DISCRETE STRUCTURE

COMPUTER ENGINEERING

MAY 2018

Q1.a) Prove by induction that the sum of the cubes of three consecutive numbers is divisible by 9. [5]

Solution :- Let the consecutive numbers be n, n+1, n+2

P(n): $n^3 + (n+1)^3 + (n+2)^3 = 9k$

LHS : $1^3 + 2^3 + 3^3 \Rightarrow 1 + 8 + 27 = 36$

36 is divisible by 9

Let us assume P(k) is true;

LHS : $k^3 + (k+1)^3 + (k+2)^3 = 9m$

$\Rightarrow (k+1)^3 + (k+2)^3 = 9m - k^3$

Let us prove P(k+1) is true;

LHS : $(k+1)^3 + (k+2)^3 + (k+3)^3$

$\Rightarrow 9m - k^3 + (k+3)^3$

$\Rightarrow 9m - k^3 + k^3 + 9k^2 + 27k + 27$

$\Rightarrow 9m + 9k^2 + 27k + 27$

$\Rightarrow 9(m + k^2 + 3k + 3)$

Which implies P(k+1) is true

Therefore, P(k) is true

Therefore, P(n) is true.

Hence for $n \in N$, the result is true.

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Q1.b) Find the generating function for the finite sequences.

i)2,2,2,2,2,2 ii)1,1,1,1,1,1 [5]

Solution:

1. Multiplying the sequence successively by $x^0, x^1, x^2, x^3, ...$

   $2x^0 + 2x^1 + 2x^2 + 2x^3 + ...$

   $2(x^0 + x^1 + x^2 + x^3 + ...)$

   $2(1-x)^{-1}$

Ans : $\frac{2}{1-x}$

2. Multiplying the sequence successively by $x^0, x^1, x^2, x^3, ...$

   $x^0 + x^1 + x^2 + x^3 + ...$

   $(x^0 + x^1 + x^2 + x^3 + ...)$

   $(1-x)^{-1}$

Ans : $\frac{1}{1-x}$

Q1.c) A box contains 6 white balls and red balls. In how many ways 4 balls can be drawn from the box if, i) they are to be of any color ii)all the balls to be of the same color. [5]

Solution :

1. There are 11 balls and 4 are to be drawn.

   This can be done in $11C_4 = 330$

2. All the balls of same color:

   4 white balls can be drawn from $6C_4$

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4 red balls can be drawn from $5C_4$

Number of ways = $6C_4 . 5C_4$ = 15 x 5 = 75

Q1.d) Find the complement of each element in D_{30} **[5]**

Sol : D_{30} = {1,2,3,5,6,10,15,30}

1	˅	30
2	˅	30
3	˅	30
5	˅	30
6	˅	30
10	˅	30
15	˅	30
30	˅	30

1	^	1
2	^	1
3	^	1
5	^	1
6	^	1
10	^	1
15	^	1
30	^	1

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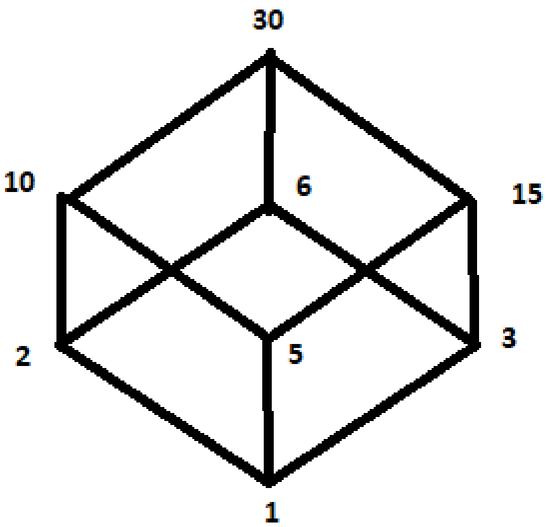
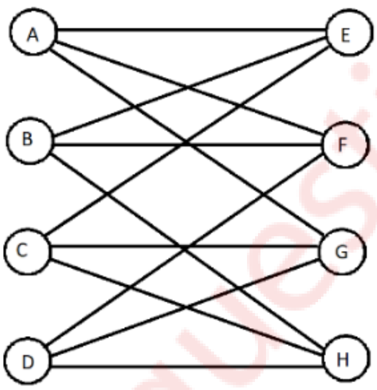
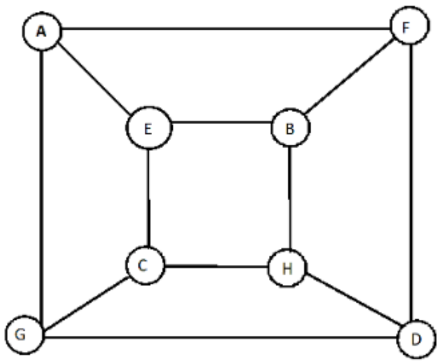


Q2.a) Define Isomorphism of graphs. Find if the following two graphs are isomorphic. If yes, find the one-to-one correspondence between the vertices. [8]



G1



G2

Solution :

If two graphs are isomorphic then:

i) They must have same number of vertices.

ii) They must have same number of edges.

iii) They must have same degree of vertices.

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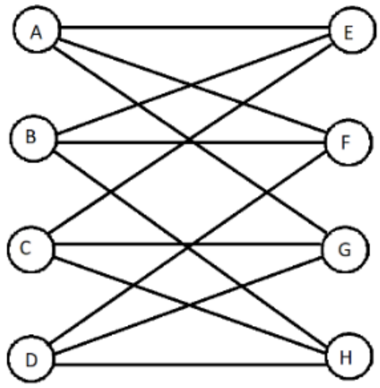
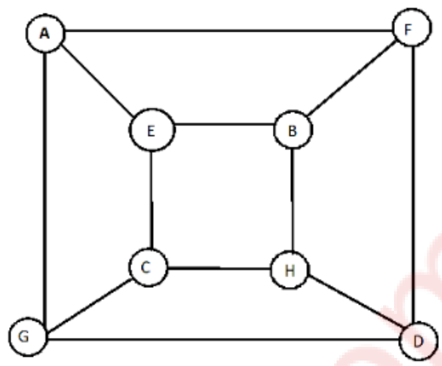


G1



G2

n(G1)=8

no. of edges=12

degree of each vertex=3

n(G2)=8

no. of edges=12

degree of each vertex=3

Ans: The graphs are isomorphic.

Q2.b) In a certain college 4% of the boys and 1% of the girls are taller than 1.8 mts. Furthermore 60% of the students are girls. I a student at random is taller than 1.8 mts, what is the probability that the student was a boy? Justify your answer. [8]

Solution:

For the convenience suppose there are 1000 students in the college.

Therefore,

	Taller than 1.8	Less than 1.8	Total
Boys	16	384	400
Girls	6	594	600
Total	22	978	1000

Since the student selected at random is found to be taller than 1.8, the student is one of 22.

But one of 22 students, 16 are boys

Therefore, the required probability = $\frac{16}{22} = \frac{8}{11}$

Ans : 8/11

Q2.c) Prove $\backsim (p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalent by developing a series of logical equivalences. [4]

Solution :

By Distributive law,

=> $\backsim ((\sim p \vee p) \wedge (p \vee q))$

=> $\backsim (T \wedge (p \vee q))$ $[(\sim p \vee p) = T]$

=> $\backsim ((p \vee q))$

=> $\backsim p \wedge \backsim q$ By Demorgan's

Q3.a) Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 with respect to multiplication modulo 7. [8]

Solution:

X_7	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

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From the table, X_7 is associative

Eg : 2 X_7 (3 X_7 5) = 2 X_7 1=2

(2 X_7 3) X_7 5) = 6 X_7 5=2

a X_7 e = a

Here e=1, identity element = 1

a X_7 a^{-1} = e

Every element has a multiplicative inverse.

Also, a X_7 e = b X_7 a

4 X_7 5 = 6

5 X_7 4 = 6

Ans : Therefore, G is abelian group.

Q3.b)LetA={1,2,3,4,5}, let R={{1,1},{1,2},{2,1},{2,2},{3,3},{3,4},{4,3},{4,4},{5,5}} and S={{1,1},{2,2},{3,3},{4,4},{4,5},{5,4},{5,5}} be the relations on A. Find the smallest equivalence containing relation R and S. [8]

Solution.

MR:

$$\begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

MS:

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

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$$M_{RUS} = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

Finding W5 by Warshall's Algorithm,

C1: 1 is at 1,2

R1: 1 is at 1,2

Put 1 in (1,1), (1,2), (2,1), (2,2)

$$W1 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C2: 1 is at 1,2

R2: 1 is at 1,2

Put 1 in (1,1), (1,2), (2,1), (2,2)

$$W2 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C3: 1 is at 3,4

R3: 1 is at 3,4

Put 1 in (3,3), (3,4), (4,3), (4,4)

$$\begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{vmatrix}$$

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$$W3 = \begin{vmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C4: 1 is at 4,5,6

R4: 1 is at 4,5,6

Put 1 in (4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)

$$W4 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

C5: 1 is at 5,6

R5: 1 is at 5,6

Put 1 in (5,5), (5,6), (6,5), (6,6)

$$W5 = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

Therefore, transitive closure = {(1,1),(1,2),(2,1),(3,3),(3,4),(4,3),(4,4),(4,5),(5,4),(5,5)}

Q3.c) Test whether the following function is one-to-one, onto or both.

f:Z→Z, $f(x) = x^2 + x + 1$ **[8]**

Sol : The set is Z

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To test whether function is injective,

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_2 - x_1)(x_1 + x_2 + 1) = 0$

Therefore, $x_2 - x_1 = 0;$ $x_1 + x_2 + 1 = 0$

$x_2 = x_1$; $x_2 = -1 - x_1$

Therefore, f is not injective.

Now, $y = x^2 + x + 1$

If y=0; $x_1 + x_2 + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1-4}}{2} \Rightarrow \text{ x is imaginary, ie, } \notin Z$$

Therefore, f is not surjective.

Ans: The function f is not one-to-one or onto

Q4.a) Show that the (2,5) encoding function e:$B^2 \rightarrow B^5$ defined by

e(00)=00000 e(01)=01110

e(10)=10101 e(11)=11011 is a group code.

How many errors will it detect and correct? [8]

Sol:

$\oplus$	**00000**	**01110**	**10101**	**11011**
00000	00000	01110	10101	11011
01110	01110	00000	11011	10101
10101	10101	11011	00000	01110
11011	11011	10101	01110	00000

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From diagonal elements, 00000 is identity element.

$x \oplus y \in N$ for $x, y \in N$

Every element has its inverse.

Therefore, e is a groupcode.

00000,01110=δ(w,x)=3

00000,10101=δ(w,y)=3

00000,11011=δ(w,z)=4

01110,10101=δ(x,y)=4

01110,11011=δ(x,z)=3

10101,11011=δ(y,z)=3

Therefore, the minimum distance = 3

k+1=3,

k=2

Ans: It can detect 2 or less errors.

Q4.b) Let H= $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ **be a parity check matrix. Determine the group code eH:**$B^3 \rightarrow B^6$

[8]

Sol:

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B^3= {000,001,010,011,100,101,110,111}

$x_1 = b_1h_{11} + b_2h_{21} + b_3h_{31}$

$x_2 = b_1h_{12} + b_2h_{22} + b_3h_{32}$

$x_3 = b_1h_{13} + b_2h_{23} + b_3h_{33}$

But $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Substituting the values;

$e(000) = 000\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 0.0 + 0.1 = 0$

$x_2 = 0.0 + 0.1 + 0.1 = 0$

$x_3 = 0.0 + 0.1 + 0.1 = 0$

$e(001) = 001\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 0.0 + 1.1 = 1$

$x_2 = 0.0 + 0.1 + 1.1 = 1$

$x_3 = 0.0 + 0.1 + 1.1 = 1$

$e(010) = 010\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 1.0 + 0.1 = 0$

$x_2 = 0.0 + 1.1 + 0.1 = 1$

$x_3 = 0.0 + 1.1 + 0.1 = 1$

$e(011) = 011\, x_1\, x_2\, x_3$

$x_1 = 0.1 + 1.0 + 1.1 = 0$

$x_2 = 0.0 + 1.1 + 1.1 = 0$

$x_3 = 0.0 + 1.1 + 1.1 = 0$

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$e(100) = 100\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 0.0 + 0.1 = 1$

$x_2 = 1.0 + 0.1 + 0.1 = 0$

$x_3 = 1.0 + 0.1 + 0.1 = 0$

$e(101) = 101\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 0.0 + 1.1 = 0$

$x_2 = 1.0 + 0.1 + 1.1 = 1$

$x_3 = 1.0 + 0.1 + 1.1 = 1$

$e(110) = 110\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 1.0 + 0.1 = 1$

$x_2 = 1.0 + 1.1 + 0.1 = 1$

$x_3 = 1.0 + 1.1 + 0.1 = 1$

$e(111) = 111\ x_1\ x_2\ x_3$

$x_1 = 1.1 + 1.0 + 1.1 = 0$

$x_2 = 1.0 + 1.1 + 1.1 = 0$

$x_3 = 1.0 + 1.1 + 1.1 = 0$

e(000) = 000000	**e(100) = 100100**
e(001) = 001111	**e(101) = 101011**
e(010) = 010011	**e(110) = 110111**
e(011) = 011100	**e(111) = 111000**

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Q4.c) How many friends must you have to guarantee that at least five of them will have birthdays in the same month? [4]

Sol : Since there are twelve months considering even distribution, if there are 48 friends, atleast 4 will have birthday in the same month.

Hence, if we have 49 friends, then five of them will have birthday in the same month.

Or by extended pigeonhole principle,

$$\left[\frac{n-1}{12}\right] + 1 = 5$$

$$\Rightarrow \left[\frac{n-1}{12}\right] = 4$$

$$\Rightarrow n - 1 = 48$$

Therefore, n=49.

Ans: Total friends needed so that at least five of them will have birthdays in the same month = 49

Q5) a] Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a * b = a + b – ab for all a, b ∈ G. Prove that (G, *) is a group (8)

Solution:-

Let a, b ∈ G

Assume, $a + b - ab = 1$

$$a - b + b - ab = 0$$

$$(a - 1)(-b + 1) = 0$$

a = 1 and b = 1; since a, b ∈ G the set of rational numbers

$$a * (b * c) = a * (b + c - bc) = a + (b + c - bc) - a(b + c + bc)$$

$$= a + b + c - bc - ab - ac - abc$$

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G is associative

$a * 0 = 0$; $0 \in G$ is identity

$a * a^{-1} = 0$; a^{-1}exists $\forall$ (x, y) $\in$ R

Q5] b) solve $a_r - 7a_r + 10a_{r-2} = 6 + 8r$ given $a_0 = 1$, $a_1 = 1$ [4mk]

Solution:-

$a_r - 7a_r + 10a_{r-2} = 0$

$r^2 - 7r + 10 = 0$

$(r-2)(r-5) = 0 \quad r = 2;5$

Roots are real rational and distinct

Let the solution be

$a_n^{(n)} = A2^n + B5$

F(r) is linear ; we assume particular solution as ar+b

$(an+b) - 7[a(n-1)+b] + 10[a(n-2)+b] - 6 - 8n = 0$

$(4a-8)n + (-13a+4b-6) = 0$

$4a - 8 = 0$

$a = 2$

$-26 + 4b - 6 = 0$

$b = 8$

Solution:- $a_n = 2(2)^n + 3(5)^n + 2n + 8$

Q5] c) Let A ={a,b,c,d,e,f,g,h}. Consider the following subsets of A

A1 = {a, b, c, d} A2 = {a, c, e, g, h} A3 = {a, c, e, g}

A4 = {b, d} A5={f,h}

Determine whether following is partition of A or not. Justify your answer.

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1. {A1,A2}   2. {A3, A4, A5}   (8)

Solution:-

For partition $A_1 \cup A_2 \cup \ldots\ldots A_n = A$

And $A_i \cap A_j = \emptyset;\ i \neq j$

Consider $A_1 = \{a,b,c,d\}$ and $A_2 = \{a,c,e,g,h\}$

$A_1 \cup A_2 = \{a,b,c,d\} \cup \{a,c,e,g,h\} = \{a,b,c,d,g,h,e\} \neq A$

$A_1 \cap A_2 = \{a,c\} \neq 0$

$\{A_1,A_2\}$ is not a partition

Consider $A_3,A_4,A_5,$

$A_3 = \{a,c,e,g\};\ A_4 = \{b,d\};\ A_5 = \{f,h\}$

$A_3 \cup A_4 \cup A_5 = \{a,b,c,d,e,f,g,h\} = A$

$A_3 \cap A_4 = \emptyset$

$A_4 \cap A_5 = \emptyset$

$A_3 \cap A_5 = \emptyset$

A_3,A_4,A_5 is a partition.

Q6] a) Draw the Hasse Diagram of the following sets under the partial order relation divides and indicate which are chains. Justify your answers.   (8)

1. **A = {2,4,12,24}**
2. **A = {1,3,5,15,30}**

Solution:-

The given partial order relation is divides

1. A = {2,4,12,24}

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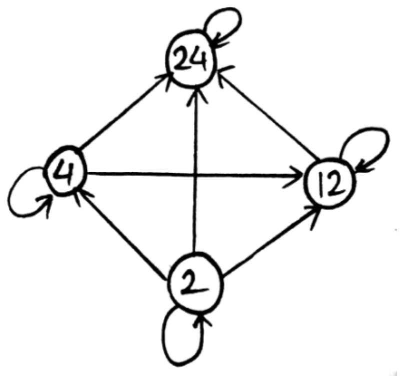
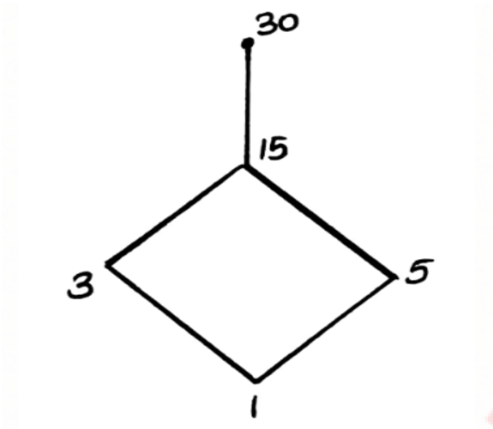
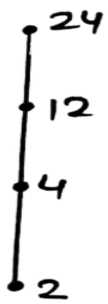
2.



The poset is a chain

3. B = {1,3,5,15,30}





The poset is not a chain

Q6] b) Let the functions f, g and h defined as follows: (8)

f: R -> R, f(x) = 2x + 3

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g: R->R, g(x) =3x + 4

h: R -> R,h(x) = 4x

find gof, fog, foh and gofoh

Solution:-

1. gof = g(f(x))
   = g(2x+3)
   = 3(2x+3)+4
   = 6x+9+4
   = 6x+13

2. fog = f(g(x))
   = f(3x+4)
   = 2(3x+4)+3
   = 6x+8+3
   = 6x+11

3. foh = f(h(x))
   = f(4x)
   = 2(4x)+3
   = 8x+3

4. gofoh = g(f(h(x)))
   = g(f(4x))
   = g(2(4x)+3)
   = g(8x+3)
   = 24x+9+4

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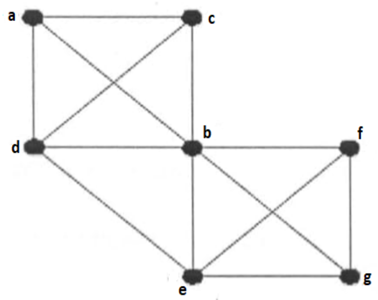
= 24x+13

Q6] c) Determine Euler Cycle and path in graph shown below (4)

Solution:-



No Eulerian path exists as 4 vertices are of odd degree i.e a, c, g, f are of odd degree = 3

The graph is not Eulerian as not every vertex has even degree.

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