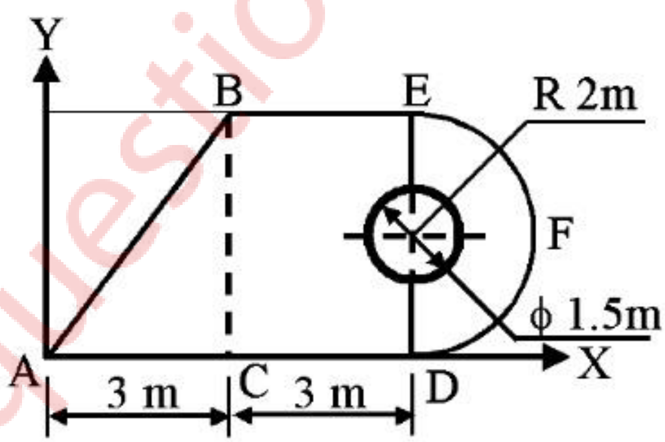
$R_1 = 869.1373 \cos 55.1501$

R_1=496.65 N(Acting towards right)

ANSWER :

Sr.no.	Point	Force
1.	R_1	496.65 N(Towards right)
2.	R_2	1463.2591 N(Towards up)
3.	R_3	869.1373 N(55.1501° in first quadrant)
4.	R_4	573.4819 N(30° in second quadrant)

Q.3(b)A circle of diameter 1.5 m is cut from a composite plate.Determine the centroid of the remaining area of plate. (6 marks)



Solution :

PART	AREA(in m^2)	X co-ordinate of centroid(m)	Y co-ordinate of centroid(m)	$A_x(m^3)$	$A_y(m^3)$
Rectangle	3 x 4 =12	$3 + \frac{3}{2} = 4.5$	2	54	24
Triangle	0.5 x 3 x 4 =6	$3 - \frac{3}{3} = 2$	$\frac{4}{3} = 1.3333$	12	8
Semicircle	$0.5 \times 2^2 \times \pi$ =6.2832	$6+\frac{4 \times 2}{3\pi}$ =6.8488	2	43.0324	12.5664
Circle (To be removed)	$-\pi r2$ $=-0.75^2 \times \pi$ =-1.7671	6	2	-10.6029	-3.5343
Total	22.5161			98.4295	41.0321

X co-ordinate of centroid ($\bar{x}$) = $\frac{\Sigma Ax}{\Sigma A} = \frac{98.4295}{22.5161}$ = 4.3715 m

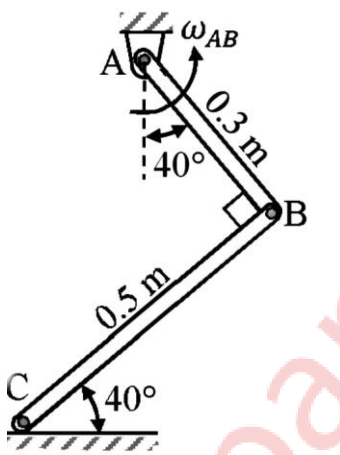
Y co-ordinate of centroid($\bar{y}$) = $\frac{\Sigma Ay}{\Sigma A} = \frac{41.0321}{22.5161}$ = 1.8223 m

Centroid is at (4.3715,1.8223)m

Q.3(c)A rod AB has an angular velocity of 2 rad/sec,counter clock wise as shown.End C of rod BC is free to move on a horizontal surface.Determine:

(1)Angular velocity of rod BC

(2)Velocity of C (6 marks)



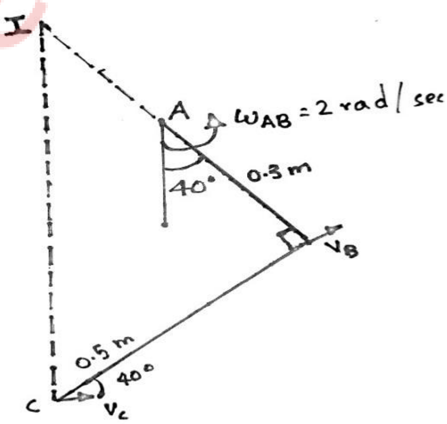
Solution :

Given : ω_{AB}=2 rad/sec (anti clockwise)

To find : ω_{BC}

v_C

Solution :



BY GEOMETRY :

Assume I to be the ICR of rod BC

In △IAB,

∠BIC=40°

∠IBC=90°

$\tan 40 = \frac{BC}{IB} = \frac{0.5}{IB}$

$\sin 40 = \frac{BC}{IC} = \frac{0.5}{IC}$

IB = 0.5959m and IC = 0.7779m

$v_B = r\omega$

$= AB x \omega_{AB}$

= 0.3 x 2

= 0.6 m/s

$\omega_{BC} = \frac{vB}{r}$

$= \frac{vB}{IB}$

$= \frac{0.6}{0.5959}$

=1.0069 rad/sec

The direction is anti-clockwise

$v_C = r\omega$

$= IC \text{ x } \omega_{BC}$

= 0.7779 x 1.0069

= 0.7832 m/s

The direction of v_C is towards right

Angular velocity of BC=1.0069 rad/sec(anto clockwise)

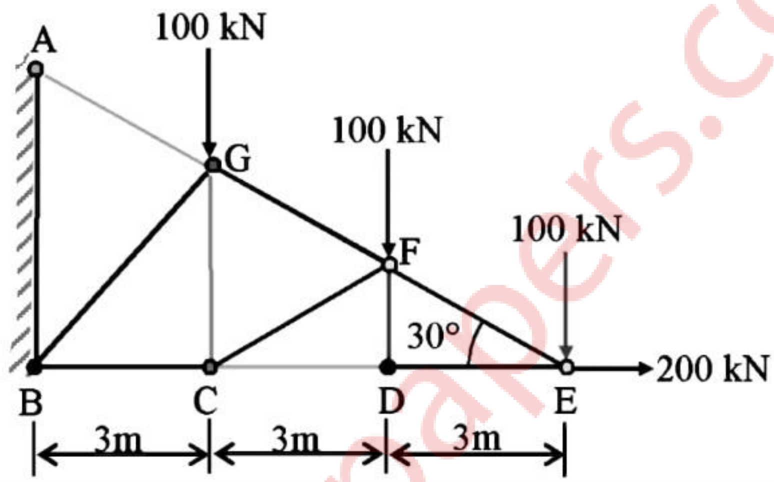
v_C=0.7832 m/s(Towards right)

Q.4(a)A truss is loaded and supported as shown.Determine the following:

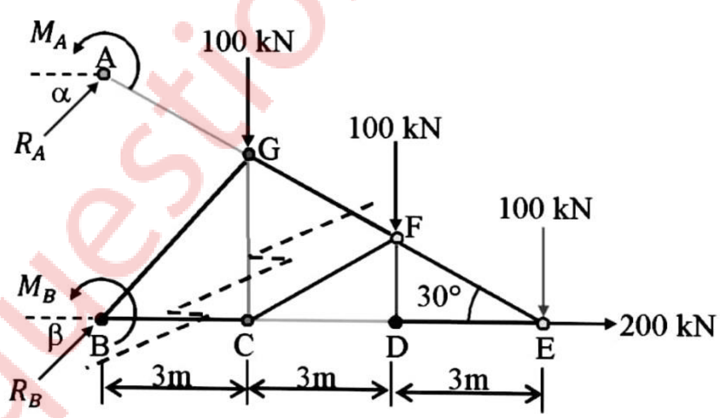
(1)Identify the zero force members,if any

(2)Find the forces in members EF,ED and FC by method of joints.

(3)Find the forces in members GF,GC and BC by method of sections (8 marks)

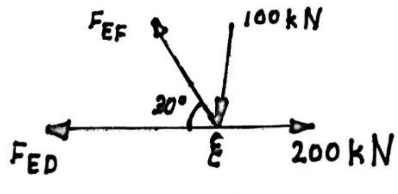


Solution:



By analysis of truss,we can say that **DE is zero force member**

METHOD OF JOINTS:



Joint E:

Applying the conditions of equilibrium

$\Sigma F_Y=0$

$F_{EF}\sin 30-100=0$

$F_{EF}=200$ kN

Applying the conditions of equilibrium

$\mathbf{\Sigma F_X=0}$

$-F_{EF}\cos 30-F_{ED}+200=0$

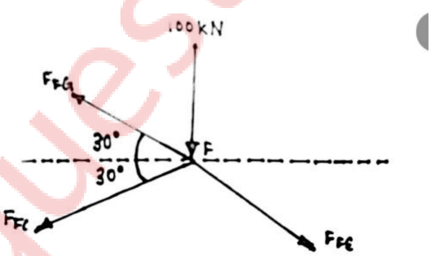
$-200\cos 30+200=F_{ED}$

$\mathbf{F_{ED}=26.7949}$ **kN**

$\triangle$FED is congruent to $\triangle$FCD

$\angle FCD=\angle FED=30^o$

JOINT F:



Applying the conditions of equilibrium

$\mathbf{\Sigma F_Y=0}$

$F_{FG}\sin 30-F_{FC}\sin 30-F_{FE}\sin 30-100=0$

$F_{FG}-F_{FC}-200=200$

F_{FG}-F_{FC}=400 ……….(1)

ΣF_X=0

-F_{FG}cos30-F_{FC}cos30+F_{FE}cos30=0

Dividing by cos30

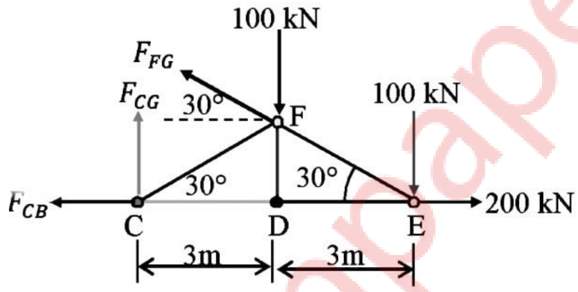
F_{FG} +F_{FC} = 200 ………(2)

Solving (1) and (2)

F_{FG}=300 kN

F_{FC}=-100 kN

METHOD OF SECTIONS:



In △FED

tan 30 = $\frac{FD}{DE}$

DE=3m

FD=$\sqrt{3}$ m

Consider the equilibrium of the truss section

ΣM_C=0

F_{FG}cos 30 x F_D + F_{FG}sin30 x CD -100 x CD -100 X CE = 0

3F_{FG}=900

F_{FG}=300 kN

Applying the conditions of equilibrium

ΣF_X=0

-F_{FG}cos30-F_{CB}+200=0

$-300\cos30+200=F_{CB}$

$\mathbf{F_{CB}=-59.8076\ kN}$

$\Sigma F_Y=0$

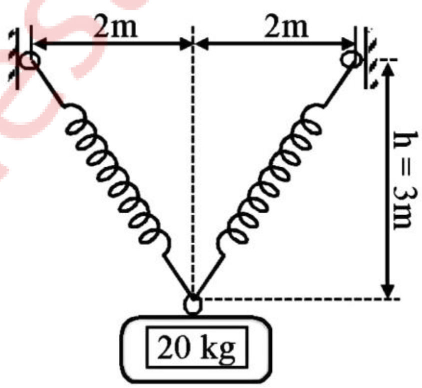
$F_{CG}+F_{FG}\sin30-100-100=0$

$\mathbf{F_{CG}=50\ kN}$

Answer :

Member of truss	Magnitude of force(kN)	Nature of force
BC	59.8076	Compression
GC	50	Tension
GF	300	Tension
FC	100	Compression
ED	26.7949	Tension
EF	200	Tension

Q.4(b)A cylinder has a mass of 20kg and is released from rest when h=0 as shown in the figure.Determine its speed when h=3m.The springs have an unstretched length of 2 m.Take k=40 N/m. (6 marks)



Solution :

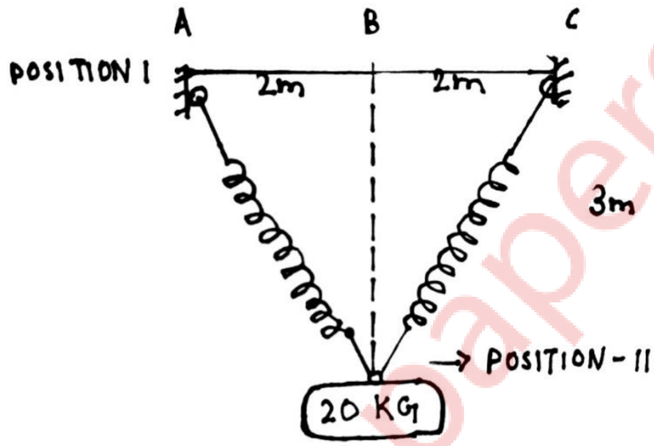
Given : m=20 kg

h=0

k=40 N/m

To find: Speed when h=3m

Solution:



POSITION 1

Un-stretched length of spring = 2 m

Extension (x_1) of spring = 0

Spring energy $=E_{S1} = \frac{1}{2} k{x_1}^2$

$=0$

$PE_1=mgh$

$=0$ J

$KE_1=0$ J

AT POSITION II:

Let h=-3m

$PE_2 = mgh = 20 \times 9.81 \times (-3)$

$=-588.6$ J

$KE_2 = \frac{1}{2} \times 20v^2$

$=10v^2$

In△ABD,

By Pythagoras theorem

$AD=\sqrt{2^2+3^2}$

$=3.6056$ m

Extension(x_2) of spring = 3.6056 - 2=1.6056 m

$E_{S2} = \frac{1}{2}kx_2^2 = \frac{1}{2} \times 40 \times 1.6056^2$

$=51.5559$ J

Applying work-energy principle

$U_{1\text{-}2} = KE_2 - KE_1$

$PE_1 - PE_2 + E_{S1} - E_{S2} = KE_2 - KE_1$

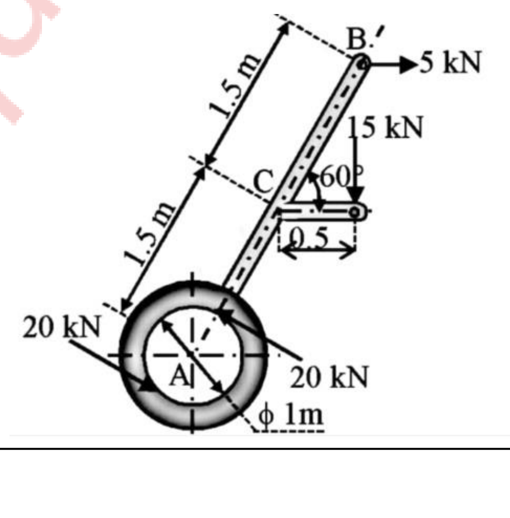
$588.6 - 51.5559 = 10v^2$

$v = 7.3283$ m/s

Speed when h=3m is 7.3283 m/s

Q.4(c)A machine part is subjected to forces as shown.Find the resultant of forces in magnitude and in direction.

Also locate the point where resultant cuts the centre line of bar AB. (6 marks)



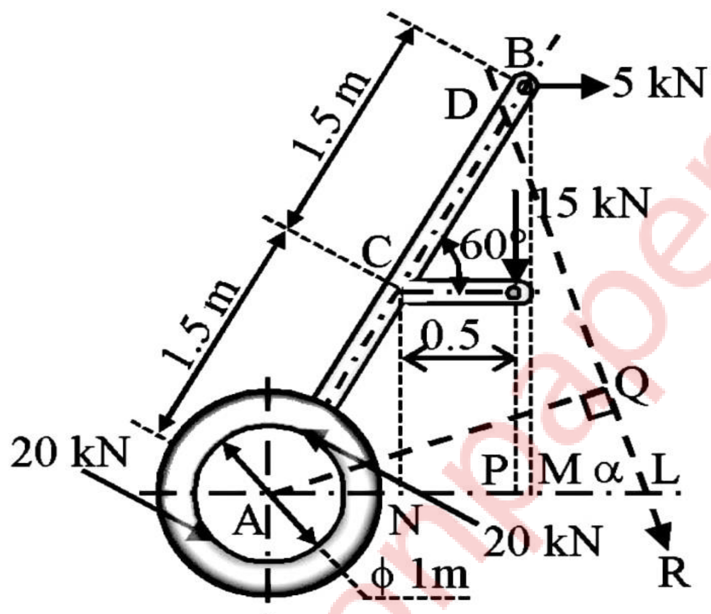
Solution:

Given : A machine subjected to various forces

To find : Resultant of forces

Point where the resultant force cuts the bar AB

Solution:



In △BAM, ∠A=60°

AB=3 m

BM=3sin60

=2.5981 m

In △CAN

AC=1.5m

AN=1.5cos60

=0.75 m

AP=AN+NP

=0.75+0.5

=1.25 m

Two 20N forces are equal and opposite in direction.Hence,they form a couple

Perpendicular distance between two 20 N forces = 1m

Moment of couple = 20 x 1

=20 kN-m (Anti clockwise)

Assume R is the resultant of the forces and it is inclined at an angle θ with horizontal

ΣF_X=5 kN

ΣF_Y=-15 kN

R=$\sqrt{F_x^2 + F_Y^2}$

=$\sqrt{5^2 + (-15)^2}$

=15.8114 kN

θ = $\tan^{-1}(\frac{R_y}{R_x})$

=$\tan^{-1}(\frac{-15}{5})$

=71.5651^o (in fourth quadrant)

Assume that the resultant cut the center line of bar AB at point D

Applying Varigon's theorem

$\Sigma M_A = \Sigma M_A^R$

-5 x BM -15 x AP +20 = R x AQ

-11.7405 = -15.8114 x AQ

AQ = 0.7425 m

In △AQL, ∠ALQ=θ

∠QAL=90 - θ

∠BAL=60

∠QAD=60 - (90 - θ)

= θ - 30

=71.5651 - 30

$=41.5651^o$

In△DAQ,cos QAD= $\frac{AQ}{AD}$

AD= $\frac{AQ}{\cos DAQ} = \frac{0.7425}{\cos 41.5651}$ = 0.9924 m

Resultant force = 15.8114 kN(at 71.5651° in fourth quadrant)

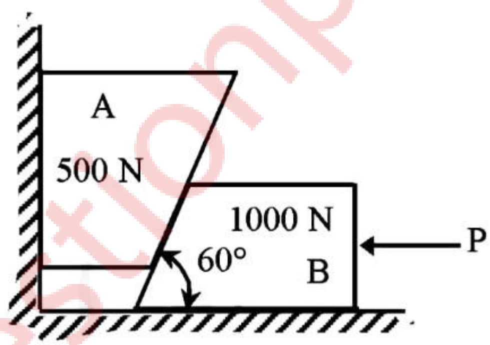
It cuts the center line of bar AB at point D such that AD=0.9924m

---

Q.5(a)Two blocks A and B are resting against the wall and floor as shown in the figure.Find the minimum value of P that will hold the system in equilibrium.

Take μ=0.25 at the floor,μ=0.3 at the wall and μ=0.2 between the blocks.

(8 marks)



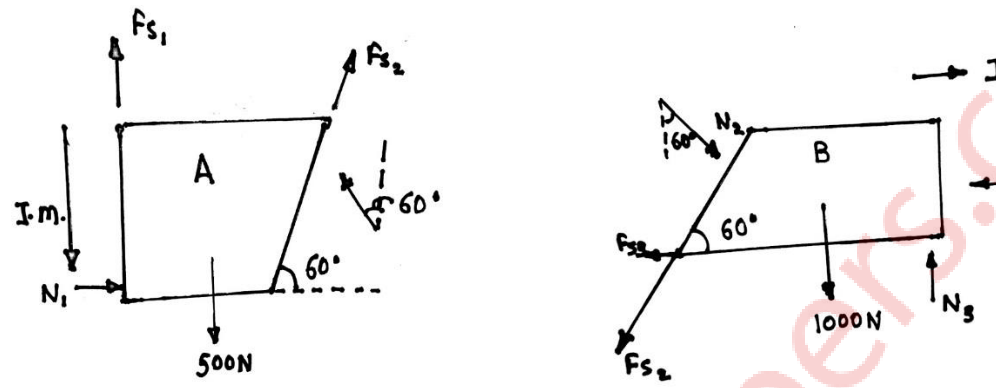
Solution:

Given : μ=0.25 at floor

μ=0.2 between blocks

To find : Minimum value of force P

Solution :



The impending motion of block A is to move down and that of block B is to move towards left

$F_{S1}=\mu 1 N_1=0.3N_1$

$F_{S2}=\mu_2 N_2=0.2N_2$

$F_{S3}=\mu_3 N_3=0.25N_3$ ……………..(1)

Block A is under equilibrium

Applying conditions of equilibrium

$\Sigma F_Y=0$

$-500+F_{S1}+F_{S2}\sin60+N_2\cos60=0$

$0.3N_1+0.6732N_2=500$ ………….(2)

Similarly,

$\Sigma F_X=0$

$N_1+F_{S2}\cos60-N_2\sin60=0$

$N_1+0.2N_2 \text{ x } 0.5 - N_2 \text{ x } 0.866 = 0$ (From 1)

$N_1-0.766N_2=0$ ……….(3)

Solving (2) and (3)

$N_1=424.1417$ N

$N_2=553.71$ N

Applying conditions of equilibrium on block B

$\Sigma F_Y = 0$

$-1000 + N3 - F_{S2}\sin 60 - N_2\cos 60 = 0$

$N_3 - 0.6732N_2 = 1000$

$N_3 = 1372.7576$ N

$\Sigma F_X = 0$

$-P - F_{S3} - F_{S2}\cos 60 + N_2\sin 60 = 0$

$-0.25N_3 - 0.2N_2 \times 0.5 + N_2 \times 0.866 = P$

P=80.9525 N

The minimum value of force P that will hold the system in equilibrium is 80.9525 N

Q.5(b)A shot is fired with a bullet with an initial velocity of 20 m/s from a point 10 m infront of a vertical wall 5 m high.

Find the angle of projection with the horizontal to enable the shot to just clear the wall.

Also find the range of the shot where the bullet falls on the ground. (6 marks)

Solution :

Given : u=20 m/s

Distance from wall=10m

Height of wall=5m

To find :Angle of projection

Range of shot

Solution:

Let α be the angle of projection of projectile

Equation of projectile is given by:

y=xtanα - $\frac{gx^2}{2u^2}$**sec²α**

(10,5) are the co-ordinates of top of wall when O is taken as origin

Substituting x=10 and y=5 in the projectile equation

$5=10\tan\alpha-\frac{g10^2}{2\ x\ 20^2}\sec^2\alpha$

1.2262tan2 α-10tan α+6.2262=0

Solving the quadratic equation

tan α=7.4758 or tan α=0.6792

$\alpha=82.381^o$ or $\alpha=34.184^o$

Range of a projectile is given by R = $\frac{u^2 sin2\alpha}{g}$

Substituting $\alpha=82.381^o$ or $\alpha=34.184^o$

$R=\frac{20^2\sin(2x82.381)}{9.81}$

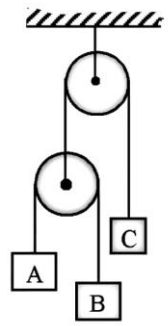
=10.7161 m

$R=\frac{20^2\sin(2\ x\ 34.184)o\backslash}{9.81}$

=37.902 m

Angle of projectile should be 82.381° or 34.184° and the corresponding ranges will be 10.7161 m and 37.902 m respectively.

Q.5(c)Three blocks A,B and C of masses 3 kg,2 kg and 7 kg respectively are connected as shown.Determine the acceleration of A,B and C.Also find the tension in the string. (6 marks)



Solution :

Given : m_A=3kg

m_B=2 kg

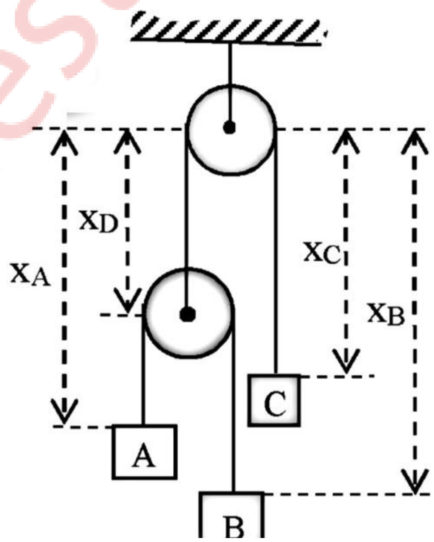
m_C=7kg

To find: Acceleration of blocks A,B and C

Solution:

Assuming the pulleys and the connecting inextensible strings are massless and frictionless

Assume x_A,x_B,x_C and x_D be the displacements of blocks A,B,C and D respectively.



Assume blocks A,B,C and D move downwards.So x_A,x_B,x_C and x_D will increase

Assume k be the length of string that remains constant irrespective of positions of A,B and C.

As the length of string is constant

$(x_A-x_D)+(x_B-x_D)+k=0$

$x_A+x_B-2x_D+k=0$

Differentiating w.r.t t

$v_A+v_B-2v_D=0$

Differentiating once again w.r.t to t

$a_A+a_B-2a_D=0$ …….(1)

and $x_D+x_C+k=0$

$x_D=-x_C-k$

Differentiating w.r.t t

$v_D=-v_C$

Differentiating once again w.r.t to t

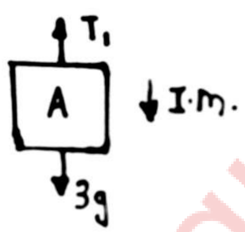
$a_D=-a_C$ ………..(2)

Substituting (2) in (1)

$a_A+a_B+2a_C=0$ …..(3)

Assume tensions T_1 and T_2 be the tensions in two strings

For block A

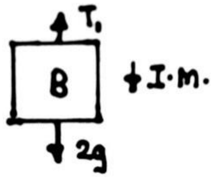


$\Sigma F=m_Aa_A$

$3g-T_1= m_Aa_A$

$T_1=3g-3a_A$ ……….(4)

For block B



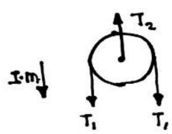
$\Sigma F = m_B a_B$

$2g - T_1 = m_B a_B$

$2g - (3g - 3a_A) = 2a_B$ (From 4)

$3a_A - 2a_B = g$ ……….(5)

For pulley D



$\Sigma F = m_B a_B$

$2T_1 - T_2 = m_D a_D$

$m_D = 0$

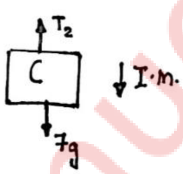
$2T_1 - T_2 = 0$

$T_2 = 2T_1$

$= 2(3g - 3a_A)$

$= 6g - 6a_A$ ………(6) (From 6)

For block C



$\Sigma F = m_B a_B$

$7g - T_2 = m_C a_C$

$7g - (6g - 6a_A) = 7a_C$ ………..(From 6)

$6a_A - 7a_C = -g$ ....(7)

Solving (3),(5) and (7)

$\mathbf{a_A = 0.4988\ m/s^2}$

$\mathbf{a_B = -4.1568\ m/s^2}$

$\mathbf{a_C = 1.8290\ m/s^2}$

From (4)

$T_1 = 3g - 3a_A$

$= 3(9.81 - 0.4988)$

$= 27.9336$ N

From (6)

$T_2 = 2T_1$

$= 55.8671$ N

Acceleration of block A=0.4988 m/s^2(Vertically downwards)

Acceleration of block B=4.1568 m/s^2(Vertically upwards)

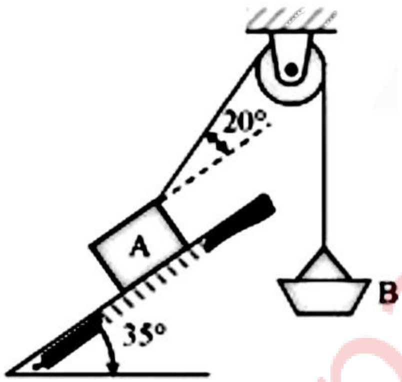
Acceleration of block C=1.8290 m/s^2(Vertically downwards)

Tension of the string T_1=27.9336 N

Q.6(a)Block A of weight 2000N is kept on the inclined plane at 35°.It is connected to weight B by an inextensible string passing over smooth pulley.

Determine the weight of pan B so that B just moves down.Assume μ=0.2.

(5 marks)



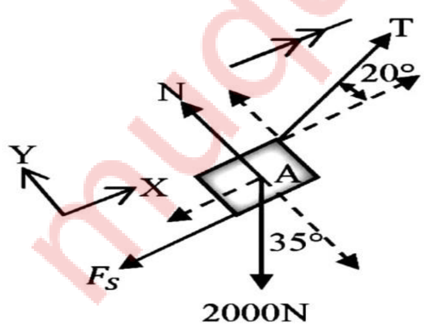
Given : Weight of block A=2000N

Angle of inclined plane = 35°

μ=0.2

To find : Weight of pan B

Solution :



The pan B is in equilibrium

Applying the conditions of equilibrium

$\Sigma F_Y=0$

$T-W_B=0$

$T=W_B$ ………..(1)

Applying the conditions of equilibrium on block A

$\Sigma F_Y=0$

$N-W_A\cos35+T\sin20=0$

From (1)

$N=2000\cos35-WB\sin20$ ………..(2)

$F_S=\mu_s N$

$F_S=0.2(2000\cos35-WB\sin20)$

$F_S=400\cos35-0.2WB\sin20$

Applying the conditions of equilibrium on block A

$\Sigma F_X=0$

$T\cos20-W_A\sin35-F_S=0$

$W_B\cos20-2000\sin35-(400\cos35-0.2W_B\sin20)=0$(From 1 and 2)

$W_B=\frac{2000sin35+400cos35}{cos20+0.2sin20}$

W_B=1462.9685 N

The weight of pan B so that pan B just moves down is 1462.9685 N

Q.6(b)A particle falling under gravity travels 25 m in a particular second. Find the distance travelled by it in the next 3 seconds. (4 marks)

Solution :

Given : Particle falls 25 m in a particular second

Sn=-25 m

u=0 m/s

To find : Distance travelled by it in next 3 seconds

Solution:

Distance travelled by the particle in nth second is

Sn = u + $\frac{1}{2}\ a\ (2n-1)$

-25 = 0- $\frac{1}{2}$ x 9.81 x (2n-1)

5.0968 = 2n-1

n = 3.0484

Considering n as an integer

n = 3 s

Using kinematical equation : s= ut + $\frac{1}{2}at^2$ ……………..(1)

S is the displacement of the particle in 3 seconds

S = 0 - $\frac{1}{2}$x9.81x3^2

S = -44.145 m

V is the displacement of particle in 6 seconds is

V = 0-$\frac{1}{2}$ x 9.81 x 6^2 (From 1)

=-176.58 m

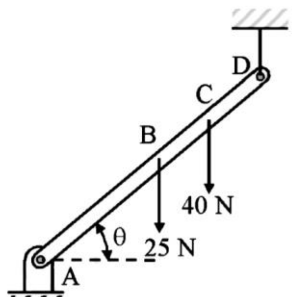
The distance travelled by particle in 4^{th},5^{th} and 6^{th} seconds = 176.58-44.145

=132.435 m

The distance travelled by particle in next 3 seconds is 132.435 m

Q.6(c)A rod AD of length 40 cm is suspended from point D as shown in figure.

If it has a weight of 25 N and also supports a load of 40N,find the tension in the cable using the method of virtual work.Take AC=30 cm.



Solution:

Given : Length of rod AD=40cm=0.4m

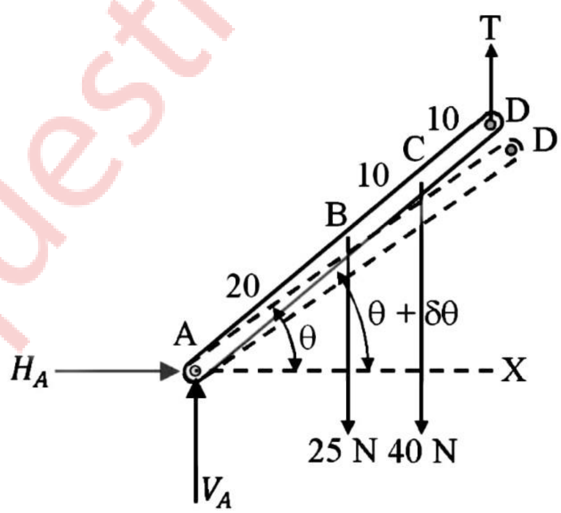
AC=0.3m

W=25N

Load on rod AD=40N

To find : Tension in the cable

Solution:



Assume rod AD have a small virtual angular displacement $\delta\theta$ in the clockwise direction

T is the tension in the cable

Assume A be the origin and AX be the X-axis

Reaction forces HA and VA do not do any virtual work

Sr. no.	**Active force**	**Co-ordinate of the point of action along the force**	**Virtual displacement**
1.	W = 25N	0.2sinθ	δy_B=0.2cosθ δθ
2.	40 N	0.3sinθ	δy_C=0.3cosθ δθ
3.	T	0.4sinθ	δy_D=0.4cos θδθ

By using the principle of virtual work,

$\delta U=0$

$-25 \times \delta y_B - 40 \times \delta y_C + T \times \delta y_D = 0$

$T \times \delta y_D = 25 \times \delta y_B + 40 \times \delta y_C$

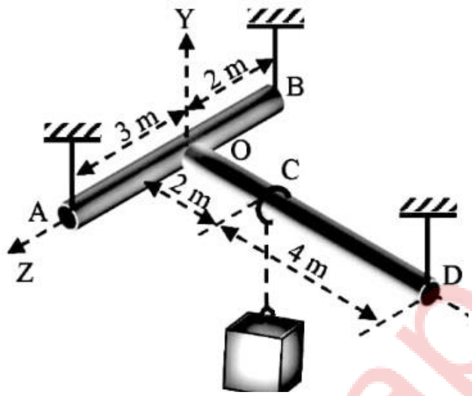
$T \times (0.4\cos\theta\,\delta\theta) = 25 \times (0.2\cos\theta\,\delta\theta) + 40 \times (0.3\cos\theta\,\delta\theta)$

Dividing by cosθ δθ and solving

T=42.5N

Tension in the cable=42.5N

Q.6(d)A T-shaped rod is suspended using 3 cables as shown.Neglecting the weight of rods,find the tension in each cable.



Solution:

Given:A T shaped suspended with cables supporting a bock of 100 N is in equilibrium

To find: Tension in the cables

Solution:

Applying the conditions of equilibrium

$\Sigma F_y = 0$

T1+T2+T3-100=0

T1+T2+T3=100 …………(1)

Consider moment about an axis which is parallel to X axis and it is passing through point A

ΣMx=0

T2 X AB – 100 XAO +T3 X AO = 0

5T2+3T3=300 ………..(2)

Consider moment about Z axis at point O

ΣMZ=0

-100 X CO+T3XDO=0

6T3=200

T3=33.3333 N ………..(3)

From (2) and (3)

5T2 + 3 x 33.3333 =300

5T2=200 N

T2=40 N ……(4)

From (1),(3) and (4)

T1+40+33.3333=100

T1=26.6667 N

T1=26.6667 N

T2=40 N

T3=33.3333 N